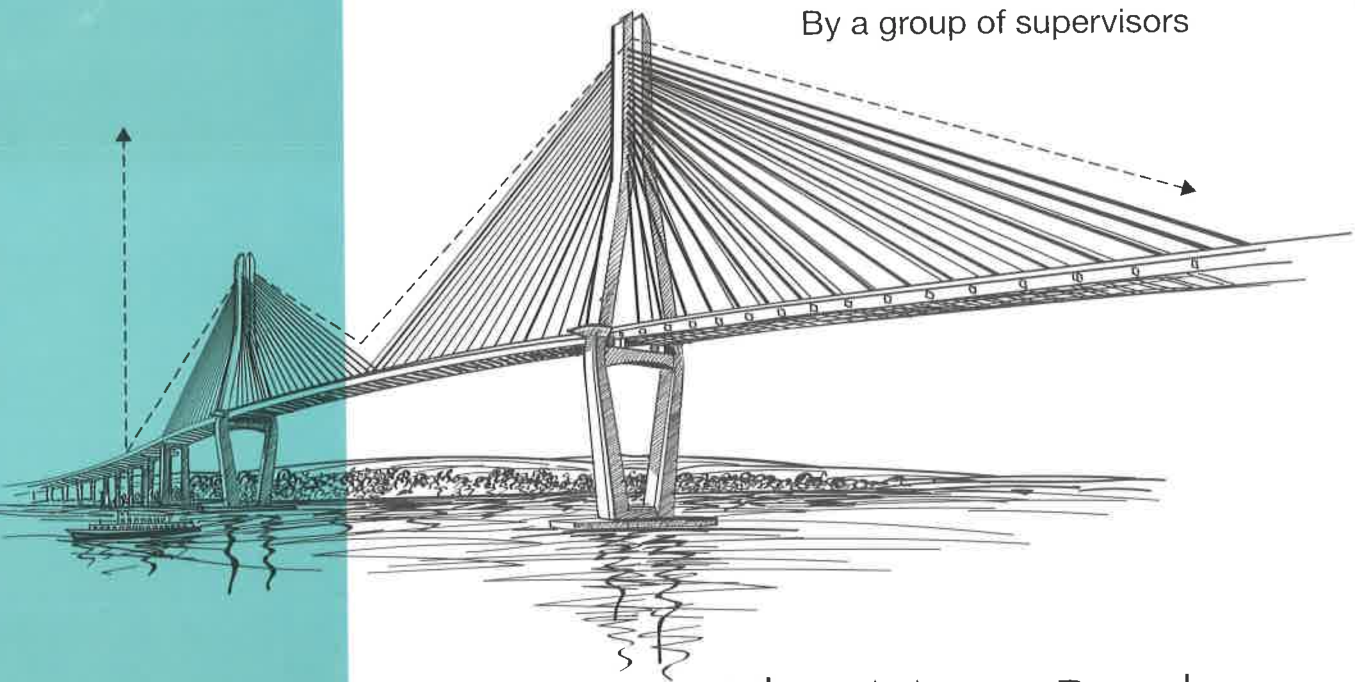
EL-MOASSER

Mathematics

Applications

By a group of supervisors



The Main Book

SCIENTIFIC SECTION

FIRST TERM

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CONTENTS

Revision on vectors

The quantities which we deal with in our life are divided into two kinds of quantities :

(1) Scalar quantities : They are the quantities which are completely determined if we knew their magnitude only as a real number

As : length , mass , time , temperature degree , volume , distance.

(2) Vector quantity : It is a quantity determined by a real number (the magnitude of this quantity) besides the direction.

i.e. The vector quantity is determined completely if we knew its magnitude and its direction.

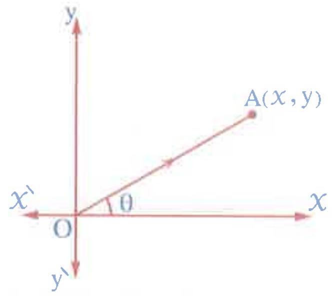
- **Directed line segment :**

  It is a straight line segment having an initial point and an ending point , with direction defined from the initial point to the ending point.

- The norm of the directed line segment $\left(\overrightarrow{AB}\right)$:
  the norm of $\overrightarrow{AB}$ is the length of $\overline{AB}$ and it is denoted by $\|\overrightarrow{AB}\|$
- The two directed line segments are equivalent if they have :
  – The same length (the norm) and the same direction.
- $\overrightarrow{AB} \neq \overrightarrow{BA}$ (they have opposite directions)
- $\overrightarrow{AB} = -\overrightarrow{BA}$
- $\|\overrightarrow{AB}\| = \|\overrightarrow{BA}\|$

- The position vector of a given point (A) with respect to the origin point (O) it is the directed line segment $\overrightarrow{OA}$, it is denoted by $\vec{A}$

For example :



In the opposite figure :

If : $\overrightarrow{OA}$ is the position vector of the point (A) = (x , y) then :

* $\|\vec{A}\|$ = the length of $\overline{OA} = \sqrt{x^2 + y^2}$

* If $\|\vec{A}\| = 1$ length unit (unity)

  , then $\vec{A}$ is called the unit vector.

* $\vec{i} = (1 , 0)$

, $\vec{j} = (0 , 1)$ are two unit vectors

(called two basic unit vectors) in the two directions of the two coordinate axes.

* $\vec{O} = (0 , 0)$ it is zero vector which has no direction and it is denoted by $\vec{O}$

* $\vec{A} = (x , y)$ is called the cartesian form of the vector $\vec{A}$

* $\vec{A} = x\,\vec{i} + y\,\vec{j}$ expresses the vector $\vec{A}$ in terms of the two basic unit vectors.

* $\vec{A} = \left(\|\vec{A}\| , \theta\right)$ is called the polar form of the vector $\vec{A}$

* θ is the measure of the angle made by the vector $\overrightarrow{OA}$ with the positive direction of x-axis , it is called the polar angle.

* $x = \|\vec{A}\| \cos\theta$, then $\cos\theta = \dfrac{x}{\|\vec{A}\|}$

* $y = \|\vec{A}\| \sin\theta$, then $\sin\theta = \dfrac{y}{\|\vec{A}\|}$

- If $\vec{A} = (x_1 , y_1)$, $\vec{B} = (x_2 , y_2)$, then :

* $\vec{A} = \vec{B}$ if and only if $x_1 = x_2$, $y_1 = y_2$

  $\vec{A} \pm \vec{B} = (x_1 \pm x_2 , y_1 \pm y_2)$

* $\overrightarrow{AB} = \vec{B} - \vec{A} = (x_2 - x_1 , y_2 - y_1)$

* $K\vec{A} = K(x , y) = (Kx , Ky)$

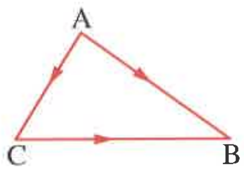
* $\vec{A} \parallel K\vec{A}$ with regarding that :

If $\boxed{K > 0}$ then → $\vec{A}$ and $K\vec{A}$ have the same direction

If $\boxed{K < 0}$ then → $\vec{A}$ and $K\vec{A}$ have opposite direction

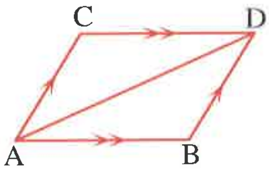
where $k \neq 0$

- **Adding and subtracting vectors geometrically :**



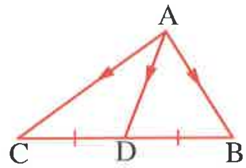
$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{O}$



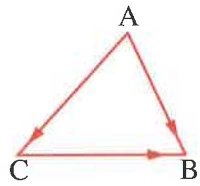
$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$



$\overrightarrow{AB} + \overrightarrow{AC} = 2\,\overrightarrow{AD}$

$\overrightarrow{BD} + \overrightarrow{CD} = \overrightarrow{O}$



$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

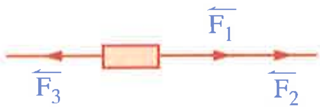
- **Physical applications :**

The resultant force $\overrightarrow{R}$

- The resultant of a set of forces acting on a body is operated as the operation of adding vectors

i.e. The resultant force

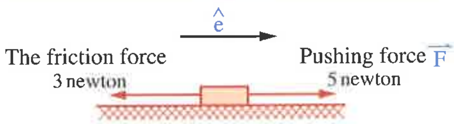
$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \ldots$



For example :

If we defined a unit vector $\hat{e}$ in the direction of the motion of the body then :

In the case of motion of the body on a rough plane



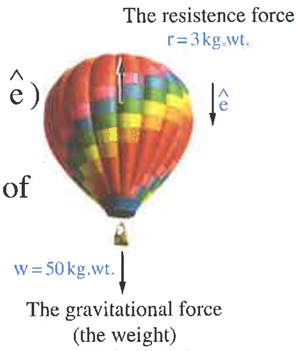
The resultant force

$\overrightarrow{R} = 5\,\hat{e} + (-3\,\hat{e}) = 2\,\hat{e}$

i.e. The magnitude of the resultant = 2 newton

- The direction of the resultant is in the direction of the motion of the body.

In the case of vertical motion



The resultant

force $= 50\,\hat{e} + (-30\,\hat{e})$

$= 20\,\hat{e}$

i.e. The magnitude of the resultant = 20 Kg.wt.

- The direction of resultant is in the direction of the weight

- If the two forces have the same magnitude and the same line of action but in two opposite directions then the resultant $\overrightarrow{R} = \overrightarrow{O}$

- If the resultant of a set of concurrent forces $= \overrightarrow{O}$ this means the set of forces are in equilibrium.

Example 1

(1) Write the vector $\overrightarrow{A} = \left(3 , -\sqrt{3}\right)$ in the polar form.

(2) Write in terms of the two basic unit vectors the vector $\overrightarrow{A}$ whose norm = 10 length unit and act in the direction of Western North.

Solution

(1) $\because \|\overrightarrow{A}\| = \sqrt{9+3} = 2\sqrt{3}$

$\therefore \cos\theta = \dfrac{x}{\|\overrightarrow{A}\|} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2} > 0$ $\quad$ $\sin\theta = \dfrac{y}{\|\overrightarrow{A}\|} = \dfrac{-\sqrt{3}}{2\sqrt{3}} = -\dfrac{1}{2} < 0$

$\therefore$ θ lies in the 4^{th} quadrant $\quad$ $\therefore \theta = 360 - 30 = 330°$

$\therefore \overrightarrow{A} = \left(2\sqrt{3} , 330°\right)$

(2) $\because \|\overrightarrow{A}\| = 10 \quad , \quad \theta = 135°$ $\quad$ $\therefore x = \|\overrightarrow{A}\| \cos\theta = 10 \cos 135° = -5\sqrt{2}$

$y = \|\overrightarrow{A}\| \sin\theta = 10 \sin 135° = 5\sqrt{2}$

$\therefore \overrightarrow{A} = \left(-5\sqrt{2} , 5\sqrt{2}\right)$ $\quad$ $\therefore \overrightarrow{A} = -5\sqrt{2}\,\overrightarrow{i} + 5\sqrt{2}\,\overrightarrow{j}$

Example 2

If the forces $\overrightarrow{F_1} = 2\overrightarrow{i} + 3\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} + \overrightarrow{j}$, $\overrightarrow{F_3} = 5\overrightarrow{i} + b\overrightarrow{j}$ act on a particle

Find the values of a and b if these forces :

(1) Their resultant = $5\overrightarrow{i} - 2\overrightarrow{j}$

(2) Are in equilibrium.

Solution

The resultant = $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \left(2\overrightarrow{i} + 3\overrightarrow{j}\right) + \left(a\overrightarrow{i} + \overrightarrow{j}\right) + \left(5\overrightarrow{i} + b\overrightarrow{j}\right)$

$= (2 + a + 5)\overrightarrow{i} + (3 + 1 + b)\overrightarrow{j}$

(1) $\because$ The resultant = $5\overrightarrow{i} - 2\overrightarrow{j}$

$\therefore (7 + a)\overrightarrow{i} + (4 + b)\overrightarrow{j} = 5\overrightarrow{i} - 2\overrightarrow{j}$

$\therefore 7 + a = 5$ $\quad$ $\therefore a = -2$

$\therefore 4 + b = -2$ $\quad$ $\therefore b = -6$

(2) $\because$ The forces are in equilibrium

$\therefore \overrightarrow{R} = \overrightarrow{O}$ $\quad$ $\therefore (7 + a)\overrightarrow{i} + (4 + b)\overrightarrow{j} = \overrightarrow{O}$

$\therefore a = -7 \quad , \quad b = -4$

UNIT 1 Statics

Lesson 1 — Forces - Resultant of two forces meeting at a point.

Lesson 2 — Forces resolution into two components.

Lesson 3 — The resultant of coplanar forces meeting at a point.

Lesson 4 — Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point (The triangle of forces rule - Lami's rule).

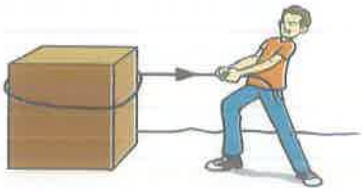
Lesson 5 — Follow : The equilibrium (Meeting lines of action of three equilibrium forces).

Lesson One — Forces - Resultant of two forces meeting at a point

The force

"The force is defined as the effect of a natural body upon another one" by pushing, attraction, pressure or repulsion". The natural body is a body consisting of material (mass) and volume not equal to zero ,

The natural bodies can be classified into two kinds :



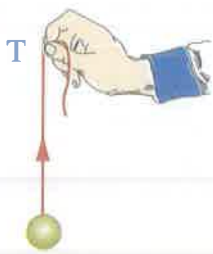
- Rigid bodies (solid bodies) : They are the bodies whose shapes do not change whatever the forces which are acting on them as solid metals, rocks, ...
- Deformable bodies : They are the bodies whose shapes can be disfigured as strings, liquids, gases, rubber and clay and our study in this unit will be contined to rigid bodies only.

Kinds of forces

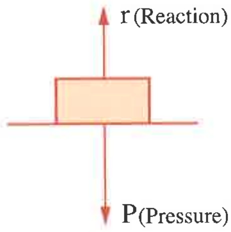
There are different kinds of forces , as :

1 Tension force (T) :

As the force in the string (or the rope) when carrying a body at it.



(Tension in the string)



(2) Pressure force (P) :

As the force that appears when a body stabilized on a surface.

(3) Reaction force (r) :

As the reaction of a smooth surface on a body stabilized on it.

(4) Attraction forces and repulsion forces :

As the forces which formed between magnetic poles , electric charges and astronomical objects.

(5) Gravitational forces (weights) :

If we let a body in the air , then it will drop down towards the Earth because the attraction force of the Earth attracts any body towards it.

This force is called Gravitational force or weights.

- Notice that : the weight (W) = the body mass × acceleration of gravity = $m \times g$

Expressing force

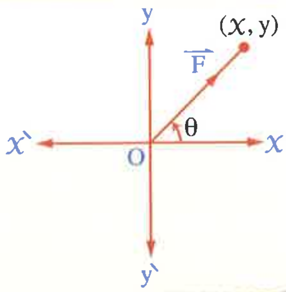
The force is a vector quantity so it can be represented by the same way as the vectors.

***i.e.* The force can be expressed as follow :**

(1) $\vec{F} = (x , y) \Rightarrow$ the cartesian form.

(2) $\vec{F} = x\,\vec{i} + y\,\vec{j} \Rightarrow$ in terms of the fundamental unit vectors.

(3) $\vec{F} = \left(\|\vec{F}\| , \theta\right) \Rightarrow$ The polar form.



Determination of the force

The force is a vector which passes through a fixed point.

i.e. It acts in a given straight line.

***i.e.* The force is determined by :**

(1) The magnitude of the force.

(2) The direction of the force.

(3) The point of action of the force.

For example :

The football player kicks the ball by a determined force (magnitude of the force) in a determined direction (direction of the force) in a certain point on the surface of the ball (Point of action of the force)

UNIT 1

The magnitude of the force

1 The measurement units :

* The magnitude of the force (The numerical value of the force) is measured by units which are called weight units.

  As : gram weight (gm.wt.) , kilogram weight (kg.wt.)

  , where $1 \text{ kg.wt.} = 1000 \text{ gm.wt.} = 10^3 \text{ gm.wt.}$

* There are other units to measure the magnitude of force (they are called absolute units)

  As : The dyne, the newton :

  , where $1 \text{ newton} = 100\,000 \text{ dyne} = 10^5 \text{ dyne}$

* The weight units connect with the absolute units by the relation :

  $1 \text{ kg.wt.} = 9.8 \text{ newton}$, $1 \text{ gm.wt.} = 980 \text{ dyne}$

  (unless something else mentioned)

2 The direction of the force :

It is the direction of the vector which represents this force and it is determined by the measure of the polar angle of the force vector in the case of the coplanar forces in the same plane.

• The polar angle is the positive directed angle which the vector makes with the positive direction of X-axis.

3 The point of action of the force :

The action of the force is determined by its point of action. If you try to open the door of a room or close it with a force near of the line of hinges , you will find difficult to rotate it.

As you are far from the line of hinges as the difficulty becomes less. As shown in the figure.

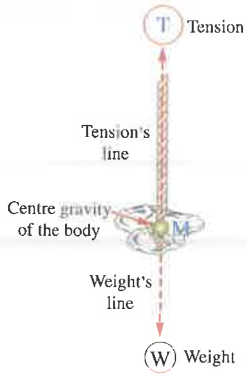
The line of action of the force

The line of action of the force is the line passing through the point of action parallel to the direction of the force.

For example :

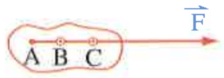
– The tension line in a string is the string itself.

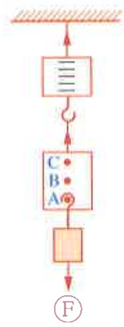
– The line of action of the weight of a body is the vertical line passing through the center gravity of the body.



Displacing (or translation) of the point of action of the force (force penetration)

If the force $\overrightarrow{F}$ acts on a rigid body and A is the point of its action , then we can displace this point to another point on the body "B" or "C" or .... on the line of action of the force without changing in its influence on the body.





i.e. Any point lying on the line of action of a force can be considered as a point of action of this force.

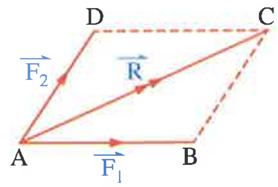
Resultant of two forces meeting at a point

The resultant of two or more forces is a single force has the same effect as the two or more forces.

Finding the resultant of two forces meeting at a point (geometrically)

This method depends on the parallelogram rule to add two forces :

If two forces ($\overrightarrow{F_1}$, $\overrightarrow{F_2}$) meeting at a point are represented in magnitude and direction by two sides of a parallelogram meeting at this point , then their resultant ($\overrightarrow{R}$) is represented in magnitude and direction by the diagonal of the parallelogram which starts from the same point.



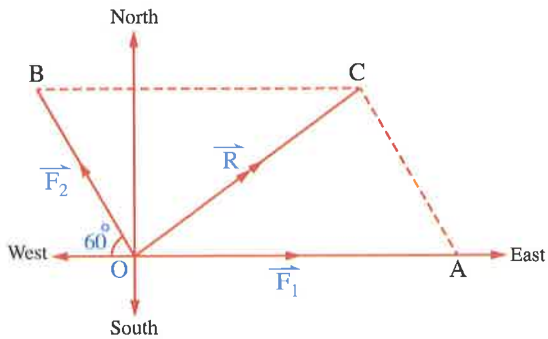
i.e. $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

Example 1

$\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two forces acting on the point O from a solid body , where $F_1 = 500$ newton and acts on the direction of East , $F_2 = 300$ newton and acts in the direction 60° North of west , find their resultant graphically.

Solution

* We use the drawing scale one cm. per 100 newton
* Draw $\overline{OA}$ to represent $\overrightarrow{F_1}$ and $\overline{OB}$ to represent $\overrightarrow{F_2}$ where $\|\overrightarrow{OA}\| = 5$ cm. , $\|\overrightarrow{OB}\| = 3$ cm.
* Then complete the parallelogram OACB , then $\overrightarrow{OC}$ represents the resultant $\overrightarrow{R}$

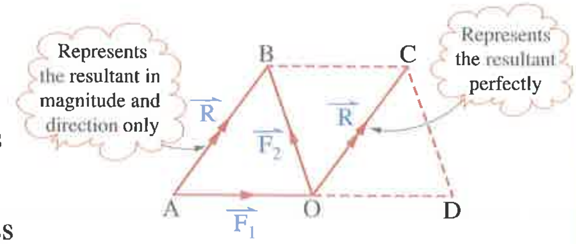


∗ By measuring , we find that $\| \overrightarrow{OC} \| = 4.4$ cm. approximatly, m ($\angle$ AOC) = 37°

$\therefore$ $\overrightarrow{R}$ acts at O and its magnitude = 440 newton in the direction 37° North of east approximatly.

Remark

If $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ act at the point O and if they are represented by the two vectors $\overrightarrow{AO}$ and $\overrightarrow{OB}$ as in the opposite figure , then according to the rule of addition of two vectors , $\overrightarrow{AB}$ represents the resultant of these two vectors. But the line of action of the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ must pass through O

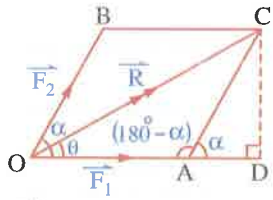


Therefore , we draw from O the directed line segment $\overrightarrow{OC}$ equivalent to $\overrightarrow{AB}$ which represents the resultant of these two forces perfectly.

Finding the resultant of two forces meeting at a point analytically

Let the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ meet at O and α is the measure of the angle between the directions of the two forces.

If $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then $\overrightarrow{OC}$ represents the resultant $\overrightarrow{R}$



Let θ be the measure of the angle between the resultant $\overrightarrow{R}$ and the force $\overrightarrow{F_1}$, then from our study of the cosine law in trigonometry , we can get the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ in magnitude and direction from the following relations :

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos\alpha} \quad , \quad \tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$$

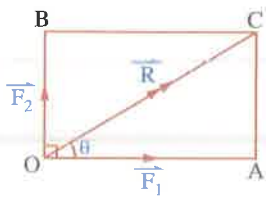
where F_1 , F_2 and R are the magnitudes of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{R}$

Special cases

1 If the two forces are perpendicular (*i.e.* $\alpha = 90°$) :

$\therefore \cos\alpha = 0$, $\sin\alpha = 1$

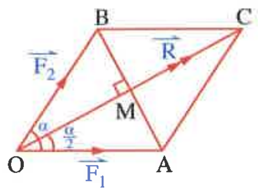
Substituting in the two previous relations , we get that :



$$R = \sqrt{F_1^2 + F_2^2} \quad , \quad \tan\theta = \frac{F_2}{F_1}$$

(2) If the two forces are equal in magnitude (*i.e.* $F_1 = F_2 = F$) :

In this case , the parallelogram OACB exchanges to a rhombus , then :



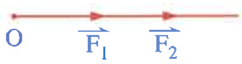
$$R = OC = 2\,OM = 2\,OA \cos\frac{\alpha}{2}$$
$$= 2\,F \cos\frac{\alpha}{2}$$

i.e. $R = 2\,F\cos\frac{\alpha}{2}$, $\theta = \frac{\alpha}{2}$ (where $\overrightarrow{R}$ bisects the angle between the two forces)

Notice that : $\alpha = 120°$, So $R = F$

(3) If the two forces have the same line of action and the same direction (*i.e.* $\alpha = 0°$) :

$\therefore \cos\alpha = 1$



Substituting :

$$\therefore R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2 \times 1} = \sqrt{(F_1 + F_2)^2} = F_1 + F_2$$

i.e. $R = F_1 + F_2$

and the direction of the resultant is the same direction of the line of action of the two forces.

* In this case , R is called the greatest or the maximum value of the resultant.

(4) If the two forces have the same line of action but in opposite directions (*i.e.* $\alpha = 180°$) :

$\therefore \cos\alpha = -1$

Substituting :

$$\therefore R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2\,F_1F_2 \times (-1)} = \sqrt{F_1^{\,2} + F_2^{\,2} - 2\,F_1F_2} = \sqrt{(F_1 - F_2)^2} = |\,F_1 - F_2\,|$$

i.e. $R = |\,F_1 - F_2\,|$

and the direction of the resultant is the direction of the greater force in magnitude.

* In this case , R is called the smallest or the minimum value of the resultant.

(5) If the two forces are equal in magnitude and have the same line of action but in opposite directions :

In this case : $F_1 = F_2 = F$, $\alpha = 180°$

$\therefore \cos\alpha = -1$ $\qquad \therefore R = \sqrt{F^2 + F^2 - 2\,F^2} = 0$

$\therefore$ $R = \text{zero}$

i.e. The resultant is zero vector.

6 If the resultant is perpendicular to the first force (*i.e.* $\theta = 90°$) :

$\because \theta = 90°$

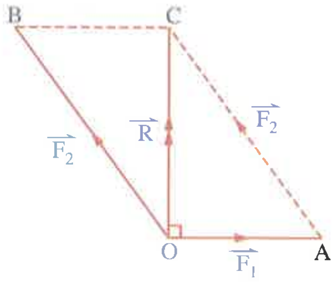
$\therefore \boxed{R^2 = F_2^2 - F_1^2}$ (Pythagoras' theorem)

, $\cot \theta = 0$ $\qquad \therefore \dfrac{F_1 + F_2 \cos \alpha}{F_2 \sin \alpha} = 0$

$\therefore \boxed{F_1 + F_2 \cos \alpha = 0}$

$\therefore \cos \alpha = \dfrac{-F_1}{F_2}$

$\therefore \alpha$ is an obtuse angle , $F_1 < F_2$



i.e. When the resultant is perpendicular to one of the two forces it is perendicular to the smallest force.

Example 2

Two forces of magnitudes 5 newton and 3 netwon act at a point and include an angle of measure 60°, find their resultant in magnitude and direction analytically.

Solution

$\because R = \sqrt{F_1^{\ 2} + F_2^{\ 2} + 2 F_1 F_2 \cos \alpha}$

$\therefore R = \sqrt{25 + 9 + 2 \times 5 \times 3 \times \cos 60°} = 7$ newton.

$\because \tan \theta = \dfrac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$ $\qquad \therefore \tan \theta = \dfrac{3 \sin 60°}{5 + 3 \cos 60°} = \dfrac{3\sqrt{3}}{13}$

$\therefore \theta \simeq 21° 47'$

$\therefore$ The magnitude of $\overrightarrow{R}$ is 7 newton and include an angle of measure $21° 47'$ with the first force.

Example 3

Two perpendicular forces act at a point such that $F_1 = 6$ newton and $F_2 = 2.5$ newton. Find their resultant in magnitude and find its direction.

Solution

$\because R = \sqrt{F_1^{\ 2} + F_2^{\ 2}}$ $\qquad \therefore R = \sqrt{(6)^2 + (2.5)^2} = 6.5$ newton.

$\because \tan \theta = \dfrac{F_2}{F_1}$ $\qquad \therefore \tan \theta = \dfrac{2.5}{6} = \dfrac{5}{12}$ $\qquad \therefore \theta = 22° 37'$

$\therefore$ The magnitude of $\overrightarrow{R}$ is 6.5 newton and include an angle of measure $22° 37'$ with the first force.

Example 4

Two forces of magnitudes 50 newton and 100 newton act at a point. Their resultant is perpendicular to the first force. Find the measure of the angle included between the two forces and the magnitude of the resultant.

Solution

$F_1 = 50$ newton , $F_2 = 100$ newton.

$\because$ The resultant is perpendicular to the first force.

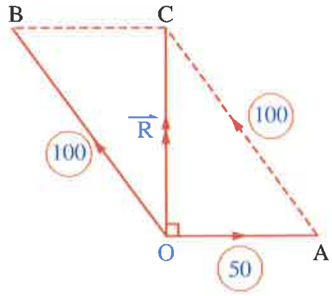
$\therefore F_1 + F_2 \cos\alpha = 0$ $\qquad \therefore 50 + 100 \cos\alpha = 0$

$\therefore \cos\alpha = \frac{-50}{100} = -\frac{1}{2}$ $\qquad \therefore \alpha = 120°$

$\because R = \sqrt{(F_1)^2 + (F_2)^2 + 2(F_1)(F_2)\cos\alpha}$

$\therefore R = \sqrt{(50)^2 + (100)^2 + 2 \times 50 \times 100 \cos 120°} = 50\sqrt{3}$ newton.

Another solution :



Let $\overrightarrow{OA}$ represents the force whose magnitude is 50 newton ,

$\overrightarrow{OB}$ represents the force whose magnitude is 100 newton.

, $\because$ the resultant is perpendicular to the first force

$\therefore \Delta$ OAC is right-angled triangle at O

$\therefore \cos A = \frac{OA}{AC} = \frac{50}{100} = \frac{1}{2}$

$\therefore m(\angle A) = 60°$

$\therefore m(\angle AOB) = 120°$ and it is the measure of the angle between the two forces.

, $\because R^2 = (100)^2 - (50)^2$ $\qquad \therefore R = \sqrt{(100)^2 - (50)^2} = 50\sqrt{3}$ newton.

Example 5

Two forces act at a point. The greatest value of their resultant = 32 kg.wt. and the smallest value of their resultant is 12 kg.wt. Find the magnitude of each of them , then find the magnitude of their resultant if the measure of the included angle between them is 60°

Solution

Let the great force = F_1 and the small force = F_2

$\therefore F_1 + F_2 = 32$ (1) , $F_1 - F_2 = 12$ (2)

From (1) and (2) : $F_1 = 22$ kg.wt. and $F_2 = 10$ kg.wt.

If $\alpha = 60°$, then $R = \sqrt{(22)^2 + (10)^2 + 2 \times 22 \times 10 \cos 60°} = 2\sqrt{201}$ kg.wt.

Example 6

Two forces are equal in magnitude. The magnitude of their resultant is $70\sqrt{3}$ newton and the measure of the angle between them is 60°. Find the magnitude of each of the two forces.

Solution

$\because$ The two forces are equal in magnitude.

$\therefore R = 2F\cos\frac{\alpha}{2}$ $\quad\quad \therefore 70\sqrt{3} = 2F\cos 30°$ $\quad\quad \therefore F = 70$ newton.

$\therefore$ The two forces are 70 netwon and 70 netwon.

Example 7

Two forces of magnitude 6 and F kg.wt. act at a particle such that the measure of the angle between them is 135°
Find the magnitude of their resultant if the line of action of the resultant inclines by an angle of measure 45° with the force F

Solution

$\because \tan\theta = \dfrac{F_1 \sin\alpha}{F_2 + F_1\cos\alpha}$

, where θ is the measure of the angle between the resultant and the force F

$\therefore \tan 45° = \dfrac{6\sin 135°}{F + 6\cos 135°}$ $\quad\quad \therefore 1 = \dfrac{3\sqrt{2}}{F - 3\sqrt{2}}$

$\therefore F - 3\sqrt{2} = 3\sqrt{2}$ $\quad\quad \therefore F = 6\sqrt{2}$ kg.wt.

, $\because R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\alpha}$

$\therefore R = \sqrt{(6)^2 + (6\sqrt{2})^2 + 2\times 6\times 6\sqrt{2}\cos 135°} = 6$ kg.wt.

Example 8

Two forces acting at a point. If their magnitudes are 4 F and 3 F , find the measure of the angle between them if the magnitude of the resultant is $\sqrt{13}$ F

Solution

$\because R^2 = (F_1)^2 + (F_2)^2 + 2(F_1)(F_2)\cos\alpha$

$\therefore (\sqrt{13}F)^2 = (4F)^2 + (3F)^2 + 2\times 4F\times 3F\times\cos\alpha$

$\therefore 13F^2 = 16F^2 + 9F^2 + 24F^2\cos\alpha$ $\quad\quad \therefore -12F^2 = 24F^2\cos\alpha$

$\therefore \cos\alpha = \dfrac{-12F^2}{24F^2} = -\dfrac{1}{2}$ $\quad\quad \therefore \alpha = 120°$

Example 9

Two forces of magnitude 7 kg.wt. and F kg.wt. act at a particle and the measure of the included angle between their directions is 120°

If the magnitude of their resultant is $7\sqrt{3}$ kg.wt.

Find the value of F and the measure of the angle which the resultant makes with the first force.

Solution

$\because R^2 = F_1^2 + F_2^2 + 2\,F_1F_2\cos\alpha$ $\therefore \left(7\sqrt{3}\right)^2 = (7)^2 + F^2 + 2\times 7\times F\cos 120°$

$\therefore 147 = 49 + F^2 - 7\,F$ $\therefore F^2 - 7\,F - 98 = 0$

$\therefore (F - 14)\,(F + 7) = 0$ $\therefore F = 14$ kg.wt.

$\because \tan\theta = \dfrac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$ $\therefore \tan\theta = \dfrac{14\sin 120°}{7 + 14\cos 120°} = \dfrac{7\sqrt{3}}{\text{zero}}$ «undefined»

$\therefore \cot\theta = \text{zero}$ $\therefore \theta = 90°$

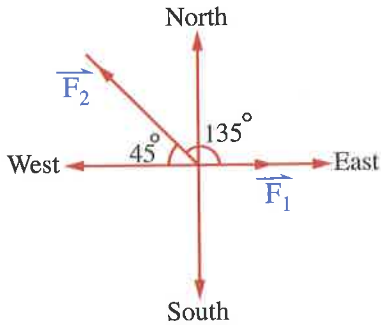
i.e. The resultant is perpendicular to the first force.

Example 10

Two forces of magnitudes 5 and $5\sqrt{2}$ kg.wt. act at a point.

The first towards East , the second is towards Western North. Prove that the magnitude of the resultant = the magnitude of the first force and find the measure of the angle which the resultant makes with each of the two forces.

Solution



$F_1 = 5$ kg.wt. , $F_2 = 5\sqrt{2}$ kg.wt.

From the figure $\alpha = 135°$

$\because R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\alpha}$

$\therefore R = \sqrt{25 + 50 + 2\times 5\times 5\sqrt{2}\times\cos 135°}$

$\therefore R = 5$ kg.wt. $= F_1$, $\because \tan\theta = \dfrac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$

$\therefore \tan\theta = \dfrac{5\sqrt{2}\sin 135°}{5 + 5\sqrt{2}\cos 135°} = \dfrac{5}{\text{zero}}$ «undefined»

$\therefore \cot\theta = \text{zero}$ $\therefore \theta = 90°$

$\therefore \overrightarrow{R}$ is perpendicular to $\overrightarrow{F_1}$

i.e. Towards North and makes an angle of measure $135° - 90° = 45°$ with $\overrightarrow{F_2}$

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1

Example 11

Two equal forces intersect at a point and the magnitude of their resultant equals 8 newton , if one of them is reversed , then the magnitude of their resultant equals 6 newtons. Find the magnitude of each force.

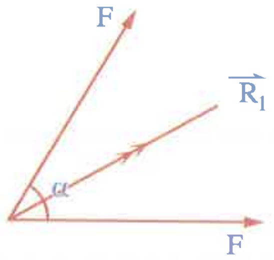
Solution

$R_1 = 2 F \cos \frac{\alpha}{2} = 8$

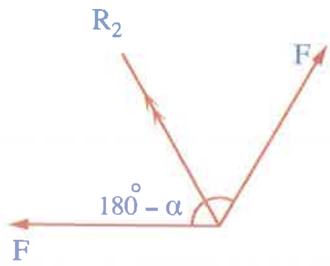
$\therefore F \cos \frac{\alpha}{2} = 4 \qquad (1)$

$, R_2 = 2 F \cos \left(\frac{180 - \alpha}{2}\right) = 6$

$\therefore F \sin \frac{\alpha}{2} = 3 \qquad (2)$



(First case)



(Second case)

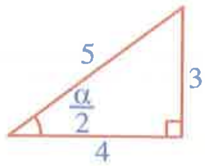
By squaring the two equations (1) , (2) and add :

$\therefore F^2 \cos^2 \frac{\alpha}{2} + F^2 \sin^2 \frac{\alpha}{2} = 16 + 9 \qquad \therefore F^2 \left(\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2}\right) = 25$

$\therefore F^2 = 25 \qquad \therefore F = 5$ newton

$\therefore$ The magnitude of each force 5 , 5 newton

• Notice that the two equations can be solved as the following :



Dividing equation (2) by equations (1) :

$\therefore \tan \frac{\alpha}{2} = \frac{3}{4} \qquad \therefore \sin \frac{\alpha}{2} = \frac{3}{5}$

, by substituting in equation (2) :

$\therefore F \times \frac{3}{5} = 3 \qquad \therefore F = 5$ newton.

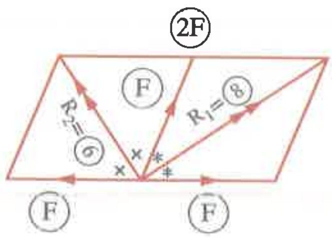
$\therefore$ The magnitude of each force is 5 newton.

Another solution (Geometrically) :

$\because$ The two forces are equal.

$\therefore R_1$, R_2 bisect the angle between the two forces.

$\therefore R_1 \perp R_2$



$\therefore (R_1)^2 + (R_2^2) = (2 F)^2$

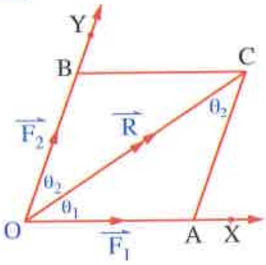
$\therefore 64 + 36 = 4 F^2$

$\therefore F^2 = 25 \qquad \therefore F = 5$

$\therefore$ The magnitude of the two forces are 5 , 5 newton.

Lesson Two — Forces resolution into two components

Resolution of a known force into two known directions



Suppose that the force $\overrightarrow{R}$ acts at a point O and it is required to resolve $\overrightarrow{R}$ into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$. Let θ_1 and θ_2 be the measure of angles of inclination of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ to the direction of $\overrightarrow{R}$

Therefore , we draw using a drawing scale the vector $\overrightarrow{OC}$ to represent the force $\overrightarrow{R}$, then we draw from O the two rays $\overrightarrow{OX}$ and $\overrightarrow{OY}$ making two angles θ_1 and θ_2 with $\overrightarrow{OC}$ and in different sides of it.

Then we draw from C two rays one is parallel to $\overrightarrow{OX}$ and the other is parallel to $\overrightarrow{OY}$ to get the parallelogram OACB as in the shown figure , thus the vector $\overrightarrow{OA}$ represents the component $\overrightarrow{F_1}$ and $\overrightarrow{OB}$ represents the component $\overrightarrow{F_2}$ and the vector $\overrightarrow{AC}$ represents $\overrightarrow{F_2}$ also

By using the sine rule on Δ OAC , where m ($\angle$ ACO) = θ_2

and sin ($\angle$ OAC) = sin [180° – ($\theta_1 + \theta_2$)] = sin ($\theta_1 + \theta_2$)

$$\therefore \quad \frac{F_1}{\sin\theta_2} = \frac{F_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1+\theta_2)}$$

i.e.

F_1 (the magnitude of the component of $\overrightarrow{R}$, which inclines by θ_1 on $\overrightarrow{R}$) $= \dfrac{R\sin\theta_2}{\sin(\theta_1+\theta_2)}$

And

F_2 (the magnitude of the component of $\overrightarrow{R}$, which inclines by θ_2 on $\overrightarrow{R}$) $= \dfrac{R\sin\theta_1}{\sin(\theta_1+\theta_2)}$

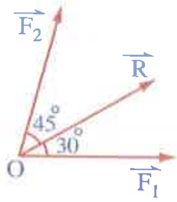
Example 1

Resolve the force of magnitude 20 newton into two components one of them inclined on the given force with an angle of measures 30° and the other force inclined by an angle of measure 45° on the other side of the force , then approximate the answer to the nearest one decimal.

Solution

$$F_1 = \frac{R \sin \theta_2}{\sin(\theta_1 + \theta_2)} = \frac{20 \sin 45°}{\sin 75°} \simeq 14.6 \text{ newton.}$$

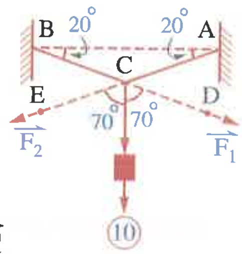
$$F_2 = \frac{R \sin \theta_1}{\sin(\theta_1 + \theta_2)} = \frac{20 \sin 30°}{\sin 75°} \simeq 10.4 \text{ newton.}$$



Example 2

In the opposite figure :

A light of weight 10 Newton is suspended by two strings $\overline{AC}$, $\overline{BC}$ fixed in two horizontal points with equal two angles , the measure of each of them is 20°



(1) Resolve the weight of the light in each of the two directions $\overrightarrow{AC}$, $\overrightarrow{BC}$

(2) What happen , if the magnitude of the components of the weight in the two directions of the strings , if its angle decreased with horizontal less than 20, and what do you deduce to the magnitude of the component of the weight, when the string becomes horizontal.

Solution

(1) The weight (10 newton) acts vertically downwards , and from the figure :

$$\frac{W_1}{\sin 70°} = \frac{W_2}{\sin 70°} = \frac{10}{\sin 140°}$$

$$\therefore W_1 = W_2 = \frac{10 \sin 70°}{\sin 140°} \simeq 15 \text{ newton.}$$

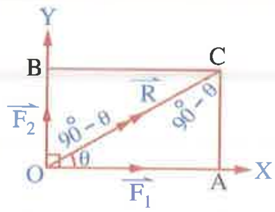
(2) If the measure of the angle decreased with horizontal less than 20° , then the magnitude of the component will increase to become unlimited when the strings are horizontal.

Resolution of the force into two perpendicular directions

Let the force $\overrightarrow{R}$ acts at the point O and we want to resolve this force into two perpendicular forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ such that $\overrightarrow{F_1}$ inclines by θ on the direction of $\overrightarrow{R}$

In this case , the parallelogram become a rectangle.

Applying the sine rule on Δ OAC we get :



$$\frac{F_1}{\sin(90° - \theta)} = \frac{F_2}{\sin\theta} = \frac{R}{\sin 90°}$$

$$\frac{F_1}{\cos\theta} = \frac{F_2}{\sin\theta} = \frac{R}{1} = R$$ Thus , $F_1 = R\cos\theta$, $F_2 = R\sin\theta$

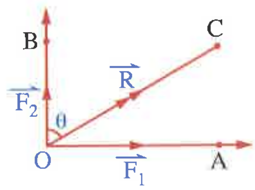
$\therefore$ F_1 (the magnitude of the component in the given direction) = $R\cos\theta$, and F_2 (the magnitude of the component in the perpendicular direction to the given direction) = $R\sin\theta$

The component $\overrightarrow{F_1}$ sometimes is called the projection of $\overrightarrow{R}$ in the direction of $\overrightarrow{OA}$ and the component $\overrightarrow{F_2}$ is called the projection of $\overrightarrow{R}$ in the direction of $\overrightarrow{OB}$

Remarks

(1) The magnitude of the component adjacent to the given angle = R cos (this angle) , the magnitude of the other perpendicular component to the previous component = R sin (this angle)

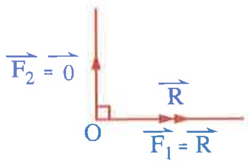
In the opposite figure :



If the component $\overrightarrow{F_2}$ inclines on the direction of $\overrightarrow{R}$ by an angle of measure θ , then

$F_2 = R\cos\theta$, $F_1 = R\sin\theta$

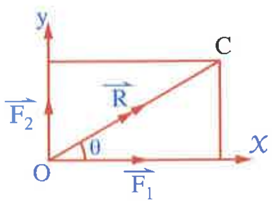
(2) The component of $\overrightarrow{R}$ in the same direction of $\overrightarrow{R}$ = The same force $\overrightarrow{R}$ and its component in the perpendicular direction to its direction = $\overrightarrow{0}$



Because in this case , the measure of the angle between $\overrightarrow{R}$ and the first component = zero , then the magnitude of the first component = $R\cos 0° = R \times 1 = R$

and the magnitude of the perpendicular component on the previous component = $R\sin 0° = R \times 0 =$ zero

(3) If $\overrightarrow{i}$ and $\overrightarrow{j}$ are two perpendicular unit vectors in the directions $\overrightarrow{OX}$ and $\overrightarrow{OY}$ where O is the origin point.



Then $\overrightarrow{F_1} = (R\cos\theta)\,\overrightarrow{i}$, $\overrightarrow{F_2} = (R\sin\theta)\,\overrightarrow{j}$

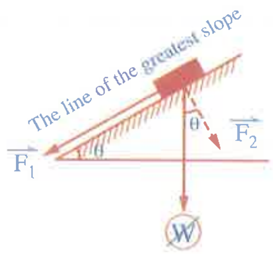
$\therefore\ \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = (R\cos\theta)\,\overrightarrow{i} + (R\sin\theta)\,\overrightarrow{j}$

(4) If $\overrightarrow{F} = (F , \theta)$, then $\overrightarrow{F} = F \cos\theta \, \overrightarrow{i} + F \sin\theta \, \overrightarrow{j}$

(5) If $\theta \in \left]0 , \frac{\pi}{2}\right[$, then the magnitude of the two components $(R\cos\theta)$, $(R\sin\theta)$ is less than the magnitude of the force (R) because $\theta \in \left]0 , \frac{\pi}{2}\right[$, thus $0 < \sin\theta < 1$, $0 < \cos\theta < 1$

(6) If a body of weight (w) is placed on a smooth inclined plane with the horizontal by an angle (θ), then we can resolve the weight (w) which acts vertically downwards into two components.

* F_1 (The magnitude of the component in the direction of the line of the greatest slope)
  $= w \sin\theta$



* F_2 (The magnitude of the component in the perpendicular direction on the plane)
  $= w \cos\theta$

Example 3

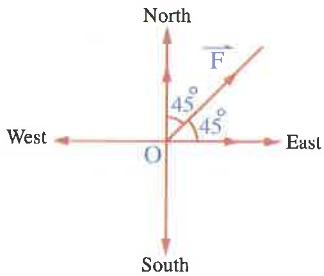
Resolve the force of magnitude $8\sqrt{2}$ newton which acts at the point O in the direction of Eastern North into two components. One of them in the East direction and the other in the North direction.

Solution

$\because$ The two components inclined on the direction of the force by angles of measures 45° and 45° , then they are perpendicular.

$\therefore$ The magnitude of the component which is in the East direction $= F\cos 45° = 8\sqrt{2} \times \frac{1}{\sqrt{2}} = 8$ newton.

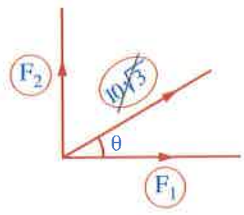
The magnitude of the component in North direction $= F\sin 45° = 8\sqrt{2} \times \frac{1}{\sqrt{2}} = 8$ newton.



Example 4

A force of magnitude $10\sqrt{3}$ kg.wt. was resolved into two perpendicular components , one of them is of magnitude 15 kg.wt. Find the magnitude of the other component.

Solution



Let the direction of the given component $(\overrightarrow{F_1})$ inclines on the direction of the force by an angle of measure θ

$\therefore$ The magnitude of this component $\overrightarrow{F_1} = 10\sqrt{3}\cos\theta$

$\therefore 15 = 10\sqrt{3}\cos\theta$ $\quad\quad$ $\therefore \cos\theta = \frac{15}{10\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\quad\quad$ $\therefore \theta = 30°$

The magnitude of the other component $F_2 = 10\sqrt{3}\sin 30° = 10\sqrt{3} \times \frac{1}{2} = 5\sqrt{3}$ kg.wt.

Another solution :

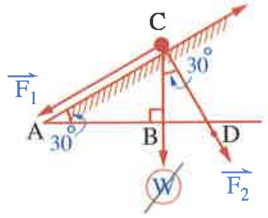
$\because R = \sqrt{F_1^2 + F_2^2}$ $\quad\quad$ $\therefore \left(10\sqrt{3}\right)^2 = (15)^2 + F_2^2$

$\therefore F_2^2 = 75$ $\quad\quad$ $\therefore F_2 = 5\sqrt{3}$ kg. wt.

Example 5

A body of weight 50 newton is placed on a smooth inclined plane by 30° with the horizontal. Find the two components of the weight in the direction of the line of the greatest slope and the direction perpendicular to it.

Solution



From the figure , we notice that : m (∠ BCD) = (∠ BAC) = 30°

$\therefore F_1$ = (the magnitude of the component in the direction of the line of the greatest slope)

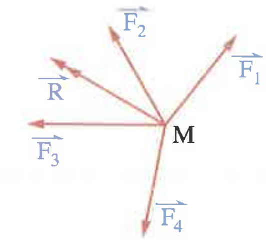
$= W \sin 30° = 50 \times \frac{1}{2} = 25$ newton.

, F_2 = (the magnitude of the component in the perpendicular direction to the plane)

$= W \cos 30° = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}$ newton.

Lesson **Three** | The resultant of coplanar forces meeting at a point

1 The geometrical method

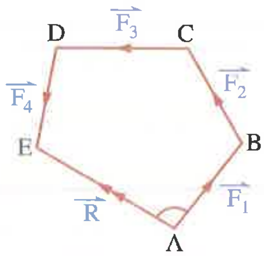


Suppose that the system of coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$, $\overrightarrow{F_4}$ acts at point M as in the opposite figure.

To find the resultant of this forces :

* Use an appropriate drawing scale.
* From any point as A draw the vector $\overrightarrow{AB}$ to represent $\overrightarrow{F_1}$ (in magnitude and direction)
* From point B draw the vector $\overrightarrow{BC}$ to represent $\overrightarrow{F_2}$
* From point C draw the vector $\overrightarrow{CD}$ to represent $\overrightarrow{F_3}$
* At last , from point D draw the vector $\overrightarrow{DE}$ to represent $\overrightarrow{F_4}$

  Match the first point (A) to the last point (E) to be the vector $\overrightarrow{AE}$ which represent the resultant ($\overrightarrow{R}$) in magnitude and direction , where $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \overrightarrow{F_4}$



* Find the length of $\overline{AE}$ and m ($\angle$ EAB) assume it is the included angle between the resultant and the first force and using the drawing scale we can find the magnitude and the direction of $\overrightarrow{R}$
* Then the resultant of the set of forces is a force of magnitude R and acts at point M in direction $\overrightarrow{AE}$

Notice that :

The vector $\overrightarrow{AE}$ which represent $\overrightarrow{R}$ has an opposite direction to the other directions of vectors which represent the forces and the polygon ABCDE is called "The force polygon"

Remark

If the first and last points are congruent in the force polygon , then $(\vec{R}) = \vec{O}$ and the set of forces are equilibrium.

i.e. The adjusted and sufficient condition to equilibrium a set of concurrent forces is a representing of these forces geometrically by the sides of a closed polygon taken in the same direction.

2 The analytical method

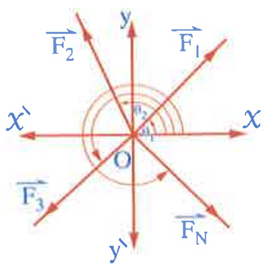
Suppose that the system of coplanar forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots, \vec{F_n}$ meet at the point O and the point O is the origin point of a coplanar cartesian axis.

and $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are the polar angles of the forces $\vec{F_1}, \vec{F_2}, F_3, \ldots, \vec{F_n}$ respectively , let $\vec{i}$ and $\vec{j}$ be the fundamental unit vectors in the two directions $\overrightarrow{OX}$ and $\overrightarrow{OY}$, then :

$\vec{F_1} = (F_1, \theta_1) = F_1 \cos\theta_1 \vec{i} + F_1 \sin\theta_1 \vec{j}$,

$\vec{F_2} = (F_2, \theta_2) = F_2 \cos\theta_2 \vec{i} + F_2 \sin\theta_2 \vec{j}$,

$\vec{F_3} = (F_3, \theta_3) = F_3 \cos\theta_3 \vec{i} + F_3 \sin\theta_3 \vec{j}$



and so on till : $\vec{F_n} = (F_n, \theta_n) = F_n \cos\theta_n \vec{i} + F_n \sin\theta_n \vec{j}$,

$\because \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$

, then by adding we get :

$\vec{R} = (F_1 \cos\theta_1 \vec{i} + F_1 \sin\theta_1 \vec{j}) + (F_2 \cos\theta_2 \vec{i} + F_2 \sin\theta_2 \vec{j})$

$+ (F_3 \cos\theta_3 \vec{i} + F_3 \sin\theta_3 \vec{j}) + \cdots + (F_n \cos\theta_n \vec{i} + F_n \sin\theta_n \vec{j})$

$= (F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + \cdots + F_n \cos\theta_n) \vec{i}$

$+ (F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + \cdots + F_n \sin\theta_n) \vec{j}$

i.e. $\boxed{\vec{R} = \left(\sum_{r=1}^{n} F_r \cos\theta_r\right)\vec{i} + \left(\sum_{r=1}^{n} F_r \sin\theta_r\right)\vec{j}}$

The expression $\left(\sum_{r=1}^{n} F_r \cos\theta_r\right)$ is called the algebraic sum of the components in the direction $\overrightarrow{OX}$ and is denoted by X and the expression $\left(\sum_{r=1}^{n} F_r \sin\theta_r\right)$ is called the algebraic sum of the components in the direction $\overrightarrow{OY}$ and is denoted by Y Hence , we can write the previous equation in the form : $\boxed{\vec{R} = X\vec{i} + Y\vec{j}}$

And let R be the magnitude of $\vec{R}$ and α is the measure of the polar angle of the resultant $\vec{R}$, then :

$\boxed{R = \sqrt{X^2 + Y^2} \text{ and } \tan\alpha = \frac{Y}{X}}$ where $\vec{R} = (R , \alpha)$

Remarks

(1) Notice the difference between X and $\vec{i}$:

- X is the algebraic sum of the components of forces in the direction of $\overrightarrow{OX}$
- $\vec{i}$ is the fundamental unit vector in the direction of $\overrightarrow{OX}$

(2) If $\boxed{X = \text{zero}}$, then $\boxed{\vec{R} = Y\vec{j}}$

and $\theta = 90°$, if $\vec{R}$ in the direction $\overrightarrow{OY}$

, $\theta = 270°$, if $\vec{R}$ in the direction $\overrightarrow{OY'}$

(3) If $\boxed{Y = \text{zero}}$, then $\boxed{\vec{R} = X\vec{i}}$

and $\theta = 0°$, if $\vec{R}$ in the direction $\overrightarrow{OX}$

, $\theta = 180°$, if $\vec{R}$ in the direction $\overrightarrow{OX'}$

(4) If $\boxed{X = \text{zero}}$ and $\boxed{Y = \text{zero}}$, then $\boxed{\vec{R} = \vec{O}}$

In this case , the set of coplanar concurrent forces are in equilibrium.

(5) To determine the direction of the resultant, consider that :

x	y	quad.	θ
+	+	1^{st}	measure of the acute angle
–	+	2^{nd}	180° – measure of the acute angle
–	–	3^{rd}	180° + measure of the acute angle
+	–	4^{th}	360° – measure of the acute angle

(6) The resultant of set of forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$

is $\boxed{\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}}$, and if $\boxed{\vec{R} = \vec{O}}$, then the set of forces are equilibrium

For Example :

If $\vec{F_1} = 5\vec{i} + 2\vec{j}$, $\vec{F_2} = -6\vec{i} + 3\vec{j}$

and $\vec{F_3} = \vec{i} - 5\vec{j}$, then $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{O}$

∴ The forces are in equilibrium.

Example 1

If the forces $\overrightarrow{F_1} = 5\,\vec{i} - 4\,\vec{j}$, $\overrightarrow{F_2} = -6\,\vec{i} + a\,\vec{j}$ and $\overrightarrow{F_3} = b\,\vec{i} + 7\,\vec{j}$ are meeting at a point and are in equilibrium. Find the value of each of : a and b

Solution

$\because$ The forces are in equilibrium $\qquad \therefore \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \vec{O}$

$\therefore (5\,\vec{i} - 4\,\vec{j}) + (-6\,\vec{i} + a\,\vec{j}) + (b\,\vec{i} + 7\,\vec{j}) = \vec{O}$ $\qquad \therefore (-1 + b)\,\vec{i} + (3 + a)\,\vec{j} = \vec{O}$

$\therefore -1 + b = 0$ $\qquad \therefore b = 1$

, $3 + a = 0$ $\qquad \therefore a = -3$

Example 2

In each of the following three figures , a set of forces meeting at a point O and their magnitudes are in newton unit.

Determine the magnitude and the direction of the resultant of each of them.

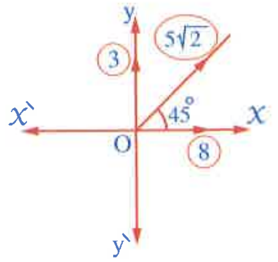


Fig. (1)

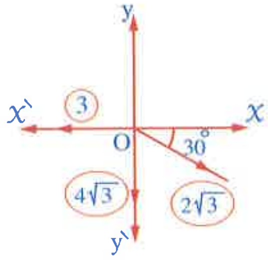


Fig. (2)

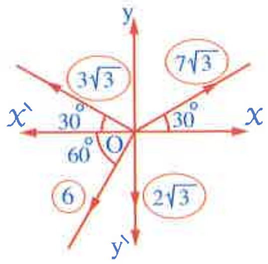


Fig. (3)

Solution

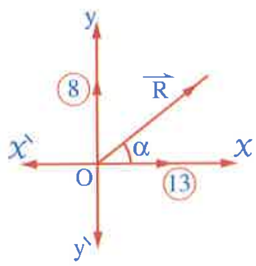
In Fig. (1) : The three forces whose magnitudes are 8 , $5\sqrt{2}$ and 3 newton and their polar angles are of measures 0° , 45° and 90° respectively.

- The algebraic sum of the components in the direction of $\overrightarrow{OX}$ is

$$X = 8 \cos 0^\circ + 5\sqrt{2} \cos 45^\circ + 3 \cos 90^\circ$$
$$= 8 \times 1 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 3 \times \text{zero} = 8 + 5 + 0 = 13 \text{ newton.}$$

- The algebraic sum of the components in the direction of $\overrightarrow{OY}$ is

$$Y = 8 \sin 0^\circ + 5\sqrt{2} \sin 45^\circ + 3 \sin 90^\circ$$
$$= 8 \times \text{zero} + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 3 \times 1 = \text{zero} + 5 + 3 = 8 \text{ newton.}$$



$\therefore \vec{R} = 13\,\vec{i} + 8\,\vec{j}$, then $R = \sqrt{X^2 + Y^2} = \sqrt{169 + 64} = \sqrt{233}$

$\therefore R \simeq 15.264$ newton , $\tan \alpha = \frac{Y}{X} = \frac{8}{13}$

$\because X > 0$, $Y > 0$

$\therefore \vec{R}$ lies in the first quadrant , using the calculator. $\qquad \therefore \alpha \simeq 31^\circ\ 36'$

In Fig. (2) : The three forces whose magnitudes are 3 , $4\sqrt{3}$ and $2\sqrt{3}$ newton and their polar angles are of measures 180° , 270° and 330° respectively.

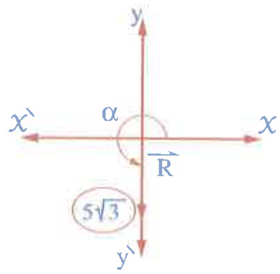
$X = 3\cos 180° + 4\sqrt{3}\cos 270° + 2\sqrt{3}\cos 330°$

$= 3 \times (-1) + 4\sqrt{3} \times 0 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = -3 + 0 + 3 = \text{zero}$

$Y = 3\sin 180° + 4\sqrt{3}\sin 270° + 2\sqrt{3}\sin 330°$

$= 3 \times 0 + 4\sqrt{3} \times (-1) + 2\sqrt{3} \times \left(-\frac{1}{2}\right)$

$= 0 - 4\sqrt{3} - \sqrt{3} = -5\sqrt{3}$ newton.



$\therefore \vec{R} = -5\sqrt{3}\ \vec{j}$

, then $R = 5\sqrt{3}$ newton and $\alpha = 270°$

In Fig. (3) : Four forces of magnitudes $7\sqrt{3}$, $3\sqrt{3}$, 6 and $2\sqrt{3}$ newton and their polar angles are of measures 30° , 150° , 240° and 270° respectively.

$X = 7\sqrt{3}\cos 30° + 3\sqrt{3}\cos 150° + 6\cos 240° + 2\sqrt{3}\cos 270°$

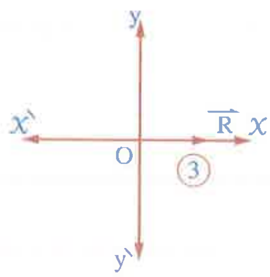
$= 7\sqrt{3} \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) + 6 \times \left(-\frac{1}{2}\right) + 2\sqrt{3} \times 0$

$= 10.5 - 4.5 - 3 + 0 = 3$ newton ,

$Y = 7\sqrt{3}\sin 30° + 3\sqrt{3}\sin 150° + 6\sin 240° + 2\sqrt{3}\sin 270°$

$= 7\sqrt{3} \times \frac{1}{2} + 3\sqrt{3} \times \frac{1}{2} + 6 \times \left(-\frac{\sqrt{3}}{2}\right) + 2\sqrt{3} \times (-1)$

$= \frac{7\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} - 3\sqrt{3} - 2\sqrt{3} = \text{zero}$



$\therefore \vec{R} = 3\ \vec{i}$, then R = 3 newton , α = zero

Another solution for the figure (3) :

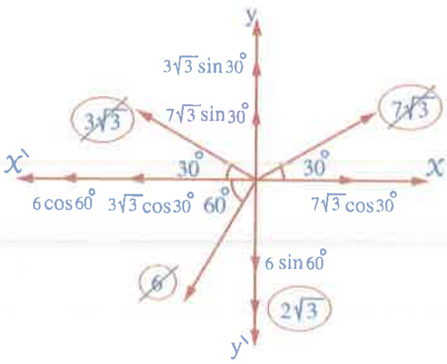
Using the analyzing of the forces into two perpendicular directions :

$\therefore X = 7\sqrt{3}\cos 30° - 3\sqrt{3}\cos 30° - 6\cos 60°$

$= 7\sqrt{3} \times \frac{\sqrt{3}}{2} - 3\sqrt{3} \times \frac{\sqrt{3}}{2} - 6 \times \frac{1}{2} = 3$ newton.

, $Y = 3\sqrt{3}\sin 30° + 7\sqrt{3}\sin 30° - 6\sin 60° - 2\sqrt{3}$

$= 3\sqrt{3} \times \frac{1}{2} + 7\sqrt{3} \times \frac{1}{2} - 6 \times \frac{\sqrt{3}}{2} - 2\sqrt{3} = 0$



$\therefore R = \sqrt{(3)^2 + (0)^2} = 3$ newton.

, $\tan \alpha = \frac{Y}{X} = \frac{0}{3} = 0 \qquad \therefore \alpha = 0°$

Example 3

Five coplanar forces meeting at a point , their magnitudes are 12 , 9 , $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt. , act in the directions : East , North , Western North , Western South and South respectively. Prove that the set of these forces are in equilibrium.

Solution

$\because$ The forces are $(12\ ,0°)\ ,(9\ ,90°)\ ,\left(5\sqrt{2}\ ,135°\right)$

$,\left(7\sqrt{2}\ ,225°\right)\ ,(7\ ,270°)$

$\therefore X = 12\cos 0° + 9\cos 90° + 5\sqrt{2}\cos 135° + 7\sqrt{2}\cos 225° + 7\cos 270°$

$= 12 \times 1 + \text{zero} + 5\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 7\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + \text{zero}$

$= 12 - 5 - 7 = \text{zero}$

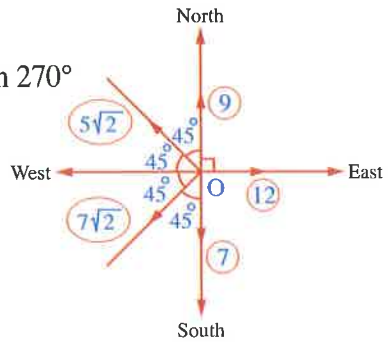
$, Y = 12\sin 0° + 9\sin 90° + 5\sqrt{2}\sin 135° + 7\sqrt{2}\sin 225° + 7\sin 270°$

$= \text{zero} + 9 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 7\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 7 \times -1$

$= 9 + 5 - 7 - 7 = \text{zero}$

$\therefore X = \text{zero}\ , Y = \text{zero}$ $\qquad \therefore \vec{R} = \vec{O}$

$\therefore$ The set of forces are in equilibrium.



Example 4

Four coplanar forces meeting at a point and their magnitudes are F , 2 F , $3\sqrt{3}$ F and 4 F kg. wt. The measure of the angle between the first and second forces is 60° and between the second and the third is 90° and between the third and the fourth is 150° Find the magnitude and the direction of $\vec{R}$

Solution

Let $\overrightarrow{OX}$ is the direction of the first force

, then the forces in polar form are $(F\ ,0°)\ ,(2\,F\ ,60°)\ ,\left(3\sqrt{3}\,F\ ,150°\right)\ ,(4\,F\ ,300°)$ respectively.

$\therefore X = F\cos 0° + 2\,F\cos 60° + 3\sqrt{3}\,F\cos 150° + 4\,F\cos 300°$

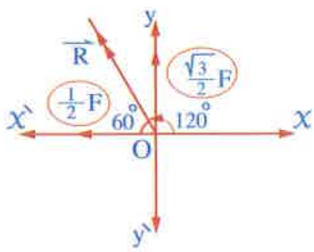
$= F \times 1 + 2\,F \times \frac{1}{2} + 3\sqrt{3}\,F \times \left(-\frac{\sqrt{3}}{2}\right) + 4\,F \times \frac{1}{2}$

$= F + F - \frac{9}{2}\,F + 2\,F = -\frac{1}{2}\,F\ ,$

$Y = F\sin 0° + 2\,F\sin 60° + 3\sqrt{3}\,F\sin 150° + 4\,F\sin 300°$

$= F \times 0 + 2\,F \times \frac{\sqrt{3}}{2} + 3\sqrt{3}\,F \times \frac{1}{2} + 4\,F \times \left(-\frac{\sqrt{3}}{2}\right)$

$= 0 + \sqrt{3}\,F + \frac{3\sqrt{3}}{2}\,F - 2\sqrt{3}\,F = \frac{\sqrt{3}}{2}\,F$



$\vec{R} = -\frac{1}{2} F \vec{i} + \frac{\sqrt{3}}{2} F \vec{j}$ $\therefore R = \sqrt{\frac{1}{4} F^2 + \frac{3}{4} F^2} = \sqrt{F^2} = F$

$\therefore R = F$, $\tan \alpha = \frac{Y}{X} = \frac{\sqrt{3} F}{2} \times -\frac{2}{F} = -\sqrt{3}$

$\because X < 0$, $Y > 0$ $\therefore \alpha = 180° - 60° = 120°$

i.e. The resultant magnitude is F and its direction between 2nd and 3rd forces making an angle of measure 30° with the 3rd force.

* Try to solve this example using the analyzing of the forces into two perpendicular directions.

Example 5

Three forces of magnitudes 2 F , 4 F , 6 F act at a point in directions parallel to the sides of an equilateral triangle in the same cyclic order. Find the magnitude and the direction of the resultant.

Solution

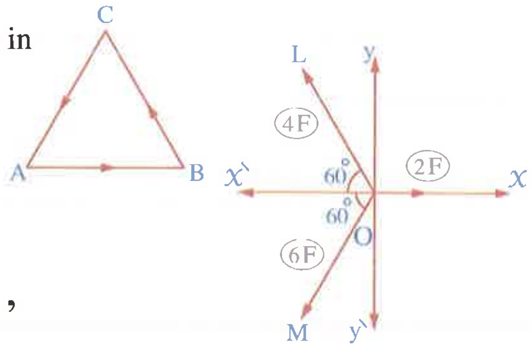
Let the forces act at the point O in the directions

$\overrightarrow{OX}$, $\overrightarrow{OL}$, $\overrightarrow{OM}$

which are parallel to the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ in the equilateral triangle ABC

, then the forces in the polar form are :

(2 F , 0°) , (4 F , 120°) , (6 F , 240°)



$\therefore X = 2 F \cos 0° + 4 F \cos 120° + 6 F \cos 240°$

$= 2 F \times 1 + 4 F \times \left(-\frac{1}{2}\right) + 6 F \times \left(-\frac{1}{2}\right) = -3 F$,

$Y = 2 F \sin 0° + 4 F \sin 120° + 6 F \sin 240°$

$= 2 F \times 0 + 4 F \times \left(\frac{\sqrt{3}}{2}\right) + 6 F \times \left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3} F$

$\therefore \vec{R} = -3 F \vec{i} - \sqrt{3} F \vec{j}$

$\therefore R = \sqrt{X^2 + y^2} = \sqrt{(-3 F)^2 + \left(-\sqrt{3} F\right)^2} = \sqrt{12 F^2} = 2\sqrt{3} F$,

$\tan \alpha = \frac{Y}{X} = \frac{-\sqrt{3} F}{-3 F} = \frac{1}{\sqrt{3}}$

, $\because$ X and Y are negative , then $\alpha = 210°$

i.e. Resultant magnitude is $2\sqrt{3}$ F and its direction between the two forces of magnitudes 6 F , 4 F making an angle of measure 30° with the force 6 F

* Try to solve this example using the resolution of the forces into two perpendicular directions.

Example 6

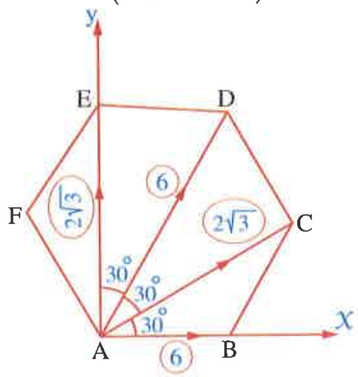
ABCDEF is a regular hexagon. Forces of magnitudes 6 , $2\sqrt{3}$, 6 , $2\sqrt{3}$ newton act along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively.

Find the magnitude and the direction of the resultant of these forces.

Solution

Suppose $\overrightarrow{OX}$ is the direction of the first force.

Then the polar form of the forces are $(6, 0°)$, $(2\sqrt{3}, 30°)$, $(6, 60°)$, $(2\sqrt{3}, 90°)$



$\therefore X = 6\cos 0° + 2\sqrt{3}\cos 30° + 6\cos 60° + 2\sqrt{3}\cos 90°$

$= 6 \times 1 + 2\sqrt{3} \times \left(\frac{\sqrt{3}}{2}\right) + 6 \times \frac{1}{2} + 2\sqrt{3} \times 0 = 12$ newton ,

$Y = 6\sin 0° + 2\sqrt{3}\sin 30° + 6\sin 60° + 2\sqrt{3}\sin 90°$

$= 6 \times 0 + 2\sqrt{3} \times \frac{1}{2} + 6 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 = 6\sqrt{3}$ newton.

$\therefore \vec{R} = 12\,\vec{i} + 6\sqrt{3}\,\vec{j}$ $\qquad \therefore R = \sqrt{X^2 + Y^2}$

$R = \sqrt{(12)^2 + \left(6\sqrt{3}\right)^2} = 6\sqrt{7}$ newton , $\tan\alpha = \frac{Y}{X} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$

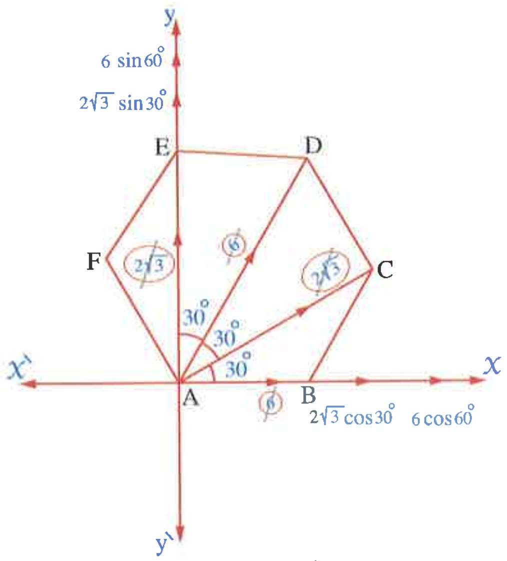
, $\because X > 0$, $Y > 0$ $\qquad \therefore \alpha = 40°\ 53'\ 36''$

i.e. The resultant magnitude is $6\sqrt{7}$ N.

and its direction between $\overrightarrow{AC}$ and $\overrightarrow{AD}$ making an angle of measure $10°\ 53'\ 36''$ with $\overrightarrow{AC}$

Another solution :

Using the resolution of the forces into two perpendicular directions :



$\therefore X = 6\cos 60° + 2\sqrt{3}\cos 30° + 6$

$= 6 \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2} + 6 = 12$ newton.

, $Y = 6\sin 60° + 2\sqrt{3}\sin 30° + 2\sqrt{3}$

$= 6 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times \frac{1}{2} + 2\sqrt{3} = 6\sqrt{3}$ newton.

$\therefore R = \sqrt{(12)^2 + \left(6\sqrt{3}\right)^2} = 6\sqrt{7}$ newton.

, $\tan\alpha = \frac{Y}{X} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$ $\qquad \therefore \alpha = 40°\ 53'\ 36''$

i.e. The resultant magnitude is $6\sqrt{7}$ N. and its direction between $\overrightarrow{AC}$ and $\overrightarrow{AD}$ making an angle of measure $10°\ 53'\ 36''$ with $\overrightarrow{AC}$

Example 7

Four coplanar forces meeting at a point their magnitudes are F_1 , $6\sqrt{2}$, $8\sqrt{2}$, F_2 gm.wt. The first force in the East direction , the second force is in the direction of Eastern North , the third is in the direction of Western North and the fourth acts in South direction.

If their resultant is 7 gm. wt. in magnitude and acts in the East direction , find the value of F_1 and F_2

Solution

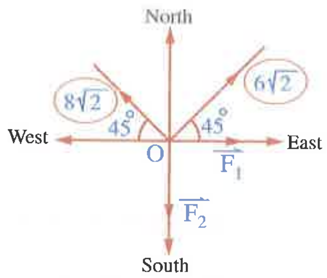
The magnitude of the resultant is 7 gm. wt. and acts towards East

$\therefore X = 7$ and $Y = $ zero

$\therefore F_1 \cos 0° + 6\sqrt{2} \cos 45° + 8\sqrt{2} \cos 135° + F_2 \cos 270° = 7$

$\therefore F_1 \times 1 + 6\sqrt{2} \times \frac{1}{\sqrt{2}} + 8\sqrt{2} \times -\frac{1}{\sqrt{2}} + F_2 \times 0 = 7$

$\therefore F_1 + 6 - 8 + 0 = 7$ $\qquad \therefore F_1 = 9$ gm. wt.



$F_1 \sin 0° + 6\sqrt{2} \sin 45° + 8\sqrt{2} \sin 135° + F_2 \sin 270° = 0$

$\therefore F_1 \times 0 + 6\sqrt{2} \times \frac{1}{\sqrt{2}} + 8\sqrt{2} \times \frac{1}{\sqrt{2}} + F_2 \times (-1) = 0$

$\therefore 6 + 8 - F_2 = 0$ $\qquad \therefore F_2 = 14$ gm. wt.

* Try to solve this example using the resolution of the forces into two perpendicular directions.

Example 8

Five coplanar forces meeting at a point their magnitudes are F , 9 , $5\sqrt{2}$, $7\sqrt{2}$, K (kg.wt.) The measure of the angle between the first force and the second force is 90° , between the second and the third is 45° , between the third and the fourth is 90° and between the fourth and the fifth 45° If the system of forces is in equilibrium , find the value of F and K

Solution

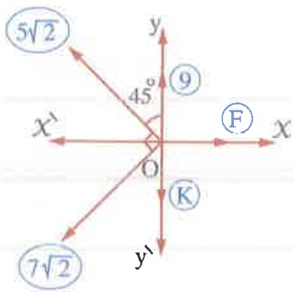
Let $\overrightarrow{OX}$ is the direction of the first force

, then the forces in the polar form are :

$(F , 0°)$, $(9 , 90°)$, $\left(5\sqrt{2} , 135°\right)$, $\left(7\sqrt{2} , 225°\right)$, $(K , 270°)$

, $\because$ the forces are in equilibrium

$\therefore X = Y = 0$



, $\because X = F\cos 0° + 9\cos 90° + 5\sqrt{2}\cos 135° + 7\sqrt{2}\cos 225° + K\cos 270°$

$\therefore F + 9\times 0 + 5\sqrt{2}\times -\frac{1}{\sqrt{2}} + 7\sqrt{2}\times -\frac{1}{\sqrt{2}} + K\times 0 = 0$ $\qquad \therefore F = 12$

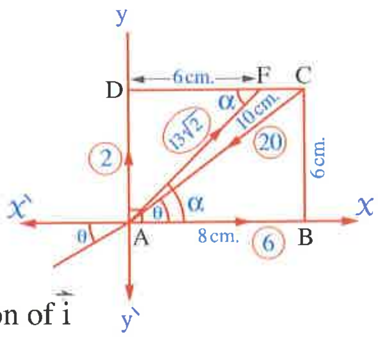
, $\because Y = F\sin 0° + 9\sin 90° + 5\sqrt{2}\sin 135° + 7\sqrt{2}\sin 225° + K\sin 270°$

$\therefore 12\times 0 + 9\times 1 + 5\sqrt{2}\times \frac{1}{\sqrt{2}} + 7\sqrt{2}\times -\frac{1}{\sqrt{2}} + K\times -1 = \text{zero}$ $\qquad \therefore K = 7$

Example 9

ABCD is a rectangle in which AB = 8 cm. , BC = 6 cm. , F $\in \overline{CD}$ where FD = 6 cm. The forces of magnitudes 6 , 20 , $13\sqrt{2}$ and 2 newton act along $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AF}$, $\overrightarrow{AD}$ repectively. Find the magnitude and the direction of the resultant of these forces.

Solution



In ΔABC : $\because (AC)^2 = 6^2 + 8^2 = 100$ $\qquad \therefore AC = 10$ cm.

$\therefore \sin\theta = \frac{6}{10} = \frac{3}{5}$, $\cos\theta = \frac{8}{10} = \frac{4}{5}$

$\because \Delta AFD$ is an isosceles triangle

$\therefore m(\angle AFD) = 45°$ $\qquad$ *i.e.* $\alpha = 45°$

Suppose $\overrightarrow{AB}$ is the direction of the first force and in the direction of $\vec{i}$

$\therefore$ The measures of the polar angles of the forces are zero° , $180° + \theta$, α , $90°$ respectively

$\therefore X = 6\cos 0° + 20\cos(180° + \theta) + 13\sqrt{2}\cos\alpha + 2\cos 90°$

$= 6\times\cos 0° + 20(-\cos\theta) + 13\sqrt{2}\times\cos 45° + 2\cos 90°$

$= 6\times 1 - 20\times\frac{4}{5} + 13\sqrt{2}\times\frac{1}{\sqrt{2}} + 2\times 0 = 6 - 16 + 13 = 3$ newton.

, $Y = 6\times\sin 0° + 20\sin(180° + \theta) + 13\sqrt{2}\sin\alpha + 2\sin 90°$

$= 6\times\sin 0° - 20\sin\theta + 13\sqrt{2}\sin 45° + 2\sin 90°$

$= 6\times 0 - 20\times\frac{3}{5} + 13\sqrt{2}\times\frac{1}{\sqrt{2}} + 2\times 1 = 3$ newton.

$\therefore R = \sqrt{X^2 + Y^2} = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$ newton ,

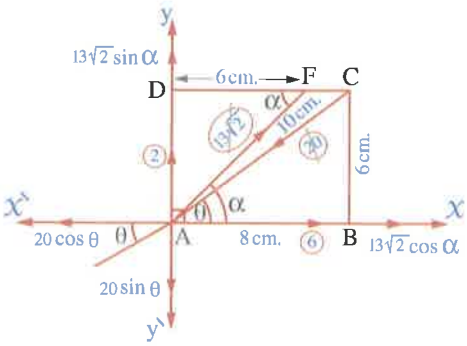
$\tan\ell = \frac{Y}{X} = \frac{3}{3} = 1$ where ℓ is the measure of the polar angle of $\vec{R}$ in this example

$\because X > 0$, $Y > 0$ $\qquad \therefore \ell = 45°$

i.e. $\vec{R}$ is in the direction of $\overrightarrow{AF}$

Another solution :

Using the analyzing of the forces into two perpendicular directions :



From Pythagoras' theorm :

AC = 10 cm.

$\therefore \sin\theta = \frac{6}{10} = \frac{3}{5}$, $\cos\theta = \frac{8}{10} = \frac{4}{5}$

, $\because$ Δ AFD is an isosceles triangle

$\therefore m(\angle\alpha) = 45°$

$\therefore X = 13\sqrt{2}\cos\alpha + 6 - 20\cos\theta$

$= 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 6 - 20 \times \frac{4}{5} = 3$ newton.

, $Y = 13\sqrt{2}\sin\alpha + 2 - 20\sin\theta$

$= 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 2 - 20 \times \frac{3}{5} = 3$ newton.

$\therefore R = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$ newton. , $\tan \ell = \frac{3}{3} = 1$

, $\because X > 0$, $Y > 0$ $\therefore \ell = 45°$

i.e. The resultant in direction of $\overrightarrow{AF}$

Lesson **Four**

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point (The triangle of forces rule - Lami's rule)

First Equilibrium of a rigid body under the action of two forces :

The conditions of equilibrium of a rigid body under the action of two forces

The rigid body is in equilibrium under the action of two forces only , if the two forces :

(1) Are equal in magnitude.

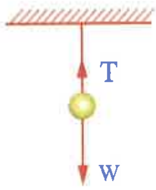
(2) Are opposite in direction.

(3) Their lines of action are on the same straight line.

★ Examples on the equilibrium of a body under the action of two forces :

(1) A body suspended by a light string :

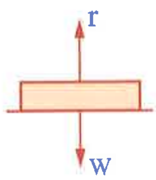
If a weight (W) is suspended by a light string. It balances under the action of two forces which are : weight (W) acting vertically downwards and the tension in the string (T) acting vertically upwards therefore : T = W



(2) A body of weight W placed on a horizontal smooth plane :

If a body of weight (W) is placed on a smooth horizontal plane.
It balances under the action of two forces which are : weight (W) acting vertically downwards and the reaction of the horizontal smooth plane (r) acting vertically upwards as shown in the figure.

, we deduce that : r = W

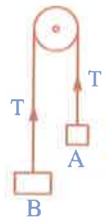


UNIT 1

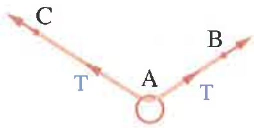
Remarks

(1) If a rigid body is acted upon by two forces equal in magnitude , opposite in direction and their lines of action are on the same straight line , they have no effect on the body neither in case of rest nor motion.

(2) **In the opposite figure :** If a string passes over a smooth pulley. and two bodies A and B are suspended from its two terminals such that the string is tensioned , then the two tensions in the two terminals of the string are equal in magnitude.

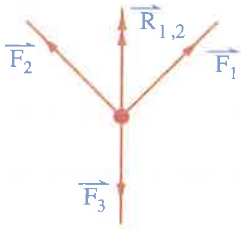


(3) **In the opposite figure :** If a string passes through a smooth ring to be suspended freely in it , then the tensions in each of the two branches of the string $\overline{AB}$, $\overline{AC}$ are equal in magnitude.



Second Equilibrium of a rigid body under the action of three forces acting at a point :

If three coplanar forces as $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are acting at a point and they are in equilibrium as shown in the figure , and if $\overrightarrow{R_{1,2}}$ is the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then the two forces , $\overrightarrow{R_{1,2}}$ and $\overrightarrow{F_3}$ are balanced. Then from the conditions of the equilibrium of two forces , we deduce that $\overrightarrow{R_{1,2}}$ and $\overrightarrow{F_3}$ are equal in magnitude , opposite in direction and they have the same line of action.



Generally : If three forces acting at a point are in equilibrium , then the resultant of any two forces of them is equal in magnitude to the third force and acts in the opposite direction of it and they have the same line of action.

Example 1

$\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three coplanar forces meeting at a point , their magnitudes are 12 , $12\sqrt{3}$ and 24 newton respectively. If these forces are balanced , find the measures of the angles among the three lines of action of the three forces.

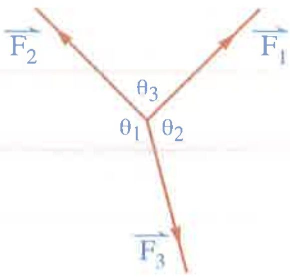
Solution

Suppose the measure of the angle between the lines of action of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ be θ_3

$\because$ The three forces are balanced.

$\therefore R_{1,2} = F_3 = 24$ and act in the opposite direction.

$\because (R_{1,2})^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta_3$



$\therefore (24)^2 = (12)^2 + \left(12\sqrt{3}\right)^2 + 2 \times 12 \times 12\sqrt{3} \cos \theta_3$

$\therefore 576 = 144 + 432 + 288\sqrt{3} \cos \theta_3$

$\therefore \cos \theta_3 = \text{zero}$ $\qquad\qquad \therefore \theta_3 = 90°$

Similarly , suppose the measure of the angle between the lines of action of $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ is θ_1 ,

$\because$ the three forces are balanced.

$\therefore R_{2,3} = F_1 = 12$ and act in the opposite direction.

$\therefore (R_{2,3})^2 = F_2^2 + F_3^2 + 2\, F_2\, F_3 \cos \theta_1$

$\therefore 144 = 432 + 576 + 2 \times 12\sqrt{3} \times 24 \cos \theta_1$ $\qquad \therefore \cos \theta_1 = -\frac{\sqrt{3}}{2}$

$\therefore \theta_1 = 150°$

$\therefore \theta_2$ (the measure of the angle between the lines of action of $\overrightarrow{F_1}$ and $\overrightarrow{F_3}$)

$= 360° - (90° + 150°) = 120°$

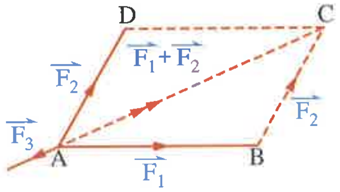
* We know that , the adjusted and sufficient condition to equilibrium of a rigid body under acting of a set of concurrent forces is a representing of these forces geometrically by the sides of a closed polygon , then we can deduce the following rule.

Rule (1)

If three forces are acting at a point and can be represented by the sides of a triangle taken in the same cyclic order , then the forces are in equilibrium.

In the opposite figure :

If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three coplanar forces meeting at A and the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ represent these forces in magnitude and direction.



$\because \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\therefore \overrightarrow{AC}$ represents the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$

but $\overrightarrow{CA}$ represents $\overrightarrow{F_3}$,

$\because \overrightarrow{AC} + \overrightarrow{CA} = \overrightarrow{O}$

$\therefore \overrightarrow{F_3}$ equals in magnitude and opposite in direction to the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$

i.e. $\overrightarrow{F_3}$ is balanced with the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$

$\therefore$ The three forces are in equilibrim

UNIT 1

Remark

The three coplanar forces acting at a point. In order to be in equilibrium , their magnitude should be formed to be lengths of sides of a triangle.

i.e. The greatest magnitude of these forces should be less than the sum of the other two magnitudes of the other two forces because in any triangle , the longest side should be less than the sum of two lengths of the other two sides.

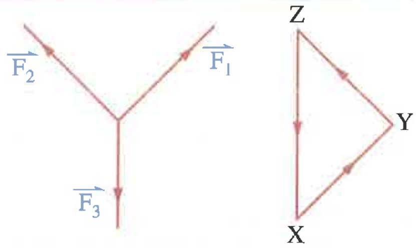
For example :

The three forces whose magnitudes are 3 , 4 and 9 force unit cannot be in equilibrium because the numbers 3 , 4 and 9 cannot be lengths of sides of any triangle because $9 > 3 + 4$ but the forces whose magnitudes are 4 , 7 , 8 could be in equilibrium, but we can not say that they are in equilibrium because that is depending on their magnitudes and their directions also.

Rule (2) The triangle of forces rule

If a rigid body is in equilibrium under the action of three forces acting at a point and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order , then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

And if we symbolized to the forces' magnitude by F_1 , F_2 and F_3 , and Δ ABC was the triangle whose sides are parallel to the lines of action of three forces then $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ represents the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ respectively in the magnitude and the direction where the body is in equilibrium , then $\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC}$ and Δ ABC is called "the triangle of forces".



It is noted that , we can draw an infinite number of similar triangles each of them is called the triangle of forces.

Example 2

Three forces of magnitudes F_1 , F_2 and 96 kg.wt. act on a particle. They are represented by the directed line segments $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively in Δ ABC , where AB = 8 cm. , BC = 10 cm. and CA = 12 cm. Find the value of each of F_1 and F_2

Solution

$\because$ The forces are represented by the directed line segments of a triangle taken in one direction

$\therefore$ The three forces are balanced , then using the triangle of forces rule we get :

$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{96}{CA} \qquad \therefore \frac{F_1}{8} = \frac{F_2}{10} = \frac{96}{12}$$

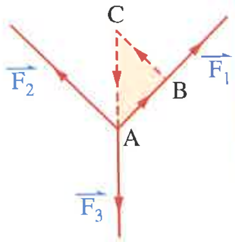
$\therefore F_1 = 64$ kg. wt. , $F_2 = 80$ kg.wt.

Important remarks :

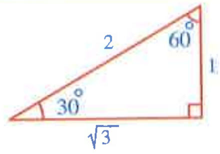
(1) It is possible to draw the triangle of forces , such that two of its sides are on the line of action of two forces and the third side is parallel to the line of action of the third force.

As in the opposite figure :

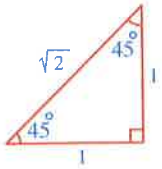
ΔABC is the triangle of forces.



(2) * If the triangle of forces of three equilibrium forces is a right-angled triangle and it has an angle of measure $30°$, then the ratio among its side lengths is $1 : 2 : \sqrt{3}$



* If the triangle of forces is isosceles right-angled triangle , then the ratio among its side lengths is $1 : 1 : \sqrt{2}$

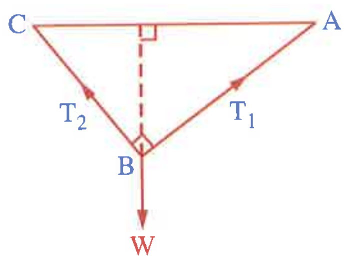


Enrich knowledge

If a triangle is drawn in which its sides are perpendicular to the directions of the equilibrium , then the ratio between each force and the length of the side perpendicular to it is equal.

In the opposite figure :

$\vec{W} \perp \overline{AC}$, $\vec{T_1} \perp \overline{BC}$, $\vec{T_2} \perp \overline{AB}$ $\qquad \therefore \dfrac{W}{AC} = \dfrac{T_1}{BC} = \dfrac{T_2}{AB}$



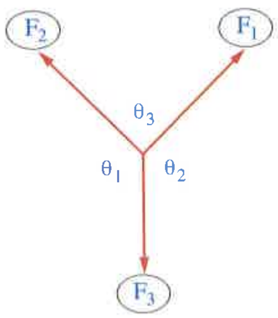
This rule is called perpendicular of forces triangle.

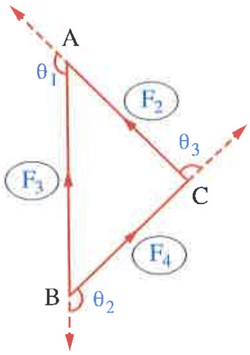
Rule (3) Lami's rule

If three coplanar forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the sine of the angle between the two other forces.

If the symbols of the magnitudes of the forces are F_1 , F_2 and F_3 , and θ_1 , θ_2 and θ_3 are the measures of the opposite angles for them respectively as shown in the opposite figure :

Then ΔBCA is the triangle of forces





$$\therefore \frac{F_1}{BC} = \frac{F_2}{CA} = \frac{F_3}{AB} \qquad (1)$$

and from sin law :

$$\therefore \frac{BC}{\sin(180-\theta_1)} = \frac{CA}{\sin(180-\theta_2)} = \frac{AB}{\sin(180-\theta_3)}$$

i.e. $\frac{BC}{\sin\theta_1} = \frac{CA}{\sin\theta_2} = \frac{AB}{\sin\theta_3} \qquad (2)$

From (1) , (2) we deduce that : $\boxed{\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}}$

Example 3

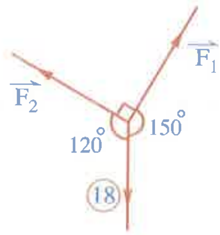
Three coplanar forces of magnitudes F_1 , F_2 and 18 newton meeting at a particle in balance. If the measure of the angle between the line of action of 1st and 2nd forces is 90° and between the 2nd and the 3rd is 120° Find the value of each of F_1 and F_2

Solution

The measure of the angle between

the 1st and 3rd forces = 360° – (90° + 120°) = 150°

Due to Lami's rule we get :



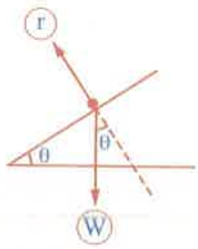
$$\because \frac{F_1}{\sin 120°} = \frac{F_2}{\sin 150°} = \frac{F_3}{\sin 90°} \qquad \therefore \frac{F_1}{\frac{\sqrt{3}}{2}} = \frac{F_2}{\frac{1}{2}} = \frac{18}{1}$$

$$\therefore F_1 = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ newton} ,$$

$$F_2 = 18 \times \frac{1}{2} = 9 \text{ newton.}$$

Equilibrium of a body placed on a smooth inclined plane

If a body of weight (W) is placed on a smooth inclined plane which inclines by an angle of measure θ with the horizontal , then the body will be under the action of two forces :



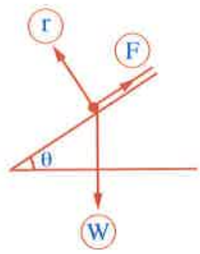
(1) The weight force ($\overrightarrow{W}$) acting vertically downwards.

(2) The reaction force ($\overrightarrow{r}$) of the inclined plane and it acts in direction perpendicular to the plane.

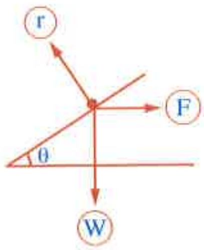
These two forces cannot be in equilibrium because they have two different lines of action. Therefore , in order to be in equilibrium , a third force must act on the body.

It may be in one of the following forms :

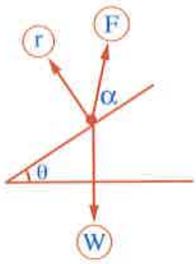
(1) The force is in the direction of the line of the greatest slope upwards.



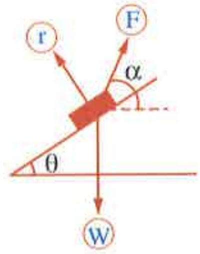
(2) The force acts horizontally.



(3) The force inclines by α with the plane upwards.



(4) The force inclines by α with the horizontal upword.



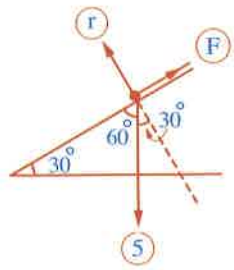
Remark

The reaction of the smooth plane $(\overrightarrow{r})$ is perpendicular to the plane.

Example 4

A body of weight 5 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30°. It is pulled up the plane under the action of a force of magnitude F which its line of action coincides the line of the greatest slope up the plane. Find F and the reaction of the plane.

Solution



The body is in equilibrium under the action of forces of magnitudes F , r and 5 kg.wt. as shown in the figure.

and the measure of the angle between the two lines of action of the two first forces = 90°

and between the 2^{nd} and the 3^{rd} = 180° – 30° = 150°

and between the 3^{rd} and the 1^{st} = 90° + 30° = 120°

Applying Lami's rule we get :

$$\frac{F}{\sin 150°} = \frac{r}{\sin 120°} = \frac{5}{\sin 90°} \qquad \textit{i.e.} \ \frac{F}{\frac{1}{2}} = \frac{r}{\frac{\sqrt{3}}{2}} = \frac{5}{1}$$

$$\therefore F = 5 \times \frac{1}{2} = 2\frac{1}{2} \text{ kg.wt.} \qquad , r = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \text{ kg.wt.}$$

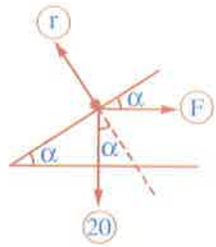
Example 5

A body of weight 20 kg.wt. is placed on a smooth plane inclined to the horizontal with an angle of measure α where cos α = $\frac{4}{5}$

The body is kept in equilibrium by a horizontal force of magnitude F

Find F and the reaction of the plane.

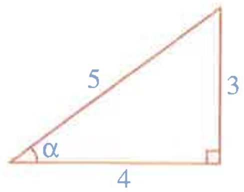
Solution



$\because$ The weight is in equilibrium under the action of forces of magnitudes F , r and 20 kg.wt. therefore , the measure of the angle between the 1^{st} and 2^{nd} forces = 90° + α and between the 2^{nd} and 3^{rd} = 180° – α and between the 3^{rd} and the first = 90°

Applying Lami's rule we get :

$$\frac{F}{\sin(180° - \alpha)} = \frac{r}{\sin 90°} = \frac{20}{\sin(90° + \alpha)}$$



i.e. $\frac{F}{\sin \alpha} = \frac{r}{1} = \frac{20}{\cos \alpha}$

, $\because \cos \alpha = \frac{4}{5}$ $\qquad$ $\therefore \sin \alpha = \frac{3}{5}$

$\therefore \frac{F}{\frac{3}{5}} = \frac{r}{1} = \frac{20}{\frac{4}{5}}$ $\qquad$ $\therefore F = 20 \times \frac{3}{5} \times \frac{5}{4} = 15$ kg.wt.

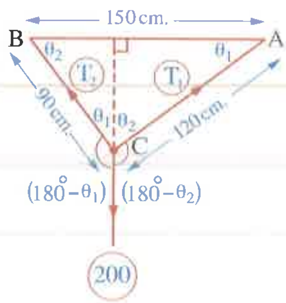
, $r = 20 \times \frac{5}{4} = 25$ kg.wt.

General examples on the equilibrium of three coplanar concurrent forces

Example 6

A weight of 200 gm.wt. is suspended by two strings of lengths 90 cm. and 120 cm. fixed in two horizontal points , the distance between them is 150 cm. Find the value of the tension in each of the two strings in case of equilibrium.

Solution



$\because (90)^2 + (120)^2 = (150)^2$

$\therefore$ Δ ABC is right-angled at C , from the figure we get :

$\sin \theta_1 = \frac{90}{150} = \frac{3}{5}$, $\sin \theta_2 = \frac{120}{150} = \frac{4}{5}$

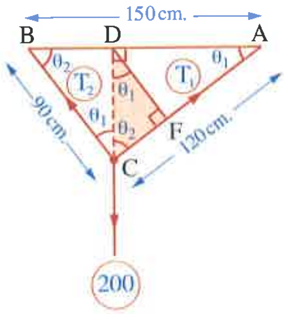
Using Lami's rule ,

$$\therefore \frac{T_1}{\sin(180^\circ - \theta_1)} = \frac{T_2}{\sin(180^\circ - \theta_2)} = \frac{200}{\sin 90^\circ}$$

$$\therefore \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{200}{\sin 90^\circ} \qquad \therefore \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}} = \frac{200}{1}$$

$$\therefore T_1 = \frac{200}{1} \times \frac{3}{5} = 120 \text{ gm.wt. }, T_2 = \frac{200}{1} \times \frac{4}{5} = 160 \text{ gm.wt.}$$

Another solution :



By using the triangle of forces rule : Draw $\overline{DF} \parallel \overline{CB}$

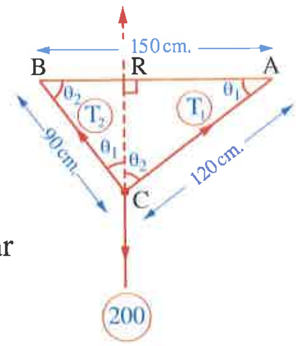
, then Δ DFC is the triangle of forces

$$\therefore \frac{T_1}{CF} = \frac{T_2}{FD} = \frac{200}{DC}$$

$$\therefore T_1 = 200 \times \frac{CF}{DC} = 200 \sin\theta_1 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$

$$, T_2 = 200 \times \frac{FD}{DC} = 200 \sin\theta_2 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}$$

Third solution : (By resolution) :



$\because$ The three forces are in equilibrium.

$\therefore$ The resultant of the two tensions = The third force and in the opposite direction

$\therefore \overrightarrow{T_1}$ and $\overrightarrow{T_2}$ are the two components of $\overrightarrow{R}$ and they are perpendicular

$\therefore T_1 = R\cos\theta_2 = 200 \times \frac{3}{5} = 120$ gm.wt. ,

$T_2 = R\sin\theta_2 = 200 \times \frac{4}{5} = 160$ gm.wt.

Fourth solution : (By perpendicular of forces triangle) :

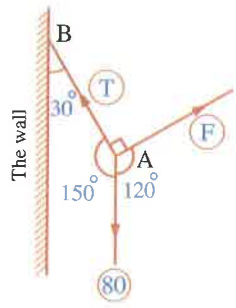
$\because \Delta$ ABC is perpendicular of forces triangle.

$$\therefore \frac{T_1}{90} = \frac{200}{150} = \frac{T_2}{120} \qquad \therefore T_1 = 120 \text{ gm. wt. }, T_2 = 160 \text{ gm. wt.}$$

Example 7

A weight of 80 gm.wt. is suspended by a string fixed in a vertical wall. The weight is pulled by a force perpendicular to the string till it becomes in equilibrium when it is inclined on the wall by an angle of measure 30° , find in case of equilibrium , the magnitude of the force and the tension in the string.

Solution

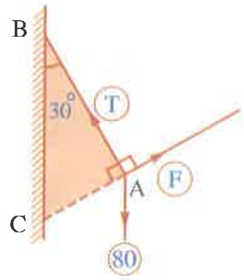


Due to Lami's rule we get , $\frac{F}{\sin 150°} = \frac{T}{\sin 120°} = \frac{80}{\sin 90°}$

$\therefore \frac{F}{\frac{1}{2}} = \frac{T}{\frac{\sqrt{3}}{2}} = 80$

$\therefore F = 80 \times \frac{1}{2} = 40$ gm.wt. , $T = 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3}$ gm.wt.

Another solution : (By triangle of forces) :



Draw the line of action of $\vec{F}$ to meet the wall at C

, then Δ CAB is the triangle of forces.

Due to the triangle of forces rule we get : $\frac{F}{CA} = \frac{T}{AB} = \frac{80}{BC}$

In Δ CAB : ∴ CA : AB : BC = $1 : \sqrt{3} : 2$

$\therefore \frac{F}{1} = \frac{T}{\sqrt{3}} = \frac{80}{2}$ ∴ F = 40 gm.wt. , $T = 40\sqrt{3}$ gm.wt.

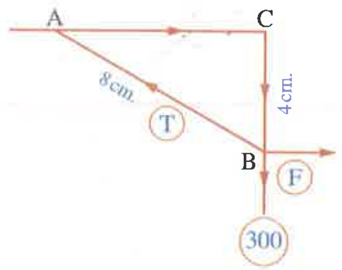
Example 8

A light string $\overline{AB}$ of length 8 cm. Its terminal A is fixed at a point. A weight of 300 gm.wt. , is suspended at the other terminal B. Find the magnitude of the needed force to keep the weight in equilibrium at a distance of 4 cm. From the horizontal line passing through A , also find the tension in the string in each of the two cases.

(1) If the force is horizontal. (2) If the direction of the force is perpendicular to $\overrightarrow{AB}$

Solution

The first case :



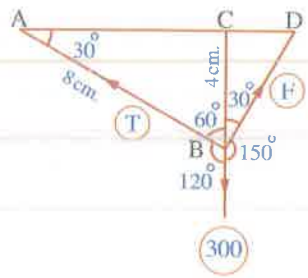
If the force is horizontal , we can take Δ ABC as the triangle of forces.

$\therefore \frac{T}{AB} = \frac{F}{AC} = \frac{300}{BC}$,

$\because AC = \sqrt{(8)^2 - (4)^2} = 4\sqrt{3}$ cm. $\therefore \frac{T}{8} = \frac{F}{4\sqrt{3}} = \frac{300}{4}$

$\therefore T = 600$ gm.wt. , $F = 300\sqrt{3}$ gm.wt.

The second case :



If the direction of the force is perpendicular to $\overrightarrow{AB}$

Due to Lami's rule :

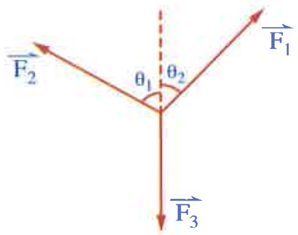
$\therefore \frac{T}{\sin 150°} = \frac{F}{\sin 120°} = \frac{300}{\sin 90°}$ $\therefore T = \frac{300 \sin 150°}{\sin 90°}$

$\therefore T = 150$ gm.wt. , $F = \frac{300 \sin 120°}{\sin 90°} = 150\sqrt{3}$ gm.wt.

Remark

If the line of action of one force of three equilibrium forces is extended to divide the angle between the two lines of action of the other two forces into two angles , then we can apply Lami's rule as follows :

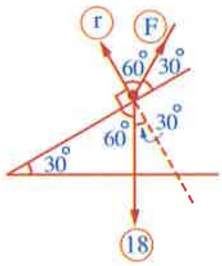
$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin (\theta_1 + \theta_2)}$$



Example 9

A body of weight 18 kg.wt. is placed on a smooth plane inclined to the horizontal by an angle of measure 30°. It is pulled upwards under the action of a force (F) inclines with the line of greatest slope of the plane by an angle of measure 30°. Find the magnitude of this force and the reaction of the plane.

Solution



The body is in equilibrium under the action of the three forces of magnitudes F , r and 18 kg.wt. where :

The measure of the angle between the 1st and 2nd forces is 60°

and between the 2nd and the 3rd is 150°

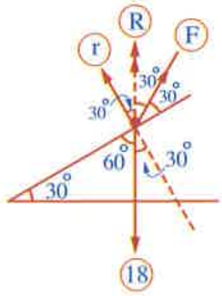
and between the 3rd and the 1st is 30° + 90° + 30° = 150° also.

Applying Lami's rule we get :

$$\frac{F}{\sin 150°} = \frac{r}{\sin 150°} = \frac{18}{\sin 60°} \qquad i.e. \ \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{18}{\frac{\sqrt{3}}{2}}$$

$$\therefore F = r = 18 \times \frac{1}{2} \div \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ kg.wt.}$$

Another solution :



$\because$ The line of action of the weight $\overrightarrow{W}$ which is the line of action of the resultant of the two forces $\overrightarrow{F}$ and $\overrightarrow{r}$ and it bisects the angle between them.

F = r ,

$\because R = 2 F \cos \frac{\alpha}{2}$ $\qquad$ $\therefore 18 = 2 F \times \cos \frac{60°}{2}$

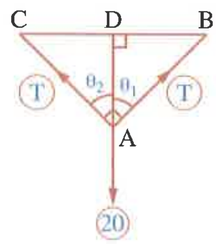
$\therefore 18 = 2 F \cos 30°$ $\qquad$ $\therefore 18 = 2 F \times \frac{\sqrt{3}}{2}$

$\therefore F = \frac{18}{\sqrt{3}} = 6\sqrt{3}$ kg.wt. $\qquad$ $\therefore F = r = 6\sqrt{3}$ kg.wt.

Example 10

A light string is fastened from its terminals at two points B and C such that $\overline{BC}$ is equilibrium horizontal. A small smooth ring of weight 20 gm.wt. slides on the string till the angle between the two branches of the string in equilibrium becomes 90° in measure. Prove that the lengths of the two branches of the string are equal , then find the value of the tension in each of them.

Solution



$\because$ The ring is smooth.

$\therefore$ The values of tension in the two branches of the string are equal.

i.e. Tension in the branch $\overline{AB}$ = tension in the branch $\overline{AC}$ = T

Using Lami's rule we get :

$\frac{T}{\sin \theta_2} = \frac{T}{\sin \theta_1} = \frac{20}{\sin 90°}$ $\qquad \therefore \sin \theta_1 = \sin \theta_2$

$\therefore \theta_1 = \theta_2 = \frac{90°}{2} = 45°$ $\qquad \therefore \frac{T}{\sin 45°} = \frac{20}{\sin 90°}$

$\therefore T = 10\sqrt{2}$

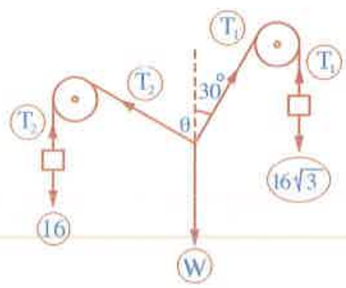
, $\because \overline{AD} \perp \overline{BC}$, $\theta_1 = \theta_2 = 45°$ $\qquad \therefore AB = AC$

$\therefore$ The lengths of the two branches of the string are equal.

Example 11

A body of weight (W) newton is suspended by two strings. The first inclines on the vertical by an angle of measure 30° and passes over a fixed smooth pulley and carries at its free end a weight $16\sqrt{3}$ newton. The second string inclines on the vertical by an angle of measure θ and passes over another fixed smooth pulley and carries at its terminal a body of weight 16 newton. Find in equilibrium case the value of W and the value of θ

Solution



Using Lami's rule :

$\therefore \frac{T_1}{\sin \theta} = \frac{T_2}{\sin 30°} = \frac{W}{\sin (\theta + 30°)}$

$\therefore \frac{16\sqrt{3}}{\sin \theta} = \frac{16}{\sin 30°} = \frac{W}{\sin (\theta + 30°)}$

$\therefore \sin \theta = \frac{16\sqrt{3} \sin 30°}{16} = \frac{\sqrt{3}}{2}$ $\qquad \therefore \theta = 60°$

$\therefore \frac{16}{\sin 30°} = \frac{W}{\sin (60° + 30°)}$ $\qquad \therefore W = 32$ newton.

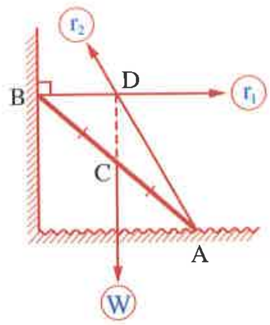
Lesson Five

Follow : The equilibrium
(Meeting lines of action of three equilibrium forces)

Rule (4)

If a rigid body is in equilibrium under the action of three coplanar non parallel forces , then the lines of action of these forces meet at a point.

For example , in the opposite figure :



If a uniform rod of weight (W) is in equilibrium on a smooth vertical wall and on rough horizontal ground then :

(1) The weight of the rod acts vertically downwards at its midpoint. (centre of gravity)

(2) The reaction of the smooth vertically wall (r_1) which is perpendicular to the wall in direction of $\overrightarrow{BD}$

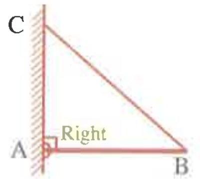
(3) The reaction of the rough ground (r_2) with unknown direction , and to determine its direction , we draw $\overrightarrow{AD}$ passes through the point D (The point of intersection between $\vec{w}$, $\vec{r_1}$)

Remarks

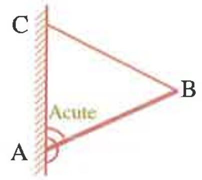
(1) The weight of the uniform sphere acts at its geometric centre (centre of gravity)

(2) If $\overline{AB}$ is a rod , its end A is attached to a hinge fixed at a vertical wall and the other end B is attached to a string fixed at the point C which lies above A exactlly and we notice that :

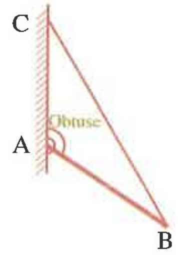
$(BC)^2 = (AB)^2 + (AC)^2$, then $\angle$ BAC is right-angled.



$(BC)^2 < (AB)^2 + (AC)^2$, then $\angle$ BAC is acute.

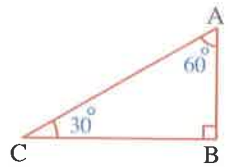


$(BC)^2 > (AB)^2 + (AC)^2$, then $\angle$ BAC is obtuse.



(3) If Δ ABC is a $30° - 60°$ triangle

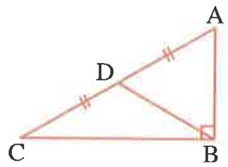
$\therefore AB = \frac{1}{2} AC$, $BC = \frac{\sqrt{3}}{2} AC$



(4) If Δ ABC is right-angled at B ,

$\overline{BD}$ is a median of it

, then $BD = \frac{1}{2} AC$



Example 1

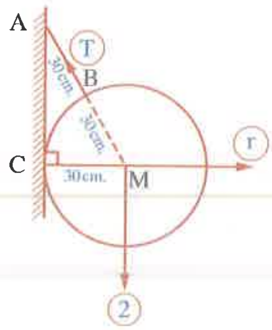
A metallic sphere of weight 2 kg.wt. and of radius length 30 cm. is suspended at a point B on its surface by a string of length 30 cm. , its other end A is fixed at a point in a vertical wall to be in equilibrium as it rests on the wall. Find the magnitude of the tension in the string and the magnitude of the reaction of the wall.

Solution

The sphere is at rest under the action of three forces :

(1) The weight of the sphere whose magnitude is 2 kg.wt. acts vertically downwards at its centre M

(2) The reaction of the wall of magnitude r , acts at the point of touch of the sphere with the wall (C) in the direction perpendicular to the wall , hence it passes through the centre of the sphere M



(3) The tension in the string of magnitude T acts in the direction of $\overrightarrow{BA}$

$\because$ The lines of action of the weight force and the reaction force meet at M

$\therefore$ The line action of the tension force in the string should pass through the point M

i.e. $\overleftrightarrow{AB}$ passes through the point M , then Δ MAC is the triangle of forces where

MA = MB + BA = 60 cm. , CM = 30 cm.

$\therefore$ Δ AMC is a right-angled triangle of (30° – 60°) angles.

$\therefore$ AC = $30\sqrt{3}$ cm. $\therefore \frac{T}{60} = \frac{r}{30} = \frac{2}{30\sqrt{3}}$

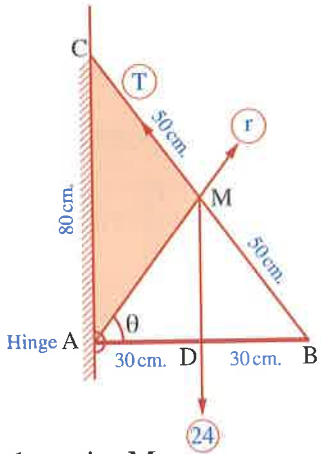
$\therefore$ T = $\frac{4\sqrt{3}}{3}$ kg.wt. , r = $\frac{2\sqrt{3}}{3}$ kg.wt.

Drill Try to solve the previous problem by Lami's rule.

Example 2

$\overline{AB}$ is a uniform rod of length 60 cm. , and its weight 24 kg.wt. , acting at (D) the midpoint of $\overline{AB}$, the end A of the rod is attached to a hinge fixed at a vertical wall. The other end B is attached to a light string , its other end is fixed at the point C on a vertical wall above A at a distance 80 cm. from A. If the rod is in equilibrium horizontally. Find the magnitude of the tension in the string and the magnitude and direction of the reaction of the hinge at A

Solution



From Δ ABC : BC = $\sqrt{(60)^2 + (80)^2}$ = 100 cm.

The rod is in equilibrium under the effect of three forces :

(1) Its weight 24 kg.wt. , acts vertically downwards at the point D (the midpoint of $\overline{AB}$)

(2) The tension in the string (T) acts in the direction of $\overrightarrow{BC}$

(3) The reaction of the hinge at A (its magnitude = r)

$\because$ The two lines of action of the weight and the tension meet at the point M

$\therefore$ The line of action of the reaction of the hinge passes through the point M also ,

$\because$ D is the midpoint of $\overline{AB}$, $\overline{MD}$ // $\overline{AC}$

$\therefore$ M is the midpoint of $\overline{BC}$

$\therefore \overline{AM}$ is a median of ΔABC which is right at A

$\therefore AM = \frac{1}{2} BC = 50$ cm.

$\because \Delta AMC$ is the triangle of forces $\therefore \frac{T}{MC} = \frac{r}{AM} = \frac{24}{CA}$

i.e. $\frac{T}{50} = \frac{r}{50} = \frac{24}{80}$ $\therefore r = T = 24 \times \frac{50}{80} = 15$ kg.wt. ,

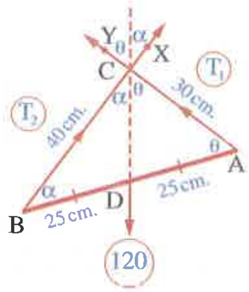
From ΔAMD : $\tan\theta = \frac{40}{30} = \frac{4}{3}$ $\therefore \theta \simeq 53^\circ\ 8'$

$\therefore$ The reaction of the hinge at A inclines to the rod with an angle of measure $53^\circ\ 8'$

Example 3

A uniform rod of length 50 cm. and weight 120 gm. wt. , is suspended at its two ends freely by two strings , their other two ends are fixed at one point.
If the lengths of the two strings are 30 cm. and 40 cm. respectively.
Find the magnitude of the tension in each of the two strings.

Solution



The rod is in equilibrium under the effect of three forces :
their magnitudes are T_1 , T_2 and 120 gm.wt , where $\overrightarrow{AC}$ is the line of action of T_1 and $\overrightarrow{BC}$ is the line of action of T_2
they are meeting at the point C

$\therefore$ The action line of the weight of the rod should pass through C also

$\because (AB)^2 = 2500$, $(AC)^2 + (CB)^2 = 900 + 1600 = 2500$ $\therefore m(\angle ACB) = 90^\circ$

$\because$ D is the midpoint of the hypotenuse $\overline{AB}$ $\therefore DC = DA = DB$

$\therefore m(\angle ACD) = m(\angle DAC)$, $m(\angle DCB) = m(\angle DBC)$

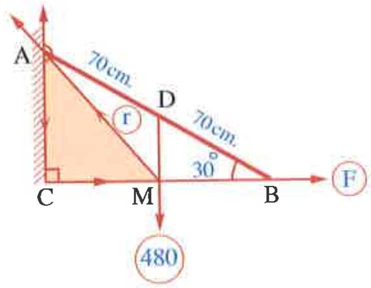
Applying Lami's rule we get : $\frac{T_1}{\sin(\alpha)} = \frac{T_2}{\sin(\theta)} = \frac{120}{\sin 90^\circ}$

$\therefore \frac{T_1}{\frac{30}{50}} = \frac{T_2}{\frac{40}{50}} = 120$ $\therefore T_1 = 120 \times \frac{30}{50} = 72$ gm.wt. , $T_2 = 120 \times \frac{40}{50} = 96$ gm.wt.

Example 4

$\overline{AB}$ is a uniform rod of length 140 cm. and weight 480 gm.wt. , its end A is attached to a hinge fixed on a vertical wall. A force $\vec{F}$ acts horizontally at the other end B to make the rod at rest at a position in which the rod inclines to the horizontal at an angle of measure 30°
Find the magnitude of $\vec{F}$, and the magnitude and the direction of the reaction of the hinge at A

Solution



The rod is at rest under the effect of three forces :

(1) Its weight 480 gm.wt. acts vertically downwards at the point D (the midpoint of $\overline{AB}$)

(2) The horizontal force $\vec{F}$ at B

(3) The reaction of the hinge at A of magnitude r

$\because$ The lines of action of the forces of weight and the horizontal force are meeting at the point M (*i.e.* in the direction of $\overrightarrow{MA}$)

$\therefore$ The line of action of the reaction of the hinge should pass through the point M also

$\therefore$ Δ CMA is the triangle of forces.

$\therefore \frac{F}{CM} = \frac{r}{MA} = \frac{480}{AC}$ $\because$ m ($\angle$ ABC) = 30°

$\therefore$ AC = $\frac{1}{2}$ AB = 70 cm. , BC = $\frac{\sqrt{3}}{2}$ AB = $70\sqrt{3}$ cm.

$\because$ D is the midpoint of $\overline{AB}$, $\overrightarrow{DM} \parallel \overline{AC}$

$\therefore$ M is the midpoint of $\overline{BC}$ $\therefore$ MC = $35\sqrt{3}$ cm.

$\therefore$ AM = $\sqrt{(70)^2 + (35\sqrt{3})^2} = 35\sqrt{7}$ cm. $\therefore \frac{F}{35\sqrt{3}} = \frac{r}{35\sqrt{7}} = \frac{480}{70}$

$\therefore$ F = $240\sqrt{3}$ gm.wt. , r = $240\sqrt{7}$ gm.wt.

, $\because$ Δ AMC is right-angled triangle at C $\therefore$ tan ($\angle$ AMC) = $\frac{AC}{MC} = \frac{70}{35\sqrt{3}} = \frac{2}{\sqrt{3}}$

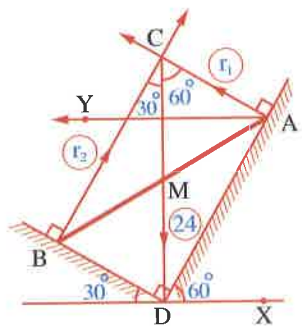
$\therefore$ m ($\angle$ AMC) $\simeq$ 49° 6́

$\therefore$ The reaction of the hinge makes an angle of measure 49° 6́ with the horizontal.

Example 5

A uniform rod rests with its two ends on two smooth planes incline to the horizontal at two angles of measures 60° and 30°. Find the measure of the angle which the rod makes with the horizontal at the equilibrium position and if the weight of the rod equals 24 newton. Determine the magnitudes of the two reactions of the two planes.

Solution



The rod is in equilibrium under the action of three forces :

(1) The weight 24 newton acts vertically downwards at the point M (the midpoint of $\overline{AB}$)

(2) The reaction of the first plane r_1

(3) The reaction of the second plane r_2

∵ The lines of action of the two forces of reactions meet at the point C

∴ The line of action of the weight of the rod passes through the same point "C" either.

If D is the point of meeting thc two planes , then ∠ A , ∠ D and ∠ B are right-angles

∴ The figure ACBD is a rectangle.

If M is the midpoint of $\overline{AB}$

∴ M is the point of intersection of the two diagonals of the rectangle.

∴ $\overline{CD}$ is a diagonal in the rectangle passing through M

, ∵ $\overline{CD}$ is vertical ∴ m (∠ CDX) = 90°

∴ m (∠ MDA) = 30°

, ∵ MD = MA ∴ m (∠ DAM) = 30°

∵ m (∠ YAD) = 60° ∴ m (∠ YAM) = 30°

∴ The rod makes an angle of measure 30° with the horizontal.

From Δ ADC : ∴ m (∠ ACD) = 60°

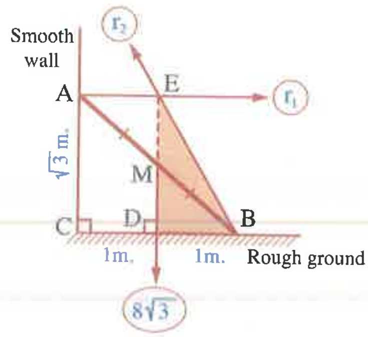
Applying Lami's rule we get : ∴ $\frac{r_1}{\sin 150°} = \frac{r_2}{\sin 120°} = \frac{24}{\sin 90°}$

∴ $r_1 = 12$ newton , $r_2 = 12\sqrt{3}$ newton.

Example 6

$\overline{AB}$ is a uniform ladder of weight $8\sqrt{3}$ kg.wt. , its upper end A rests on a smooth vertical wall and its lower end B rests on rough horizontal ground such that the upper end is far from the surface of the ground by $\sqrt{3}$ metre and the lower end is far from the wall by 2 metre. Find the magnitude of pressure on each of the wall and the ground in case of equilibrium.

Solution



The ladder is in equilibrium under the effect of three forces :

(1) The weight of the ladder of magnitude $8\sqrt{3}$ kg.wt. , acts vertically downwards at the midpoint of the ladder (M)

(2) The reaction of the smooth vertical wall of magnitude r_1 which is perpendicular to the wall at A

(3) The reaction of the rough ground of magnitude r_2

∵ The lines of action of the two forces of the weight and the reaction of the wall meet at the point E

$\therefore$ The line of action of the reaction of the ground must pass through the point E

, then Δ DBE is the triangle of forces where :

$DE = AC = \sqrt{3}$ metre. , $BD = \frac{1}{2} BC = 1$ metre

, $BE = \sqrt{(1)^2 + (\sqrt{3})^2} = 2$ metre

Applying the triangle of forces rule we get :

$\frac{r_1}{DB} = \frac{r_2}{BE} = \frac{8\sqrt{3}}{ED}$ $\therefore \frac{r_1}{1} = \frac{r_2}{2} = \frac{8\sqrt{3}}{\sqrt{3}}$

$\therefore r_1 = 8$ kg.wt. , $r_2 = 16$ kg.wt.

$\therefore$ The pressure on the wall = 8 kg.wt. , the pressure on the ground = 16 kg.wt.

Remark

The pressure of the two ends of the ladder on the floor and the wall equals in magnitude the reactions of the floor and the wall on the two ends of the ladder.

Example 7

$\overline{AB}$ is a uniform rod of length 120 cm. and weight 15 kg.wt. , its end A is attached to a hinge fixed at a point on a vertical wall.
The rod is kept in equilibrium horizontally by attaching it at a point C on it where AC = 80 cm. by a string. The other end of the string is fixed at a point D on the vertical wall above A and at a distance $80\sqrt{3}$ cm. from it. Calculate the magnitude of each of the tension in the string and the reaction of the hinge.

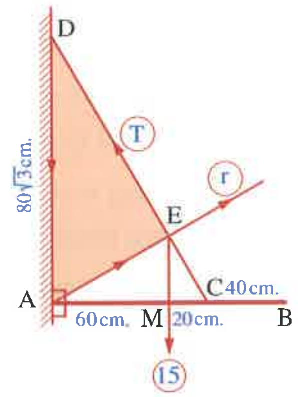
Solution

The rod is kept in equilibrium under the effect of three forces :

(1) Its weight 15 kg.wt. , acts vertically downwards at M (the midpoint of $\overline{AB}$)

(2) The tension force in the string (T)

(3) The reaction of the hinge (r)



$\because$ The lines of action of the weight and the tension are meeting at the point E

$\therefore$ The line of action of the reaction of the hinge passes through the point E also.

$\therefore \Delta$ AED is the triangle of forces

$, \because CD = \sqrt{(80)^2 + (80\sqrt{3})^2} = 160$ cm.

$\because \Delta CME \sim \Delta CAD$: $\therefore \frac{CM}{CA} = \frac{CE}{CD} = \frac{ME}{AD}$

$\therefore \frac{20}{80} = \frac{CE}{160} = \frac{ME}{80\sqrt{3}}$ $\therefore CE = 40$ cm. , $ME = 20\sqrt{3}$, $ED = 120$ cm.

$\therefore AE = \sqrt{(60)^2 + (20\sqrt{3})^2} = 40\sqrt{3}$ cm.

Applying the triangle of forces rule : $\therefore \frac{r}{40\sqrt{3}} = \frac{T}{120} = \frac{15}{80\sqrt{3}}$

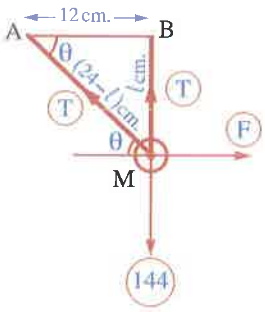
$\therefore$ r = 7.5 kg.wt. $\qquad \therefore T = 7.5\sqrt{3}$ kg.wt.

Example 8

A string of length 24 cm. , is fixed from its ends at two pins A and B in a horizontal line , the distance between them is 12 cm. the string passes inside a smooth ring to be suspended in it , its weight is 144 dyne , then the ring is pulled horizontally by a force $\vec{F}$ till it becomes down B directly.

Find the magnitude of tension in each of the two branches of the string , find also the magnitude of $\vec{F}$

Solution



Supposing that BM = ℓ cm.

$\therefore$ MA = $(24 - \ell)$ cm. $\qquad$, $\because$ m (∠ ABM) = 90°

$\therefore (24 - \ell)^2 = \ell^2 + (12)^2 \qquad \therefore 576 - 48\ell + \ell^2 = \ell^2 + 144$

$\therefore \ell = 9$ cm. $\qquad \therefore$ MB = 9 cm. , AM = 15 cm.

Let m (∠ MAB) = θ

$\therefore \cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$

$\because$ The ring is smooth.

$\therefore$ The tension in the two branches $\overline{MB}$ and $\overline{MA}$ are equal in magnitude.

$\therefore$ The ring is in equilibrium under the effect of four forces which are :

(F , 0°) , (T , 90°) , (T , 180° – θ) , (144 , 270°)

Notice that

The ring is in equilibrium under the effect of four forces , so it will be solved by using the method you studied in lesson 3

X = 0 , Y = 0

, $\because$ X = 0

$\therefore$ F cos 0° + T cos 90° + T cos (180° – θ) + 144 cos 270° = zero

$\therefore$ F × 1 + T × 0 + T × – cos θ + 144 × 0 = zero

$\therefore F - \frac{4}{5}T = 0 \qquad (1)$

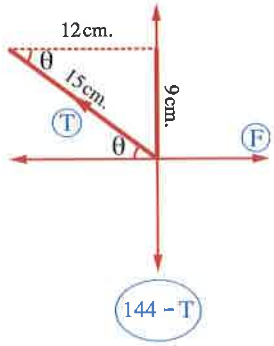
, $\because$ Y = 0

$\therefore$ F sin 0° + T sin 90° + T sin (180° – θ) + 144 × sin 270° = zero

$\therefore$ F × 0 + T × 1 + T sin θ + 144 × – 1 = zero $\qquad \therefore T + \frac{3}{5}T - 144 = $ zero

$\therefore T\left(1 + \frac{3}{5}\right) = 144 \qquad \therefore$ T = 90 dyne.

Substituting in (1) : $\therefore F = \frac{4}{5} \times 90 = 72$ dyne.



Another solution :

$\because$ The weight and the vertical tension

on the same straight line

$\therefore$ Their resultant can be calculated as (144 – T)

Using lami's rule

$$\therefore \frac{F}{\sin(90^\circ + \theta)} = \frac{T}{\sin 90^\circ} = \frac{144 - T}{\sin(180^\circ - \theta)}$$

$$\therefore \frac{F}{\cos\theta} = \frac{T}{1} = \frac{144 - T}{\sin\theta}$$

$$\therefore \frac{F}{\left(\frac{4}{5}\right)} = T = \frac{144 - T}{\left(\frac{3}{5}\right)}$$

$$\therefore \frac{3}{5}T = 144 - T$$

$$\therefore \frac{8}{5}T = 144$$

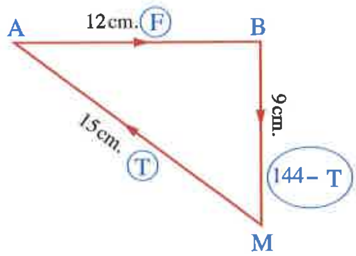
$\therefore T = 90$ dyne , $F = 90 \times \frac{4}{5} = 72$ dyne

Third solution :

$\because$ The weight and the vertical tension on the same straight line

$\therefore$ Their resultant can be calculated as (144 – T)

and the Δ ABM becomes the triangle of forces



$$\therefore \frac{144 - T}{9} = \frac{T}{15} = \frac{F}{12}$$

$$\therefore \frac{144 - T}{3} = \frac{T}{5}$$

$\therefore 3T = 720 - 5T$

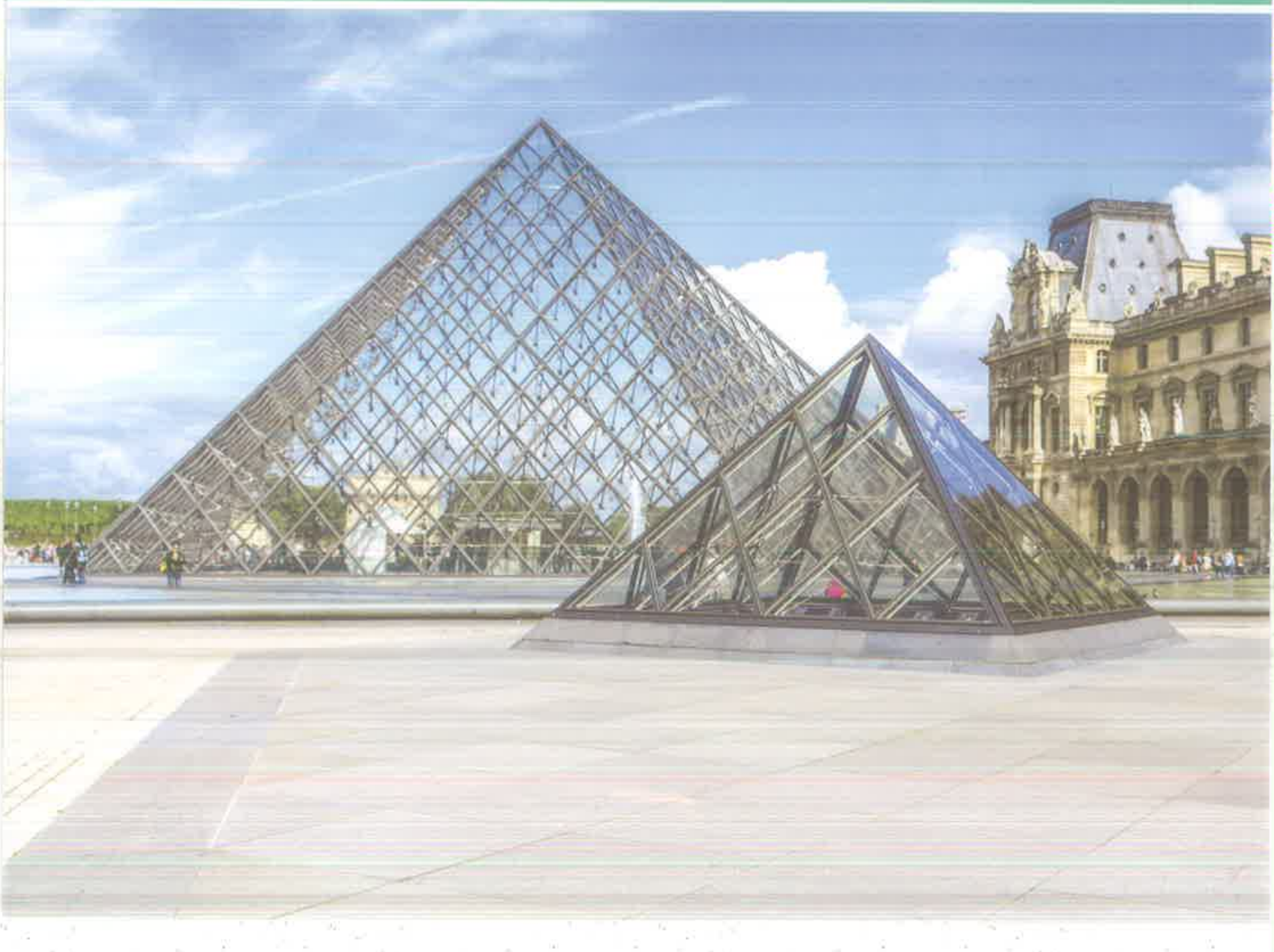
$\therefore 8T = 720$

$\therefore T = 90$ dyne

$$\therefore \frac{90}{15} = \frac{F}{12}$$

$\therefore F = 72$ dyne

UNIT 2 Geometry and Measurement

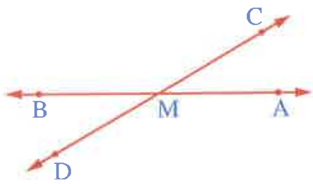


Lesson One — The straight lines and the planes in the space

Geometrical concepts and axioms

1 The straight line :

Is an infinite set of points , and we can determine it exactly if we know any two different points on it.



For example :

In the opposite figure :

The two points "A , B" passing through one and only one straight line which is $\overleftrightarrow{AB}$ while the two points "C , D" passing through another straight line which is $\overleftrightarrow{CD}$

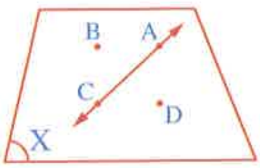
i.e. The straight line is determined by two different points on it.

Remark

$\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

2 The plane :

Is an infinite set of points represents a surface with no ends where any straight line passing through two points on it lies completely on that surface and we denote it by capital letters as X , Y or .... and we can denote it using at least three non-collinear points on the plane as : ABC



i.e. The plane is determined by three distinct non-collinear points.

UNIT 2

Remarks

(1) The geometrical shapes as the triangle , the square , the circle and ... is an infinite set of points and these shapes are called planed geometrical shapes because each of them is a subset of its plane.

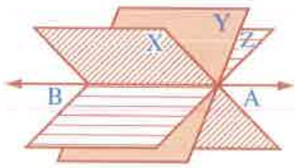
(2) Where the plane extends into infinity in all directions , we can represent it using a planed geometrical shape on it as a square or a circle or a parallelogram or ...

3 The space :

Is an infinite set of points , it contains all the geometric figures and planes , surfaces and solids while the solids as the sphere and the cylinder and the cube , ... etc. are a set of infinite points but we can't contain it in one plane , but we can contain it in the space and the faces of these solids formed from some planed parts as the cube or non-planed as the sphere.

Remarks

- Any point in the space passing through it an infinite number of the straight lines.
- Any point in the space passing through it an infinite number of the planes.



- Any two points in the space passing through them one and only one straight line.
- Any two points in the space passing through them an infinite number of the planes.

Determination of the plane in the space

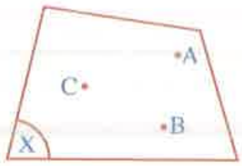
The plane is determined in each of the following cases :

1 Three distinct non-collinear points :

In the opposite figure :

The points A , B , C are non-Collinear so that we can determine the plane (X) or ABC from that we can deduce :

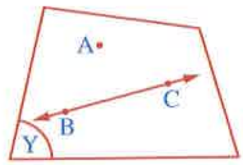
There is one and only one plane which passes through three non-collinear points.



2 A straight line and a point not belonging to it :

In the opposite figure :

$A \notin \overleftrightarrow{BC}$, then the point A and the straight line $\overleftrightarrow{BC}$ determine the plane (Y) or ABC

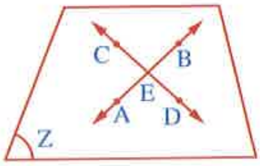


3 Two intersecting straight lines :

In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

, then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ determine the plane (Z)

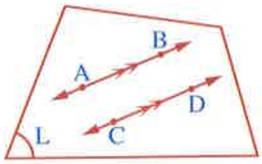


4 Two parallel and non-coincident straight lines :

In the opposite figure :

$\overleftrightarrow{AB} // \overleftrightarrow{CD}$, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \varnothing$

, then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ determine the plane (L)

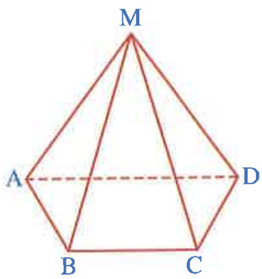


Example 1

In the opposite figure :

If $M \notin$ the plane ABCD

Find :



(1) Four straight lines passing through the point (A)

(2) Three planes passing through the point (A)

(3) The straight lines passing through the two points A and B together.

(4) Two planes each of them passing through the two points A and B together.

(5) Four planes passing through the point (M)

(6) The number of the planes which determine the solid in the figure.

Solution

(1) $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{AM}$

(2) ABCD , ABM , ADM

(3) $\overleftrightarrow{AB}$

(4) ABCD , ABM

(5) MAB , MBC , MCD , MAD

(6) Five planes.

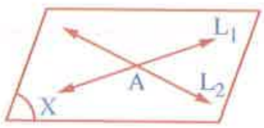
UNIT 2

Relative positions of lines and planes in the space

1 The relative positions of two different straight lines in the space :

Intersecting straight lines

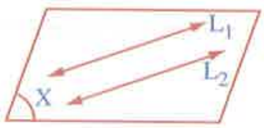
Two straight lines lie on the same plane and have one point in common.



- L_1 , L_2 are intersecting.
- $L_1 \cap L_2 = \{A\}$
- They lie in one plane.

Parallel straight lines

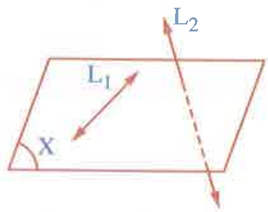
Two straight lines lie on the same plane and doesn't have any common point.



- $L_1 // L_2$
- $L_1 \cap L_2 = \varnothing$
- They lie in one plane.

Skew straight lines

Two straight lines can't be lie in one plane.



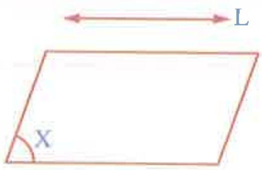
- L_1 , L_2 are skew.
- $L_1 \cap L_2 = \varnothing$
- They do not lie in one plane.

Notice that

The two skew straight lines are not parallel and not intersecting because they are not in one plane.

2 The relative positions for the straight line and the plane in the space :

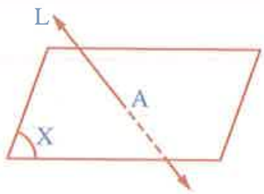
The straight line is parallel to the plane



- The straight line L // the plane X

i.e. $L \cap X = \varnothing$

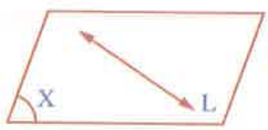
The straight line intersects the plane



- The straight line L intersects the plane X in one point.

i.e. $L \cap X = \{A\}$

The straight line lies completely in the plane



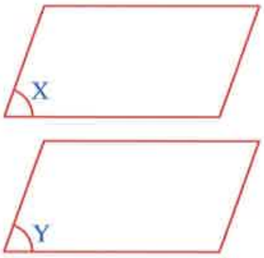
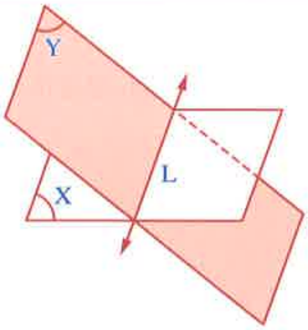
- The straight line L lies completely in the plane X ($L \subset X$)

i.e. $L \cap X = L$

Notice that

If a straight line intersects a plane in more than one point , then the straight line lies completely in the plane.

3 The relative positions for two different planes in the space :

Two parallel planes	Two intersecting planes	Two coincident planes
 The plane X // the plane Y *i.e.* $X \cap Y = \varnothing$	 The two planes intersecing at a straight line L *i.e.* $X \cap Y = L$	 The two planes are in common in all points (Coincide) *i.e.* $X \cap Y = X = Y$

Remarks

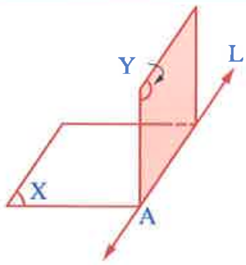
(1) If there are two planes have a common point , then they have in a common a straight line passing through this point.

In the opposite figure :

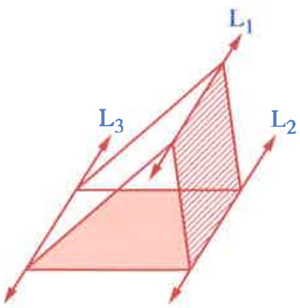
The two planes X and Y have a common point (A) then :

the two planes X and Y have a common straight line (L)

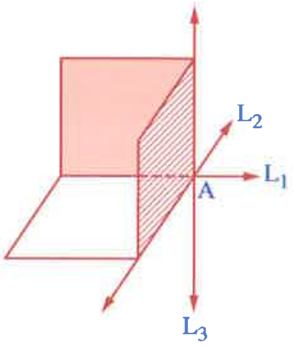
i.e. $X \cap Y = L$ where $A \in L$



(2) If there are three planes are intersected "each two with each other" , then their intersected straight lines will be parallel or intersecting at one point.



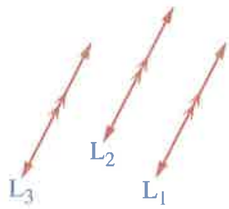
$L_1 // L_2 // L_3$



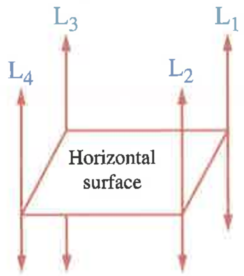
$L_1 \cap L_2 \cap L_3 = \{A\}$

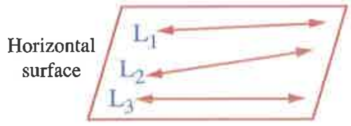
(3) If two straight lines are parallel to a third in the space , then they all are parallel.

i.e. If $L_1 // L_3$, $L_2 // L_3$, then $L_1 // L_2$



(4) All vertical straight lines in the space are parallel but not all horizontal straight lines in the space are parallel.

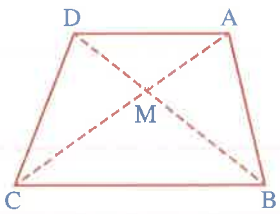


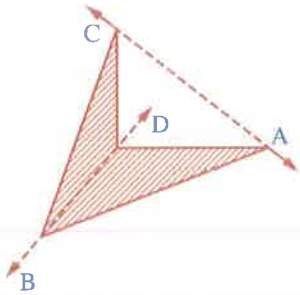


All the vertical straight lines L_1 , L_2 , L_3 and L_4 are parallel.

The horizontal straight lines L_1 , L_2 and L_3 are not parallel.

(5) If the straight lines contain the diagonals of a quadrilateral intersected at a point , then all its sides lie in one plane.





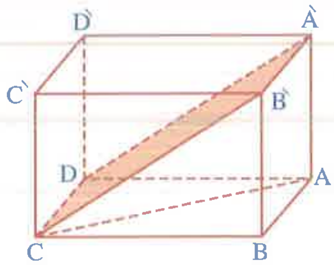
The sides of the quadrilateral ABCD lie in one plane , because $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$

The sides of the quadrilateral ABCD doesn't lie in one plane , because $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \varnothing$ $\left(\overleftrightarrow{AC} , \overleftrightarrow{BD} \text{ are skew}\right)$

Example 2

In the opposite figure :

ABCD À B̀ C̀ D̀ is a cuboid , complete the following :



(1) $\overleftrightarrow{BC}$ // the plane ....

(2) $\overleftrightarrow{AB}$ and .... are skew.

(3) The plane AB B̀ À // the plane ....

(4) The plane $ABB'A'$ $\cap$ the plane $A'B'CD$ = ……….

(5) The plane ABC $\cap$ the plane $A'B'CD$ = ……….

(6) The plane $AA'D'D$ $\cap$ the plane $ABB'A'$ $\cap$ the plane $ABCD$ = ……….

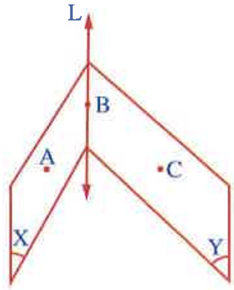
Solution

(1) $AA'D'D$ or $A'B'C'D'$ (2) $\overleftrightarrow{DD'}$ or $\overleftrightarrow{A'D'}$ (Find other answers) (3) $DCC'D'$

(4) $\overleftrightarrow{AB}$ (5) $\overleftrightarrow{DC}$ (6) $\{A\}$

Example 3

In the opposite figure :



The plane X $\cap$ the plane Y = the straight line L

$A \in X$, $C \in Y$, $B \in L$

Choose the correct answer from those given :

(1) The plane ABC $\cap$ the plane X = ………. ($\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, $\{B\}$)

(2) The plane ABC $\cap$ the plane Y = ………. ($\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, $\{B\}$)

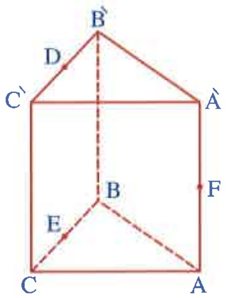
(3) The plane ABC $\cap$ the plane X $\cap$ the plane Y = ………. ($\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, $\{B\}$)

Solution

(1) $\overleftrightarrow{AB}$ (2) $\overleftrightarrow{BC}$ (3) $\{B\}$

Example 4

In the opposite figure :



If the plane ABC // the plane $A'B'C'$, D and E are the midpoints of $\overline{B'C'}$ and $\overline{BC}$ respectively , $F \in \overline{AA'}$

1 Find four planes passing through the point (A)

2 Choose the correct answer from those given :

(1) A ………. the plane DFE ($\in$, $\notin$, $\subset$, $\not\subset$)

(2) $\overleftrightarrow{AA'}$ ………. the plane DFE ($\in$, $\notin$, $\subset$, $\not\subset$)

(3) $\overleftrightarrow{AA'}$, $\overleftrightarrow{BC}$ are two ………. straight lines.(parallel , skew , intersecting , perpendicular)

(4) The plane DFE $\cap$ The plane $A'B'C'$ = ………. ($\overleftrightarrow{A'D}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{FE}$)

(5) The plane DEF $\cap$ The plane $ABB'A'$ = ………. ($\overleftrightarrow{AA'}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{A'D}$, $\overleftrightarrow{DE}$)

(6) The plane AEA' $\cap$ The plane $B'BCC'$ = ………. ($\overleftrightarrow{DE}$, $\overleftrightarrow{AE'}$, $\overleftrightarrow{A'D}$, $\overleftrightarrow{AA'}$)

Solution

1 The planes are : $AA'B'B$, $AA'C'C$, $AA'E$, ABC

2 (1) $\in$ (2) $\subset$ (3) skew

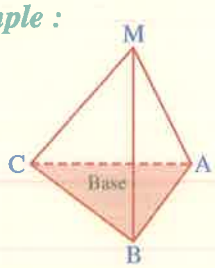
(4) $\overleftrightarrow{A'D}$ (5) $\overleftrightarrow{AA'}$ (6) $\overleftrightarrow{DE}$

Lesson Two — The pyramid

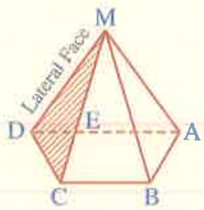
The definition of the pyramid

Is a solid has one base as a polygon and all its other faces are triangles with a common vertex and the pyramid is called a triangular , quadrilateral, pentagonal or .... according to the number of sides of the polygon of its base.

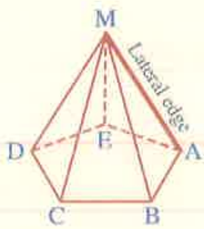
For example :



triangular pyramid, its base is a triangle

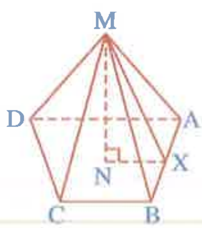


quadrilateral pyramid, its base is a quadrilateral



pentagonal pyramid, its base is a pentagon

By using the opposite figure : we can explain some concepts of the pyramid :



- MABCD is a quadrilateral pyramid.

  its lateral faces are the triangles MAB, MBC, MCD, MAD, and its base is a polygon ABCD

- The lateral faces of the pyramid : are always triangles but the base could be a triangle or a quadrilateral or a pentagon or ....
- The vertex of the pyramid : Is the common point for all lateral faces of the pyramid.

  In the figure : The point "M" is the vertex of the pyramid MABCD

- The lateral edge of the pyramid : Is a line segment joining between the vertex of the pyramid and any vertex of its base vertices. as ($\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$, as in the figure)
- The height of the pyramid : Is the distance between the vertex of the pyramid and its base surface.

  i.e. Is the length of the perpendicular from the vertex of the pyramid to its base surface.

  (MN is the height of the pyramid as in the figure)
- The slant height of the pyramid : Is the distance between the vertex of the pyramid and one of its base sides.

  i.e. Is the length of the perpendicular line segment from the vertex to one of the base sides of the pyramid.

  ($\overline{MX}$ is a slant height of the pyramid MABCD where $\overline{MX} \perp \overline{AB}$)

Remarks

- The perpendicular straight line to a plane is perpendicular to any straight line in that plane , then the perpendicular straight line to the base of the pyramid is perpendicular to any straight line in it.
- The regular polygon is a polygon in which its sides are equal in length and its angles are equal in measure.
- The geometrical centre of any regular polygon is the centre of inscribed circle or the circumcircle for it.
- The geometrical centre of the parallelogram and its special cases is the point of intersection of the diagonals.
- The geometrical centre of the triangle is the point of intersection of its medians.

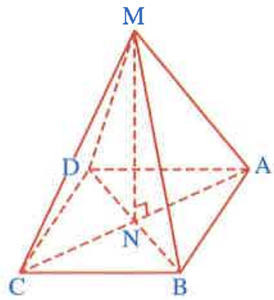
Special cases of the pyramid

1 The right pyramid :

The pyramid is right if the position of perpendicular from the vertex of the pyramid to the base is passing through its geometrical centre.

For example :

- In the pyramid MABCD as in the figure : If N is the geometrical centre of the base ABCD and $\overline{MN} \perp$ the plane of the base ABCD , then the pyramid MABCD is called a right pyramid.



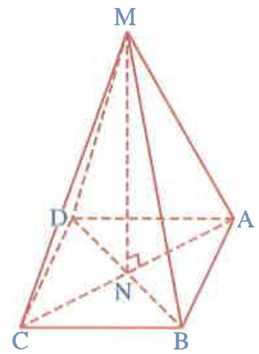
2 The regular pyramid :

Is the pyramid in which its base is a regular polygon whose centre is the position of the perpendicular from the vertex of the pyramid to the base.

i.e. A right pyramid of a base as a regular polygon.

For example :

In the pyramid MABCD as in the figure :
if N is the geometrical centre of the regular base ABCD "square" and $\overline{MN} \perp$ the plane of the base , then the pyramid MABCD is a regular pyramid.



Properties of the regular pyramid :

(1) Its lateral edges are equal in length.

(2) Its slant heights are equal in length.

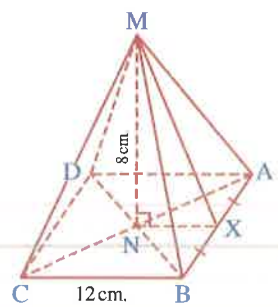
(3) Its lateral faces are congruent isosceles triangles.

Remarks

- Every regular pyramid is a right pyramid but not vice verse.
- Not necessary that the lateral edges of the right pyramid are equal in length.
- Not necessary that the slant heights of the right pyramid are equal in length.
- The regular triangular pyramid is called a triangular pyramid of regular faces if its all faces are equilateral triangles and any one of them is its base.

Example 1

MABCD is a regular quadrilateral pyramid , the length of its base side is 12 cm. and its height length equals 8 cm. Find the length of its slant height.



Solution

Let X is a midpoint of $\overline{AB}$

$\because$ MABCD is a regular quadrilateral pyramid

$\therefore$ MA = MB $\qquad \therefore \overline{MX} \perp \overline{AB}$

$\therefore \overline{MX}$ is the slant height of the pyramid

In Δ ABC : X is a midpoint of $\overline{AB}$, N is a midpoint of $\overline{AC}$

$\therefore X N = \frac{1}{2} BC \qquad \therefore X N = 6$ cm.

$\because \overline{MN} \perp$ The plane ABCD $\qquad \therefore \overline{MN} \perp \overline{XN}$

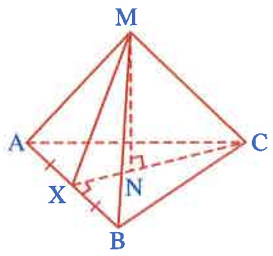
$\therefore \Delta$ MXN is right at N

$\therefore (XM)^2 = (XN)^2 + (MN)^2 \qquad \therefore (XM)^2 = 36 + 64 = 100 \qquad \therefore XM = 10$ cm.

Example 2

MABC is a regular triangular pyramid with base Δ ABC, the length of its base length is 6 cm. and the length of its height is 4 cm. Find the length of its edge and its slant height.

Solution



Let X is the midpoint of $\overline{AB}$

$\because$ MABC is a regular triangular pyramid

$\therefore$ Δ ABC is an equilateral triangle

, $\because$ X is the midpoint of $\overline{AB}$ $\quad\therefore \overline{CX} \perp \overline{AB}$

$\therefore$ Δ BXC is a right-angled triangle at X

$\therefore (XC)^2 = (BC)^2 - (BX)^2 = 36 - 9 = 27$

$\therefore XC = \sqrt{27} = 3\sqrt{3}$ cm.

, $\because$ N is the centre of Δ ABC

$\therefore$ N is the point of intersection of the medians of Δ ABC

$\therefore NX : NC = 1 : 2$ $\quad\therefore NX = \sqrt{3}$ cm. , $NC = 2\sqrt{3}$ cm.

$\because \overline{MN} \perp$ The plane ABC $\quad\therefore \overline{MN} \perp \overline{XC}$

$\therefore$ Δ MNC is right-angled triangle at N

$\therefore (MC)^2 = (MN)^2 + (NC)^2 = 16 + 12 = 28$ $\quad\therefore MC = \sqrt{28} = 2\sqrt{7}$ cm.

$\therefore$ The length of the edge of the pyramid $= 2\sqrt{7}$ cm.

, $\because$ Δ MNC is right-angled triangle at N

$\therefore (MX)^2 = (MN)^2 + (NX)^2 = 16 + 3 = 19$

$\therefore MX = \sqrt{19}$ cm.

, $\because$ X is a midpoint of $\overline{AB}$ $\quad\therefore \overline{MX} \perp \overline{AB}$

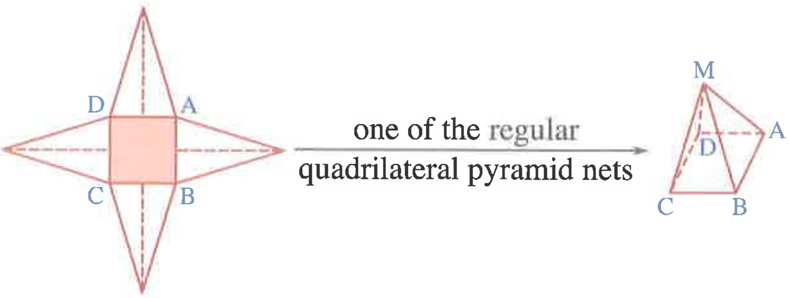
$\therefore \overline{MX}$ is the slant height of the pyramid $= \sqrt{19}$ cm.

UNIT 2

Solids net

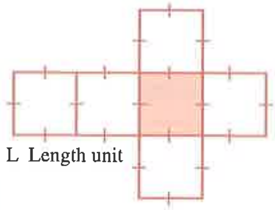
The solids net is used to make the solids by tracking the shape of the solid on the surface of the plane and folding this plane to form the solid.

The pyramid net :

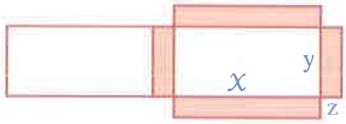


Remember

(1) one of the cube nets



(2) one of the cuboid nets



From the net of the regular quadrilateral pyramid we note that :

(1) It has 5 faces , four of them are lateral faces and one face to the base.

(2) It has 8 edges , four of them are lateral edges.

(3) It has 5 vertices , one of them (M) is called the vertex of the pyramid.

Enrich your knowledge

Euler relation : For any solid in which its base as a polygon , then :

(Number of its faces + number of its vertices = number of its edges + 2)

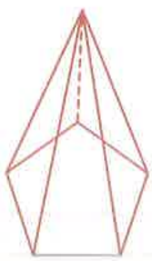
For example : In the pentagonal pyramid : number of its faces = 6 faces ,

number of its vertices = 6 vertices , number of its edges = 10 edges)

i.e. Number of its faces + number of its vertices = 6 + 6 = 12

, number of its edges + 2 = 10 + 2 = 12

∴ Number of faces + number of vertices = number of edges + 2

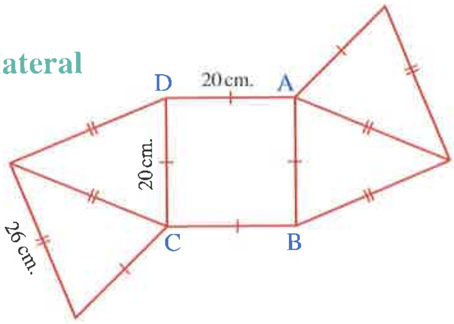


Example 3

The opposite net represents a net of a regular quadrilateral pyramid. Find :

(1) Height of the pyramid.

(2) The slant height of the pyramid.



Solution

The net is representing a regular quadrilateral pyramid its base ABCD is a square , its vertex M , and its height $\overline{MN}$, where N is the point of intersection of the diagonals of the base.

$\because$ ABCD is a square

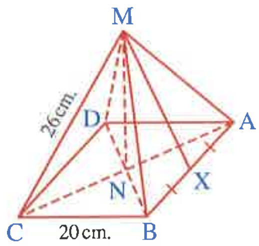
$\therefore$ The length of its diagonal = its side length $\times\sqrt{2}$

$\therefore AC = 20 \times\sqrt{2} = 20\sqrt{2}$ cm.

$\therefore AN = 10\sqrt{2}$ cm.

$\because$ MABCD is a right quadrilateral pyramid

$\therefore \overline{MN} \perp$ The plane of the base ABCD $\qquad \therefore \overline{MN} \perp \overline{AN}$



$\therefore \Delta$ ANM in which m ($\angle$ ANM) = 90°

$\therefore (MN)^2 = (AM)^2 - (AN)^2 = (26)^2 - \left(10\sqrt{2}\right)^2 = 476$

$\therefore MN = \sqrt{476} = 2\sqrt{119}$ cm.

$\therefore$ The height of the pyramid $= 2\sqrt{119}$ cm.

Let X is a midpoint of $\overline{AB}$ $\qquad \therefore AX = 10$ cm.

, $\because MA = MB$ $\qquad \therefore \overline{MX} \perp \overline{AB}$

$\therefore \Delta$ AXM in which m ($\angle$ AXM) = 90°

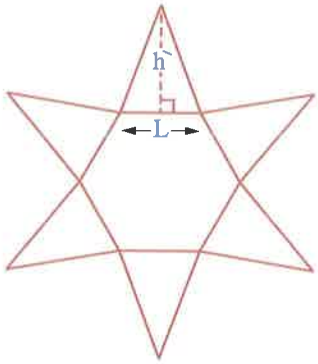
$\therefore (MX)^2 = (AM)^2 - (AX)^2 = (26)^2 - (10)^2 = 576$ $\qquad \therefore MX = \sqrt{576} = 24$ cm.

$\therefore$ The slant height of the pyramid = 24 cm.

The lateral area of regular pyramid - the total area of pyramid - the volume of pyramid :

* The lateral area of the pyramid = the sum of areas of the lateral faces
* The lateral area of the regular pyramid = $\frac{1}{2}$ base perimeter $\times$ slant height
* The total surface area of the pyramid = lateral area + area of the base
* The volume of the pyramid = $\frac{1}{3}$base area $\times$ height

Finding the lateral area of regular pyramid



If the side length of the base in regular pyramid is L and the number of its base sides is n and its lateral height h̀ , then from the net of this pyramid we find it has n congruent faces each one is an isosceles triangle and the area of each triangle $= \frac{1}{2} \times L \times \grave{h}$

$\therefore$ The lateral area of the regular pyramid

$$= \frac{1}{2} L \times \grave{h} \times n ,$$

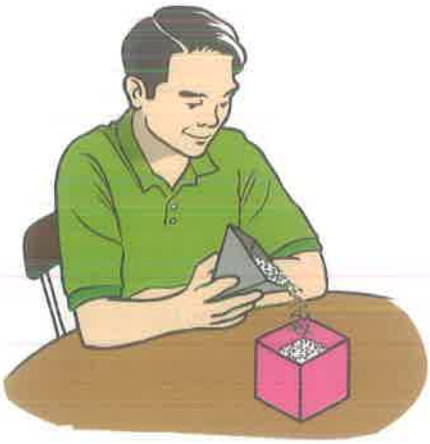
, $\because$ perimeter of the base $= n \times L$

$\therefore$ The lateral area of the regular pyramid

$= \frac{1}{2} \times$ base perimeter $\times$ slant height

Finding the volume of the pyramid

Experiment activity



* Bring a hollow vessel in the shape of a right prism , and another one in the shape of a right pyramid where be their bases are congruent and they have the same height as in the opposite figure.
* Fill the pyramid vessel with **grains** of rice or sand then put it in the prism.
* Repeat this process three times and you will note that : The prism will filled completely with the **grains** and that means :

Volume of the pyramid $= \frac{1}{3}$ volume of the prism has the same base and height.

$\because$ The volume of the prism = the base area $\times$ height

$\therefore$ Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

Remarks

(1) In the triangular pyramid of regular faces :
double the square of its edge length = 3 times the square of its height.

i.e. $2 L^2 = 3 h^2$ Where L = edge length , h = the height

(2) The total surface area to the triangular pyramid of regular faces = $L^2\sqrt{3}$ where L is the length of edge.

(3) The volume of the triangular pyramid of regular faces = $\frac{\sqrt{2}}{12}L^3$ where L is the edge length.

Example 4

A regular quadrilateral pyramid the length of its base diagonal is $60\sqrt{2}$ cm. and its slant height is 50 cm. Find :

(1) The height of the pyramid.

(2) L.S.A and T.S.A of the pyramid.

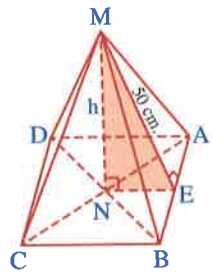
(3) Volume of the pyramid.

Solution

* Let MABCD is a regular quadrilateral pyramid , the diagonals of its base intersected at N

, the length of the base side = $\frac{60\sqrt{2}}{\sqrt{2}}$ = 60 cm.

, E is the midpoint of $\overline{AB}$



(1) $\because$ The quadrilateral pyramid is regular.

$\therefore$ Its base as a square shape.

, $\overline{MN} \perp$ the plane ABCD $\quad\therefore \overline{MN} \perp \overline{EN}$

$\therefore \Delta$ MEN is right angled at N ,

$\because$ E is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{AC}$

$\therefore$ EN = $\frac{1}{2}$ BC = 30 cm. $\quad\therefore h = \sqrt{50^2 - 30^2} = 40$ cm.

(2) The L.S.A of the pyramid = $\frac{1}{2}$ × base perimeter × slant height

$= \frac{1}{2} \times (60 \times 4) \times 50 = 6000 \text{ cm}^2.$

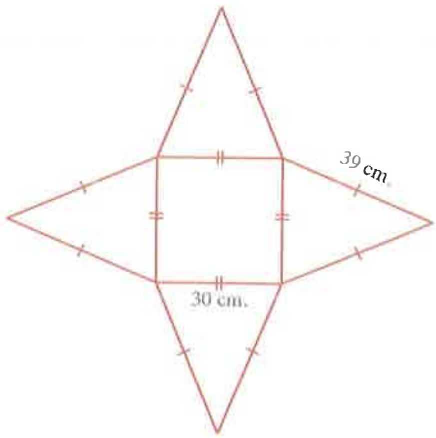
$\because$ The area of the base = 60 × 60 = 3600 cm².

$\therefore$ The T.S.A of the pyramid = L.S.A + the base area = 6000 + 3600 = 9600 cm².

(3) Volume of the pyramid = $\frac{1}{3}$ base area × height = $\frac{1}{3} \times 3600 \times 40 = 48000 \text{ cm}^3.$

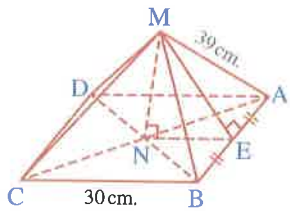
Example 5

Use the opposite net to describe the formed solid , then find its total surface area and its volume.



Solution

The net represents a regular quadrilateral pyramid , its base as a square of side length = 30 cm. and the length of its lateral edge = 39 cm.



, let the pyramid MABCD , N is the point of intersection of the diagonals of the base , E is the midpoint of $\overline{AB}$

, $\because$ the lateral face MAB is an isosceles triangle.

$\therefore$ $\overline{ME}$ is slant height.

, AE = 15 cm.

$\therefore$ Δ AEM which is right-angled at E :

$\therefore$ $ME = \sqrt{(MA)^2 - (AE)^2} = \sqrt{(39)^2 - (15)^2} = 36$ cm.

$\because$ MN $\perp$ the plane ABCD $\qquad \therefore \overline{MN} \perp \overline{EN}$

, $\because$ $EN = \frac{1}{2} BC = 15$ cm.

$\therefore$ In Δ MEN which is right-angled at N :

$MN = \sqrt{(ME)^2 - (EN)^2} = \sqrt{(36)^2 - (15)^2} = 3\sqrt{119}$ cm.

$\therefore$ The L.S.A. of the pyramid $= \frac{1}{2} \times$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (30 \times 4) \times 36 = 2160$ cm^2.

, area of the base $= 30 \times 30 = 900$ cm^2.

$\therefore$ The T.S.A. of the pyramid = L.S.A + the base area

$= 2160 + 900 = 3060$ cm^2.

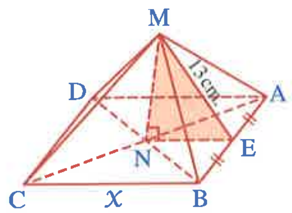
, $\because$ the volume of the pyramid $= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3} \times 900 \times 3\sqrt{119} = 900\sqrt{119}$ cm^3.

Example 6

MABCD is a regular quadrilateral pyramid , its total surface area = 360 cm^2. and its slant height = 13 cm. Find the length of its base edge , then find its volume.

Solution



Let the edge length of the squared base = x cm.

, $\because$ the T.S.A of the pyramid = 360 cm^2.

$\therefore$ The base area + L.S.A = 360

$\therefore x \times x + \frac{1}{2} \times 4x \times 13 = 360$

$\therefore x^2 + 26x - 360 = 0$

$\therefore (x + 36)(x - 10) = 0$

$\therefore x = -36$ (refused) or $x = 10$

$\therefore$ The edge length of the pyramid base = 10 cm.

$\because EN = \frac{1}{2} BC = 5$ cm.

, $\because \Delta$ MEN is right - angled at N

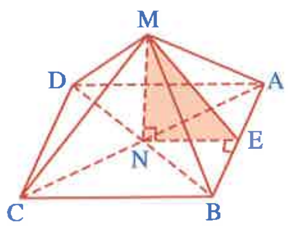
$\therefore MN = \sqrt{13^2 - 5^2} = 12$ cm.

$\therefore$ Volume of the pyramid = $\frac{1}{3}$ base area × height = $\frac{1}{3} \times (10)^2 \times 12 = 400$ cm^3.

Example 7

A regular quadrilateral pyramid of volume 48 cm^3. and the length of its base edge = 6 cm. , find its total surface area.

Solution



Let MABCD is a regular quadrilateral pyramid

, N is the intersection point of its base diagonal

, E is the midpoint of $\overline{AB}$

$\because$ Volume of the pyramid = 48 cm^3.

$\therefore \frac{1}{3}$ × the base area × the height = 48

$\therefore \frac{1}{3} \times 6 \times 6 \times h = 48$,

$\therefore h = 4$ cm.

$\therefore$ The height of the pyramid = MN = 4 cm.

, $\because EN = \frac{1}{2} BC = 3$ cm.,

$\because \Delta$ MEN is right - angled at N

$\therefore ME = \sqrt{4^2 + 3^2} = 5$ cm.

$\therefore$ The T.S.A of the pyramid = L.S.A. + the base area

$= \frac{1}{2}$ base perimeter × slant height + base area

$= \frac{1}{2} \times (4 \times 6) \times 5 + 6 \times 6 = 96$ cm^2.

Example 8

MABC is triangular pyramid of regular faces , the length of each edge of its edges equals $8\sqrt{3}$ cm. Find :

(1) The slant height of the pyramid.

(2) Height of the pyramid.

(3) T.S.A of the pyramid.

(4) Volume of the pyramid.

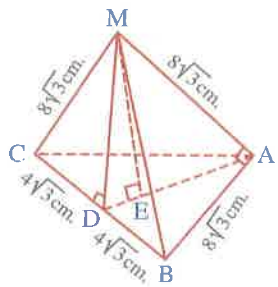
Solution

$\because$ The triangular pyramid is a regular faces.

$\therefore$ Each face is an equilateral triangle.

$\therefore$ The slant height of the pyramid

$= MD = AD = 8\sqrt{3} \sin 60° = 12$ cm.



, $\because$ E is the point of intersection of the medians of Δ ABC

$\therefore AE = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8$ cm.

, $\because \overline{ME} \perp$ the plane ABC

$\therefore \overline{ME} \perp \overline{AD}$

$\therefore \Delta$ MAE is right-angled at E

$\therefore ME = \sqrt{(8\sqrt{3})^2 - (8)^2} = 8\sqrt{2}$ cm.

$\therefore$ The height of the pyramid $= 8\sqrt{2}$ cm.

Notice that

$\because$ The triangular pyramid is a regular faces.

$\therefore 2 L^2 = 3 h^2$

$\therefore 2 \times (8\sqrt{3})^2 = 3 h^2$

$\therefore h = 8\sqrt{2}$

$\therefore$ Height of the pyramid $= 8\sqrt{2}$ cm.

, $\because$ L.S.A. of the pyramid

$= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (3 \times 8\sqrt{3}) \times 12 = 144\sqrt{3}$ cm^2.

$\because$ Area of the base $= \frac{1}{2} \times 8\sqrt{3} \times 8\sqrt{3} \sin 60° = 48\sqrt{3}$ cm^2.

$\therefore$ T.S.A. of the pyramid $= 144\sqrt{3} + 48\sqrt{3} = 192\sqrt{3}$ cm^2.

, $\because$ volume of the pyramid $= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3} \times 48\sqrt{3} \times 8\sqrt{2} = 128\sqrt{6}$ cm^3.

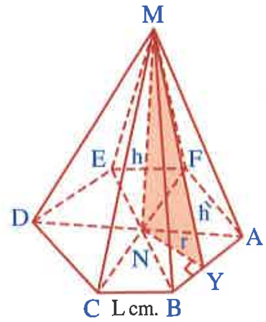
Example 9

A regular hexagonal pyramid in which the sum of areas of its lateral faces is seven times its base area.

Prove that : The volume of the pyramid $= 8 r^3$

Where (r) is the radius of the inscribed circle of the base.

Solution



Let the edge length of the base of the pyramid = L cm.

, height of the pyramid = h , and its slant height = $\grave{h}$

$\because$ The sum of areas of its lateral faces = 7 × base area

$\therefore \frac{1}{2}$ × base perimeter × slant height = 7 × base area

$\therefore \frac{1}{2} \times 6\,L \times \grave{h} = 7 \times \frac{1}{2} \times L \times r \times 6$

$\therefore 3\,L\,\grave{h} = 21\,Lr \qquad \therefore \boxed{\grave{h} = 7\,r}$

, $\because \overline{MN} \perp$ the plane ABCDEF

$\therefore \overline{MN} \perp \overline{NY}$

$\therefore \Delta$ MNY is right-angled at N

$\therefore h = \sqrt{\grave{h}^2 - r^2} = \sqrt{(7r)^2 - r^2} = 4\sqrt{3}\,r$

, $\because$ volume of the pyramid = $\frac{1}{3}$ × base area × height

$= \frac{1}{3} \times \frac{1}{2} \times L \times r \times 6 \times 4\sqrt{3}\,r = 4\sqrt{3}\,L\,r^2$

, $\because r = L \sin 60°$ $\qquad \therefore L = \frac{2}{\sqrt{3}}\,r$

$\therefore$ Volume of the pyramid = $4\sqrt{3} \times \frac{2}{\sqrt{3}}\,r \times r^2 = 8\,r^3$

Example 10

MABC is a triangular pyramid its vertex M is at distance $4\sqrt{5}$ cm. from the base ABC where AB = 7 cm. , BC = 8 cm. , AC = 9 cm. Find the volume of the pyramid.

Solution

> **Remember that**
>
> The area of $\Delta ABC = \sqrt{S\,(S-AB)(S-BC)(S-AC)}$
> where : S is half the perimeter of Δ ABC

$\because$ The perimeter of Δ ABC

= 7 + 8 + 9 = 24 cm.

$\therefore$ Half the perimeter = 12 cm.

$\therefore$ The area of Δ ABC

$= \sqrt{12\,(12-7)(12-8)(12-9)}$

$= 12\sqrt{5}$ cm².

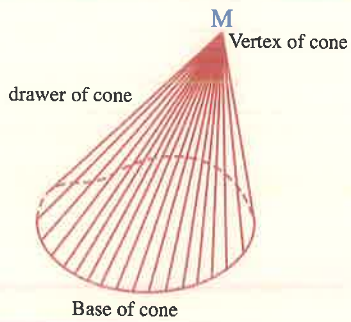
$\therefore$ The volume of the pyramid = $\frac{1}{3}$ × base area × height

$= \frac{1}{3} \times 12\sqrt{5} \times 4\sqrt{5} = 80$ cm³.

Lesson Three — The cone

The definition of the cone

Is a solid has only one base as a closed curve and one vertex , and its lateral surface formed from line segments drawn from its vertex to its curved base and each of them is called drawer of the cone.

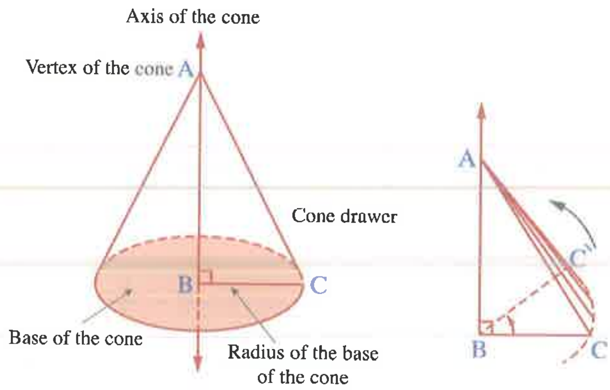


The right circular cone

Is the solid formed from the rotation of a right-angled triangle with complete rotation about one of its right sides as an axis or is the space formed from the folding of a circular sector where their two radii coincide on each other.

In the opposite figure :

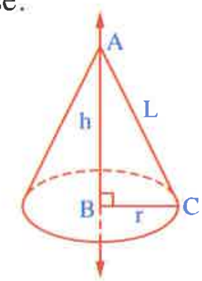
Δ ABC is right angled triangle at B , if it is rotated about the axis $\overleftrightarrow{AB}$ with a full turn , the formed solid is called right circular cone , and the point A is called vertex of the cone , $\overline{AC}$ is the drawer of the cone , $\overleftrightarrow{AB}$ is axis of the cone , surface of circle B is the base of the cone.



Properties of the right circular cone :

(1) The axis of the right circular cone is perpendicular to the plane of the base.

i.e. $\overleftrightarrow{AB} \perp$ the plane of circle B



(2) The height of the right circular cone is the length of the line segment joining between the vertex of the cone and the centre of its base and its length is always less than the length of the drawer of the cone.

If the length of $\overline{AB}$ = h length unit , the length of $\overline{AC}$ = L length unit.

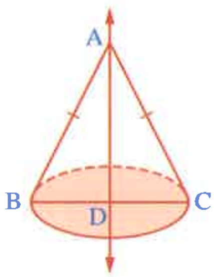
Then the height of the cone (h) = $\sqrt{L^2 - r^2}$, then : $h < L$

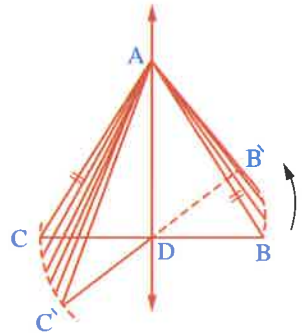
Remark

The right circular cone can be formed by the rotation of an isosceles triangle about its axis of symmetry by a half turn.

In the opposite figure :

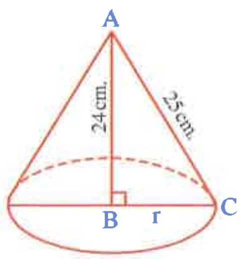
If Δ ABC is isosceles in which AB = AC , $\overleftrightarrow{AD}$ is the axis of symmetry of Δ ABC and the triangle ABC is rotated about $\overleftrightarrow{AD}$ by a half turn , then the formed solid is a right circular cone its base is the circle D , and its drawer is $\overline{AB}$ or $\overline{AC}$, its height is $\overline{AD}$ and its vertex is the point A





Example 1

A right circular cone , the length of its drawer is 25 cm. and its height is 24 cm. Find the perimeter and the area of the base of the cone. $\left(\pi = \frac{22}{7}\right)$



Solution

$\because \overleftrightarrow{AB} \perp$ the plane of circle B $\qquad \therefore \overleftrightarrow{AB} \perp \overline{BC}$

$\therefore m(\angle ABC) = 90°$ $\qquad \therefore (BC)^2 = (AC)^2 - (AB)^2 = (25)^2 - (24)^2 = 49$

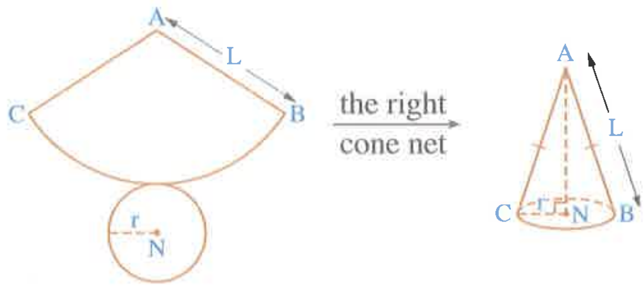
$\therefore BC = 7$ cm. $\qquad \therefore$ r (the radius of the base) = 7 cm.

$\therefore$ The perimeter of the base $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

, the area of the base $= \pi r^2 = \frac{22}{7} \times 49 = 154$ cm^2

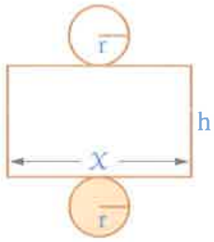
UNIT
2

The right cone net :



Remember

One of the right circular cylinder nets



From the net of the right cone we note that :

(1) AB = AC = L , where L is the length of drawer of the cone.

(2) The circular sector ABC represents the lateral surface of the cone and the length of $\overset{\frown}{BC}$ = perimeter of the circle N = $2 \pi r$

(3) Surface of the circle N represents the base of the cone.

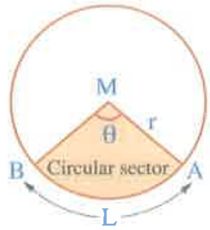
Remember that

The circular sector is a part of the surface of a circle bounded by two radii and an arc of the circle.

* Area of the circular sector = $\frac{1}{2}$ L r

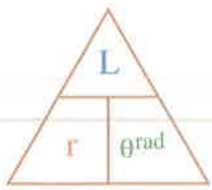
(where L is the length of the arc of the sector)

* Area of the circular sector = $\frac{1}{2}\, \theta^{rad}\, r^2$ (where θ^{rad} is the radian measure of the sector angle)



* Area of the circular sector = $\frac{x^\circ}{360^\circ} \times \pi r^2 = \frac{x^\circ}{360^\circ} \times$ area of the circle

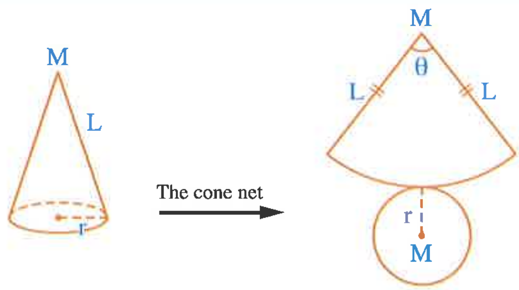
(where x° is degree measure of the sector angle)



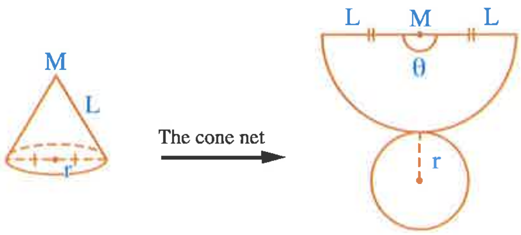
* Perimeter of the sector = 2 r + L length unit.

Important remarks

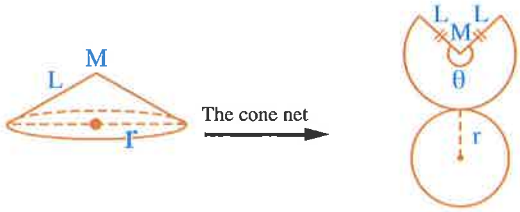
(1) If $L > 2r$, then the cone net as shown , $0° < \theta < 180°$



(2) If $L = 2r$, then the cone net as shown , $\theta = 180°$



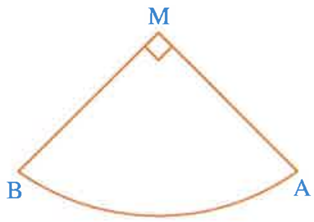
(3) If $L < 2r$, then the cone net as shown , $180° < \theta < 360°$



Example 2

In the opposite figure :

A piece of paper as a circular sector , the area of its surface $= 25\pi\ cm^2$.
and the measure of its central angle equals 90° folded to touch $\overline{MA}$ and $\overline{MB}$ and formed a cone. Find the height of the cone to nearest tenth.



Solution

$\because$ Area of the sector $= \frac{1}{2}\theta^{rad} r^2$

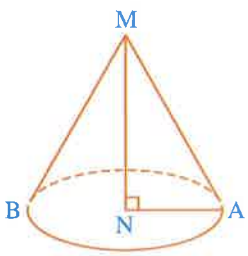
$\therefore \frac{1}{2}\theta^{rad} r^2 = 25\pi$

$\therefore \frac{1}{2} \times \frac{\pi}{2} \times r^2 = 25\pi$ $\qquad \therefore r^2 = 100$

$\therefore r = 10$ cm. $\qquad \therefore MA = 10$ cm.

, $\because$ area of the sector $= \frac{1}{2} L r$

$\therefore \frac{1}{2} \times L \times 10 = 25\pi$ $\qquad \therefore L = 5\pi$ cm.



$\therefore$ The length of $\overset{\frown}{AB} = 5\pi$ cm.

$\therefore$ The circumference of the circle N $= 5\pi$ cm.

$\therefore 2\pi r_N = 5\pi$ $\qquad\qquad$ $\therefore r_N = 2.5$ cm.

$\therefore \Delta$ ANM in which m ($\angle$ ANM) $= 90°$, NA $= 2.5$ cm.

, MA $= 10$ cm.

$\therefore$ MN $= \sqrt{(MA)^2 - (NA)^2} = \sqrt{(10)^2 - (2.5)^2} \simeq 9.7$ cm.

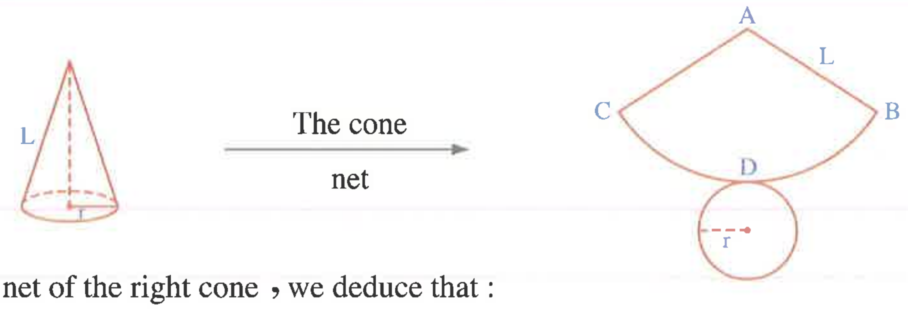
$\therefore$ Height of the cone $= 9.7$ cm.

The lateral area - total area - volume of a right cone :

If (r) is the radius of the cone base , (L) is the cone drawer, (h) is the height, then :

- The lateral surface area (L.S.A.) of the right cone $= \pi L r$
- The total surface area (T.S.A.) of the right cone $= \pi r (L + r)$
- Volume of the right cone $= \frac{1}{3} \pi r^2 h$

Finding the lateral surface area and total surface area of the right cone



From the net of the right cone , we deduce that :

The lateral surface area of the right cone = the area of sector ABC

$= \frac{1}{2} \times$ length of $\overset{\frown}{BC} \times AB$

$= \frac{1}{2} \times$ perimeter of the cone base $\times AB$

$= \frac{1}{2} \times 2\pi r \times L$

$= \pi L r$

$\therefore$ The lateral surface area of the right cone $= \pi L r$

, $\because$ the total surface area of the right cone = the lateral surface area + the base area

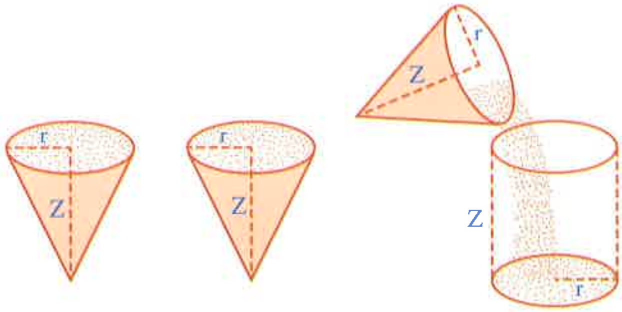
$= \pi L r + \pi r^2$

$\therefore$ The total surface area of the right cone $= \pi r (L + r)$

Finding the volume of the right cone

Experiment activity

* Bring a hollow vessel as a right circular cylinder and another one as a right circular cone where be their bases are congruent and they have the same height as in the opposite figure.



* Fill the cone vessel with grains of rice or sand then empties it in the cylinder vessel.
* Repeat this process three times and you will note that : the cylinder will filled completely with the grains and that means :

The volume of the cone = $\frac{1}{3}$ volume of the cylinder has the same base and height

, $\because$ the volume of the cylinder = base area × height

$\therefore$ The volume of the right cone = $\frac{1}{3}$ base area × height

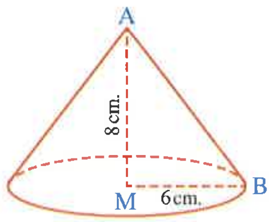
$= \frac{1}{3} \pi r^2 h$

Example 3

A right circular cone , the length of its base diameter is 12 cm. and its height is 8 cm. , find :

(1) The L.S.A. (2) The T.S.A. (3) The volume.

Solution



$\because \overline{AM} \perp$ the circle plane

$\therefore \overline{AM} \perp \overline{MB}$

$\therefore \Delta$ MAB is right-angled at M

, $\because r = \frac{12}{2} = 6$ cm.

$\therefore AB = \sqrt{(8)^2 + (6)^2} = 10$ cm. $\therefore L = 10$ cm.

$\therefore$ The lateral surface area (L.S.A.) $= \pi r L = \pi \times 6 \times 10 = 60 \pi$ cm^2

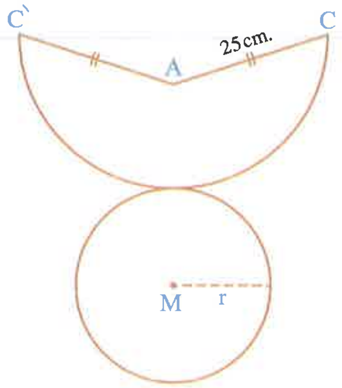
, area of the base $= \pi r^2 = 36 \pi$ cm^2

$\therefore$ The total surface area (T.S.A) $= 60 \pi + 36 \pi = 96 \pi$ cm^2

, volume of the cone $= \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 36 \times 8 = 96 \pi$ cm^3

Example 4

Use the opposite net to describe the formed solid and if the length of the arc $\overset{\frown}{CC'} = 30\pi$ cm. Find the volume of this solid and its total surface area.

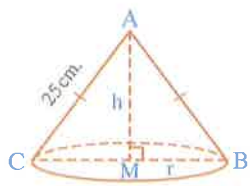


Solution

The net represents a right cone

, $\because$ the length of $\overset{\frown}{CC'} = 30\pi$

$\therefore 2\pi r = 30\pi$ $\therefore r = 15$ cm.



, $\because \Delta ABM$ is right-angled at M

$\therefore h = \sqrt{25^2 - 15^2} = 20$ cm.

$\therefore$ The volume $= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3} \times \pi \times (15)^2 \times 20 = 1500\pi$ cm^3.

, the total surface area $= \pi r (L + r) = \pi \times 15 \times (15 + 25)$

$= 600\pi$ cm^2.

Example 5

A flask in the shape of a cone of capacity 6.16 litres. and height 30 cm. Find the length of the radius of its base. $\left(\pi \simeq \frac{22}{7}\right)$

Solution

$\because$ The capacity of the flask = 6.16 litres.

$\therefore$ Volume of the right cone $= 6.16 \times 1000$ cm^3.

$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 30 = 6160$

$\therefore r = 14$ cm.

> **Remember that**
> 1 litre = 1000 millilitre
> = 1000 cm^3. = 1 dm^3.

Example 6

A pure gold alloy as a right cone of radius length 3 cm. and lateral surface area = 15 π cm². Find the gold density if the mass of the alloy = 727 gm. "π = 3.14"

Solution

$\because$ The L.S.A. of the cone = 15π

$\therefore \pi L r = 15\pi$

$\therefore \pi \times L \times 3 = 15\pi$

$\therefore L = 5$ cm.

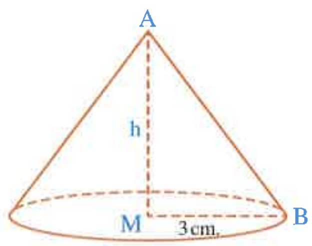
$\because \Delta ABM$ is right-angled at M

$\therefore h = AM = \sqrt{5^2 - 3^2} = 4$ cm.

$\because$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi = 37.68$ cm³.

$\therefore$ The density $= \frac{\text{mass}}{\text{volume}} = \frac{727}{37.68} \simeq 19.3$ gm./cm³.



Remember that

The density $= \frac{\text{mass}}{\text{volume}}$

Example 7

A regular octagonal pyramid of silver , its base side length is 6 cm. and its height 30 cm. , melted and convert into a circular right cone whose length of its base radius is 9 cm. if 10% of the silver was missed through the melting process. Find the height of the cone to nearest one decimal place.

Solution

$\because$ The area of the regular octagon $= \frac{8}{4} x^2 \cot \frac{\pi}{8}$

$= 2 \times (6)^2 \cot 22° 30' \simeq 173.82$ cm².

$\therefore$ Volume of the pyramid $= \frac{1}{3}$ base area × height

$= \frac{1}{3} \times 173.82 \times 30 = 1738.2$ cm³.

$\therefore$ Volume of the silver in the cone

$= \frac{90}{100} \times 1738.2 \simeq 1564.4$ cm³.

$\therefore \frac{1}{3}\pi \times (9)^2 \times h \simeq 1564.4$

$\therefore h \simeq 18.4$ cm.

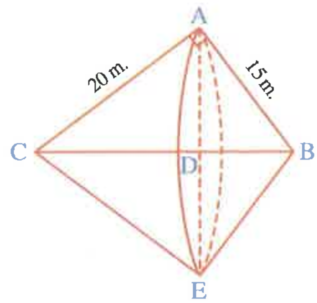
Remember that

Area of the regular polygon whose number of sides = n , and the length of its side x equals $\frac{n}{4} x^2 \cot \frac{\pi}{n}$

Example 8

Δ ABC is right-angled at A , AB = 15 m. , AC = 20 m. , if the triangle is rotated a complete rotation around $\overline{BC}$, describe the formed solid , then find the cost of painting this solid with a material resistant to erosion , if the cost of one square metre = 10 pounds and find the volume of this solid. " $\pi = \frac{22}{7}$ "

Solution



The solid will be as a two right cones with the same base.

From the figure :

Δ ABC is right angled at A , $\overline{AD} \perp \overline{BC}$

$\therefore$ BC $= \sqrt{(15)^2 + (20)^2} = 25$ metres.

, AD $= \frac{15 \times 20}{25} = 12$ metres.

, BD $= \sqrt{(15)^2 - (12)^2} = 9$ metres.

, CD $= 25 - 9 = 16$ metres.

According to the first cone whose vertex is B :

L = 15 m. , r = 12 m. , h = 9 m.

$\therefore$ The L.S.A. $= \pi L r = \pi \times 15 \times 12 = 180 \pi$ m^2.

, the volume $= \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times (12)^2 \times 9 = 432 \pi$ m^3.

According to the second cone whose vertex is C :

L = 20 m. , r = 12 m. , h = 16 m.

$\therefore$ The L.S.A. $= \pi L r = \pi \times 20 \times 12 = 240 \pi$ m^2.

, the volume $= \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times (12)^2 \times 16 = 768 \pi$ m^3.

$\therefore$ The total painting area = the sum of the lateral surface areas of the two cones

$= 180 \pi + 240 \pi = 420 \pi = 420 \times \frac{22}{7} = 1320$ m^2.

$\therefore$ The cost of the painting $= 1320 \times 10 = 13200$ L.E.

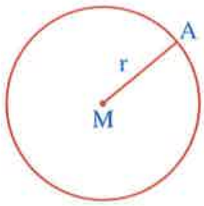
, volume of the solid = sum of volumes of the two cones $= 432 \pi + 768 \pi = 1200 \pi$

$= 1200 \times \frac{22}{7} = 3771 \frac{3}{7}$ m^3.

Lesson Four — The circle

Definition of the circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

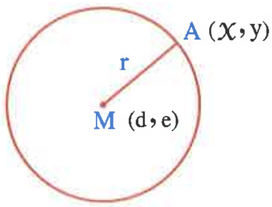


- The fixed point is called "the centre of the circle". (M)
- The constant distance is called "the radius length of the circle". (r)
- The circle is usually denoted by (C)

, where $C = \{A : MA = r , r > 0\}$

First Equation of the circle (In terms of its centre coordinates and radius length)

If $A = (x , y)$ is a point on the circle whose centre $M (d , e)$, and the length of its radius $= r$, in the perpendicular cartesian coordinates using "the distance between two points" rule we find :



$$\sqrt{(x-d)^2+(y-e)^2} = r$$

i.e. $(x-d)^2 + (y-e)^2 = r^2$ "Equation of the circle"

UNIT 2

Remarks

(1) If the centre of the circle is the origin point (0 , 0) , then the equation of the circle is :

$$x^2 + y^2 = r^2$$

(2) The position of the point (x_1 , y_1) in respect to the circle

$C : (x - d)^2 + (y - e)^2 = r^2$

* If $(x_1 - d)^2 + (y_1 - e)^2 = r^2$, then the point lies on the circle.
* If $(x_1 - d)^2 + (y_1 - e)^2 > r^2$, then the point lies outside the circle
* If $(x_1 - d)^2 + (y_1 - e)^2 < r^2$, then the point lies inside the circle

(3) Two circles are congruent if the lengths of their radii are equal.

For example : If the circle equation of C_1 is : $x^2 + y^2 = 49$

, the circle equation of C_2 is $(x - 3)^2 + (y - 4)^2 = 49$

, then $r_1 = r_2 = \sqrt{49} = 7$ length unit, then the two circles are congruent, and the circle C_2 is the image of the circle C_1 by translation (3 , 4)

Where the image of point (x , y) by translation (a , b) is : $(x + a , y + b)$

Second The general form of the circle equation

The general form of the circle equation is :

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

Where the centre $(M) = (-L , -K) = (-\frac{1}{2}$ coefficient of x , $-\frac{1}{2}$ coefficient of y)

, $r = \sqrt{L^2 + K^2 - C}$, $L^2 + K^2 - C > 0$

For example : The circle whose equation is : $x^2 + y^2 + 8x - 4y - 16 = 0$

its centre = $(-\frac{1}{2}$ coefficient of x , $-\frac{1}{2}$ coefficient of y) = (– 4 , 2)

, $r = \sqrt{L^2 + k^2 - c} = \sqrt{16 + 4 - (-16)} = 6$ length unit.

• We can deduce the general form of the circle equation as follows :

We know that : The circle whose centre (d , e) , the length of its radius $= r$ is :

$(x - d)^2 + (y - e)^2 = r^2$

i.e. $x^2 + y^2 - 2dx - 2ey + d^2 + e^2 - r^2 = 0$

Let $M(D , K) = (-L , -K)$ $\therefore x^2 + y^2 + 2Lx + 2Ky + L^2 + K^2 - r^2 = 0$

, $\because$ L , K and r are constants. $\therefore L^2 + K^2 - r^2 = C$ (constant)

$\therefore$ The general form of the circle equation is :

$$\boxed{x^2 + y^2 + 2Lx + 2Ky + C = 0}$$

Remarks

(1) The general form of the circle equation $x^2 + y^2 + 2Lx + 2Ky + C = 0$ is :

* An equation of 2^{nd} degree in x , y
* Free of the term xy *i.e.* Coefficient of xy = zero
* Coefficient of x^2 = coefficient of $y^2 = 1$

(2) To be the equation of the 2^{nd} degree in x , y represents a circle , it must satisfy the three conditions in the previous remark and $L^2 + K^2 - C > 0$

(3) To identify the centre or the radius length of a circle using the general form must be the coefficient of x^2 = the coefficient of $y^2 = 1$, so we need to divide by this coefficient if is not equals 1

Special cases

1 Equation of the circle passing through the origin point :

$\boxed{x^2 + y^2 + 2Lx + 2Ky = 0}$ The equation is free of the absolute term *i.e.* $(C = 0)$

2 Equation of the circle whose centre lies on x-axis :

$\boxed{x^2 + y^2 + 2Lx + C = 0}$ The equation is free of the term containing y *i.e.* $(K = 0)$

3 Equation of the circle whose centre lies on y-axis :

$\boxed{x^2 + y^2 + 2Ky + C = 0}$ The equation is free of the term containing x *i.e.* $(L = 0)$

4 Equation of the circle touching x-axis :

If the circle whose centre $(-L , -K)$

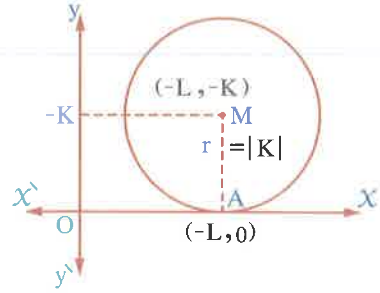
touches x-axis then :

the point of tangency A is : $(-L , 0)$ and $r = |K|$

$\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - K^2 = L^2$

Then the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + L^2 = 0$$



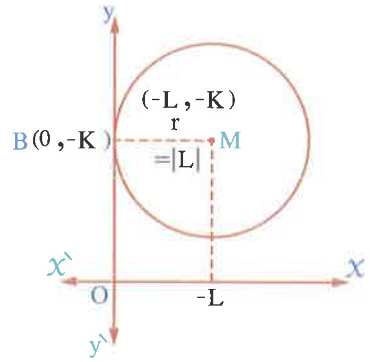
5 Equation of the circle touching y-axis :

If the circle whose centre $(-L , -K)$ touches y-axis , then the point of tangency B is $(0 , -K)$ and $r = |L|$

$\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - L^2 = K^2$

, then the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + K^2 = 0$$



6 Equation of the circle touching the two coordinates :

If the circle whose centre is $(-L , -K)$ touches the two coordinates axes then $r = |L| = |K|$

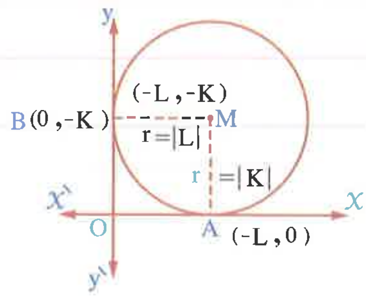
$\therefore C = L^2 + K^2 - r^2 = r^2 + r^2 - r^2 = r^2$

$$\therefore \quad C = L^2 = K^2 = r^2$$

and the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

where $|L| = |K| = r$, $C = L^2 = K^2 = r^2$



Remember that :

(1) The position of a straight line with respect to a circle (D) whose centre (M) and let $\overrightarrow{MC} \perp L$ and intersects it at C

* If $MC < r$, then L is a secant to the circle at two points.
* If $MC = r$, then L is a tangent to the circle.
* If $MC > r$, then L is outside the circle and doesn't intersect it at any point.

(2) If M , N are two circles of radii r_1 , r_2 respectively (where $r_1 > r_2$)

If the two circles M and N	Then
(1) Distant	$MN > r_1 + r_2$
(2) Touching externally	$MN = r_1 + r_2$
(3) Intersecting	$r_1 - r_2 < MN < r_1 + r_2$
(4) Touching internally	$MN = r_1 - r_2$
(5) One inside the other	$MN < r_1 - r_2$
(6) Concentric	MN = zero

(3) The tangent to a circle is perpendicular to the radius drawn from the point of tangency.

(4) The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

(5) The two tangent-segments drawn to a circle from a point outside it are equal in length.

(6) If $A = (x_1 , y_1)$, $B = (x_2 , y_2)$ then the midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$

(7) The equation of the straight line passing through (x_1 , y_1)

and its slope (m) is : $\boxed{\frac{y - y_1}{x - x_1} = m}$

(8) The length of the perpendicular from the point (x_1 , y_1) on the straight line whose equation : $a x + b y + C = 0$ equals $\frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$

Example 1

Find the general form of the equation of the circle whose centre is (– 2 , 3) and its radius length is 5 length units.

Solution

The equation of the circle is : $(x + 2)^2 + (y - 3)^2 = (5)^2$

$\therefore x^2 + y^2 + 4x - 6y - 12 = 0$ "After simplify"

Another solution :

$\because L = 2$, $K = -3$, $C = L^2 + K^2 - r^2 = (2)^2 + (-3)^2 - (5)^2 = -12$

$\therefore$ The general form of the equation of the circle is $x^2 + y^2 + 4x - 6y - 12 = 0$

"The same form we obtained before"

Example 2

Find the equation of the circle whose centre is the origin point and its diameter length = $6\sqrt{2}$ length unit , then prove that the circle passing through the point $(\sqrt{2}, -4)$

Solution

The equation of the circle is : $x^2 + y^2 = \left(3\sqrt{2}\right)^2$

, then $x^2 + y^2 = 18$

by substitute by the point $\left(\sqrt{2}, -4\right)$

$\therefore$ L.H.S. $= \left(\sqrt{2}\right)^2 + (-4)^2 = 18 =$ R.H.S.

$\therefore$ The point $\left(\sqrt{2}, -4\right) \in$ the circle.

Example 3

Find the equation of the circle whose centre $M = (3, -2)$ and passing through the point $A = (-1, 1)$

Solution

$r = MA = \sqrt{(3+1)^2 + (-2-1)^2} = 5$ length unit.

$\therefore$ The equation of the circle is : $(x-3)^2 + (y+2)^2 = 25$

Example 4

Find the equation of the circle whose diameter $\overline{AB}$ where $A = (4, -1)$, $B = (-2, 1)$

Solution

$\because$ The centre of the circle M is the midpoint of $\overline{AB}$

$\therefore M = \left(\frac{4-2}{2}, \frac{-1+1}{2}\right) = (1, 0)$

, $\because r = MA = \sqrt{(4-1)^2 + (-1-0)^2} = \sqrt{10}$ length unit.

$\therefore$ The equation of the circle is : $(x-1)^2 + (y-0)^2 = (\sqrt{10})^2$

, then : $(x-1)^2 + y^2 = 10$

Example 5

Find the centre and the length of the radius for each of the following circles :

(1) $x^2 + y^2 - 2x + 4y - 4 = 0$

(2) $x^2 + y^2 - 4y - 9 = 0$

(3) $7x^2 + 7y^2 + 42x - 14y + 28 = 0$

Solution

(1) $\because x^2 + y^2 - 2x + 4y - 4 = 0$ $\therefore L = -1, K = 2, C = -4$

$\therefore$ The centre $= (-L, -K) = (1, -2)$

$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(-1)^2 + (2)^2 - (-4)} = 3$ length unit.

(2) $\because x^2 + y^2 - 4y - 9 = 0$

$\therefore L =$ zero , $K = -2$, $C = -9$

$\therefore$ The centre $= (-L, -K) = (0, 2)$

$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(0)^2 + (-2)^2 - (-9)} = \sqrt{13}$ length unit.

Notice that

$L = 0$ because the coefficient of $x = 0$

(3) By dividing by 7 to make the coefficient of x^2 = the coefficient of $y^2 = 1$

$\therefore$ The equation will be : $x^2 + y^2 + 6x - 2y + 4 = 0$

$\therefore L = 3$, $k = -1$, $C = 4$

$\therefore$ The centre $= (-L, -K) = (-3, 1)$

$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(3)^2 + (-1)^2 - 4} = \sqrt{6}$ length unit.

Example 6

Find the equation of the circle whose centre $(3, -4)$ and touches x-axis

Solution

$\because L = -3$, $K = 4$, $\because$ the circle touches x-axis

$\therefore r = |K|$, $C = L^2$

$\therefore r = 4$ length unit , $C = 9$ "We can find C using the relation : $C = L^2 + K^2 - r^2$"

$\therefore$ Equation of the circle is : $x^2 + y^2 - 6x + 8y + 9 = 0$

Example 7

Find the equation of the circle whose the length of its radius is 5 units and touches y-axis at the point $(0, 3)$

Solution

$\because$ The circle touches y-axis at the point $(0, 3)$

$\therefore$ The centre $= (-L, 3)$, $r = |L|$ length unit. *i.e.* $|L| = 5$

$\therefore L = \pm 5$, $C = K^2 = 9$

$\therefore$ There are two circles one of them has a centre $(-5, 3)$ and its equation :

$x^2 + y^2 + 10x - 6y + 9 = 0$ and the other has a centre $(5, 3)$ and its equation :

$x^2 + y^2 - 10x - 6y + 9 = 0$

Example 8

Find the equation of the circle which touches the two coordinate axes , and its centre is the point $(-4, 4)$

Solution

$\because$ The circle touches the two coordinate axes

$\therefore C = L^2 = K^2 = 16$

$\therefore$ The equation is : $x^2 + y^2 + 8x - 8y + 16 = 0$

Example 9

Determine which of the following equations represent a circle :

(1) $x^2 + 3y^2 - 2x + 4y + 5 = 0$

(2) $2x^2 - xy + 2y^2 + 5x - y - 2 = 0$

(3) $x^2 + y^2 + 7x - y + 8 = 0$

(4) $2x^2 + 2y^2 - 6x + 4y + 9 = 0$

(5) $x^2 + y^2 - 16x + 12y + 100 = 0$

(6) $x^2 + 6x - 8y - 7 = 0$

Solution

(1) $\because$ The coefficient of $x^2 \neq$ the coefficient of y^2

$\therefore$ The equation doesn't represent a circle.

(2) $\because$ The equation has a term contains xy

$\therefore$ The equation doesn't represent a circle.

(3) The coefficient of x^2 = the coefficient of y^2 and the equation is free of a term contains xy

$\therefore$ The equation may represent a circle

$\because 2L = 7, 2K = -1$, $\therefore L = \frac{7}{2}, K = \frac{-1}{2} = , C = 8$

$\therefore L^2 + K^2 - C = \frac{49}{4} + \frac{1}{4} - 8 = \frac{9}{2} > 0$

$\because$ The equation is represent a circle whose centre $\left(\frac{-7}{2}, \frac{1}{2}\right)$

, $r = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$ length unit.

(4) $\because$ The coefficient of x^2 = The coefficient of y^2 and the equation is free of a term contains $x\,y$

$\therefore$ The equation may represent a circle.

Multiply by $\frac{1}{2}$ to make the coefficient of x^2 = the coefficient of $y^2 = 1$

$\therefore x^2 + y^2 - 3x + 2y + \frac{9}{2} = 0$

$\because 2L = -3 \quad , \quad 2k = 2$

$\therefore L = \frac{-3}{2} \quad , \quad k = 1 \quad , \quad c = \frac{9}{2}$

$\therefore L^2 + K^2 - C = \frac{9}{4} + 1 - \frac{9}{2} = \frac{-5}{4} < 0$

$\therefore$ The equation doesn't represent a circle.

(5) $\because$ The coefficient of x^2 = the coefficient of y^2 , and the equation is free of a term contains $x\,y$

$\therefore$ The equation may represent a circle.

$\therefore 2L = -16 \quad , \quad 2k = 12 \qquad \therefore L = -8 \quad , \quad k = 6 \quad , \quad c = 100$

$\therefore L^2 + K^2 - C = 64 + 36 - 100 = \text{zero}$

$\therefore$ The equation doesn't represent a circle.

(6) $\because$ The equation is free of the term y^2

$\therefore$ The equation doesn't represent a circle.

Example 10

Find the equation of the circle whose radius length = 3 units , and the two equations of its diameters are $x + y = 2$, $2x - y = 7$

Solution

The centre of the circle is the point of intersection of the two straight lines :

$x + y = 2 \quad (1) \qquad , \qquad 2x - y = 7 \quad (2)$

by adding $\quad \therefore 3x = 9 \quad \therefore x = 3$

by substitute $\quad \therefore y = -1$

$\therefore$ The centre is the point $(3 , -1)$

$\therefore L = -3 \quad , \quad K = 1 \quad , \quad C = L^2 + K^2 - r^2 = 9 + 1 - 9 = 1$

$\therefore$ Equation of the circle is : $x^2 + y^2 - 6x + 2y + 1 = 0$

Example 11

A circle whose centre M $(-2 , 7)$, and the length of its radius r = 5 units , state which of the following points lies on the circle , inside the circle and outside the circle.

$A = (-1 , 3) \quad , \quad B = (0 , -5) \quad , \quad C = (2 , 4)$

Solution

$\because$ Equation of the circle is : $(x+2)^2+(y-7)^2=25$

by substitute by the points A , B and C in the L.H.S. of the equation :

$\because (-1+2)^2+(3-7)^2=17<r^2$ $\therefore$ Point A $(-1, 3)$ lies inside the circle.

, $\because (0+2)^2+(-5-7)^2=148>r^2$ $\therefore$ Point B $(0, -5)$ lies outside the circle.

, $\because (2+2)^2+(4-7)^2=25=r^2$ $\therefore$ Point C $(2, 4)$ lies on the circle.

Example 12

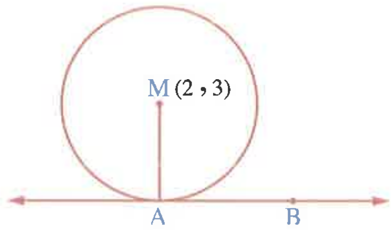
Find the equation of the circle whose centre $M=(2, 3)$ and the straight line $3x+4y+2=0$ is a tangent at the point A

Solution

$\because \overline{MA}$ is a radius , $\overleftrightarrow{AB}$ is tangent to the circle.

$\therefore \overline{MA} \perp AB$

$\therefore MA = \dfrac{|3\times 2+4\times 3+2|}{\sqrt{3^2+4^2}} = 4$ length unit.



$\therefore r=4$ length unit.

$\therefore$ Equation of the circle is :

$(x-2)^2+(y-3)^2=16$

Example 13

Determine the position of the circle $C_1 : (x-3)^2+(y-2)^2=4$ with respect to the circle $C_2 : x^2+y^2+2x+2y+1=0$

Solution

$\because C_1 : (x-3)^2+(y-2)^2=4$

$\therefore$ The centre $M_1=(3, 2)$, $r_1=\sqrt{4}=2$ length unit.

, $C_2 : x^2+y^2+2x+2y+1=0$

The centre $M_2=(-1, -1)$, $r_2=\sqrt{1+1-1}=1$ length unit.

$\therefore r_1+r_2=2+1=3$ length unit.

, $M_1M_2=\sqrt{(3+1)^2+(2+1)^2}=5$ length unit.

$\therefore M_1M_2>r_1+r_2$ $\therefore$ The two circles are distant.

Example 14

In the opposite figure :

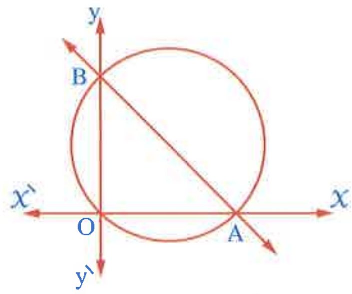
If the equation of $\overleftrightarrow{AB}$

is : $6x + 8y - 48 = 0$ intersects.

The two coordinate axes at A and B ,

Find the equation of the circle passing

through the points A , O and B



Solution

$\because m(\angle AOB) = 90°$

$\therefore \overline{AB}$ is a diameter of the circle $\because$ The equation of $\overleftrightarrow{AB}$ is $6x + 8y = 48$

i.e. $\frac{x}{8} + \frac{y}{6} = 1$

$\therefore$ The straight line cuts x-axis at the point $A = (8 , 0)$

, cuts y-axis at the point $B = (0 , 6)$

Let M be the centre of the circle.

$\therefore$ M is the midpoint of $\overline{AB} = \left(\frac{8+0}{2} , \frac{0+6}{2}\right) = (4 , 3)$

, $AB = \sqrt{8^2 + 6^2} = 10$ length unit.

$\therefore r = 5$ length unit.

$\therefore$ Equation of the circle is $(x - 4)^2 + (y - 3)^2 = 25$

Example 15

Find the area of the equilateral triangle whose vertices passing through the circle :
$x^2 + y^2 + 6x - 2y - 15 = 0$
"Where each unit in the plane represent 4 cm."

Solution

$L = 3 , K = -1 , C = -15$

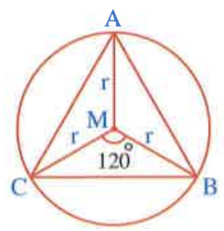
$\therefore r = \sqrt{L^2 + k^2 - C} = \sqrt{9 + 1 + 15} = 5$ length unit , M is the centre of the circumcircle of ΔABC

If ΔABC is equilateral by drawing

$\overline{MA} , \overline{MB} , \overline{MC}$, then :

$m(\angle BMC) = \frac{360°}{3} = 120°$, then :

Area of $\Delta ABC = 3 \times$ area of ΔMBC



$= 3 \times \frac{1}{2}$ MB $\times$ MC $\times \sin(\angle$ BMC)

$= \frac{3}{2} r^2 \sin 120°$

$= \frac{3}{2} \times 25 \sin 60° = \frac{3}{2} \times 25 \times \frac{\sqrt{3}}{2} = \frac{75\sqrt{3}}{4}$ square unit.

$\because$ Each length unit in the plane represent 4 cm.

$\therefore$ The square unit represent $4^2 = 16$ cm^2.

$\therefore$ Area of Δ ABC $= \frac{75\sqrt{3}}{4} \times 16 = 300\sqrt{3}$ cm^2.

Remark

If the number of sides of a regular polygon = n sides , the length of the radius to the circle passing through its vertices = r , then

Area of the regular polygon $= \frac{n}{2} r^2 \sin\left(\frac{360°}{n}\right)$

For example :

The regular hexagon polygon whose drawn inside a circle of radius length 8 cm. , its area equals :

$\frac{6}{2} \times (8)^2 \times \sin\left(\frac{360°}{6}\right)$

$= 3 \times 64 \times \sin 60°$

$= 3 \times 64 \times \frac{\sqrt{3}}{2} = 96\sqrt{3}$ square unit.

Example 16

Find the cartesian equation of the circle passing through the points A = (6 , 3) , B = (2 , 3) and C = (4 , 1) , then determine its centre and length of its radius.

Solution

Let the equation is : $x^2 + y^2 + 2 L x + 2 K y + C = 0$

$\because$ The points A , B and C lies on the circle.

$\therefore 36 + 9 + 12 L + 6 k + c = 0$ *i.e.* $12 L + 6 K + C = -45$ (1)

$, 4 + 9 + 4 L + 6 K + C = 0$ *i.e.* $4 L + 6 K + C = -13$ (2)

$, 16 + 1 + 8 L + 2 K + C = 0$ *i.e.* $8 L + 2 K + C = -17$ (3)

by subtracting (2) from (1)

$\therefore 8 L = -32$ $\therefore L = -4$

and by subtracting (3) from (1)

$\therefore\ 4L + 4K = -28$ $\qquad\qquad$ $\therefore\ L + K = -7$

$\therefore\ -4 + K = -7$ $\qquad\qquad$ $\therefore\ K = -3$

by substitute in (3)

$\therefore\ -32 - 6 + c = -17$ $\qquad\qquad$ $\therefore\ C = 21$

$\therefore$ The equation is : $x^2 + y^2 - 8x - 6y + 21 = 0$

, where the centre = $(4, 3)$

, $r = \sqrt{16 + 9 - 21} = \sqrt{4} = 2$ length unit.

Example 17

Find the equation of the circle whose touches x-axis and passing through the points $(-1, 2), (-3, 4)$

Solution

$\because$ The circle touches x-axis $\qquad\qquad$ $\therefore\ r = |K|, C = L^2$

Let the equation of the circle is :

$x^2 + y^2 + 2Lx + 2Ky + L^2 = 0$

$\therefore$ The circle passing through the point $(-1, 2)$ then :

$1 + 4 - 2L + 4K + L^2 = 0$ $\qquad\qquad$ $\therefore\ L^2 - 2L + 4K = -5$ $\qquad$ (1)

$\because$ The circle passing through the point $(-3, 4)$ then :

$9 + 16 - 6L + 8K + L^2 = 0$ $\qquad\qquad$ $\therefore\ L^2 - 6L + 8K = -25$ $\qquad$ (2)

by multiplying the equation (1) × 2 then subtracting from the equation (2) :

$\therefore\ L^2 - 2L = -15$

$\therefore\ L^2 + 2L - 15 = 0$

$(L - 3)(L + 5) = 0$

$\therefore\ L = 3$ or $L = -5$

$\therefore\ K = -2$ or $K = -10$

$\therefore$ There are two circles in one of them $L = 3$, $K = -2$ and its equation is :

$x^2 + y^2 + 6x - 4y + 9 = 0$

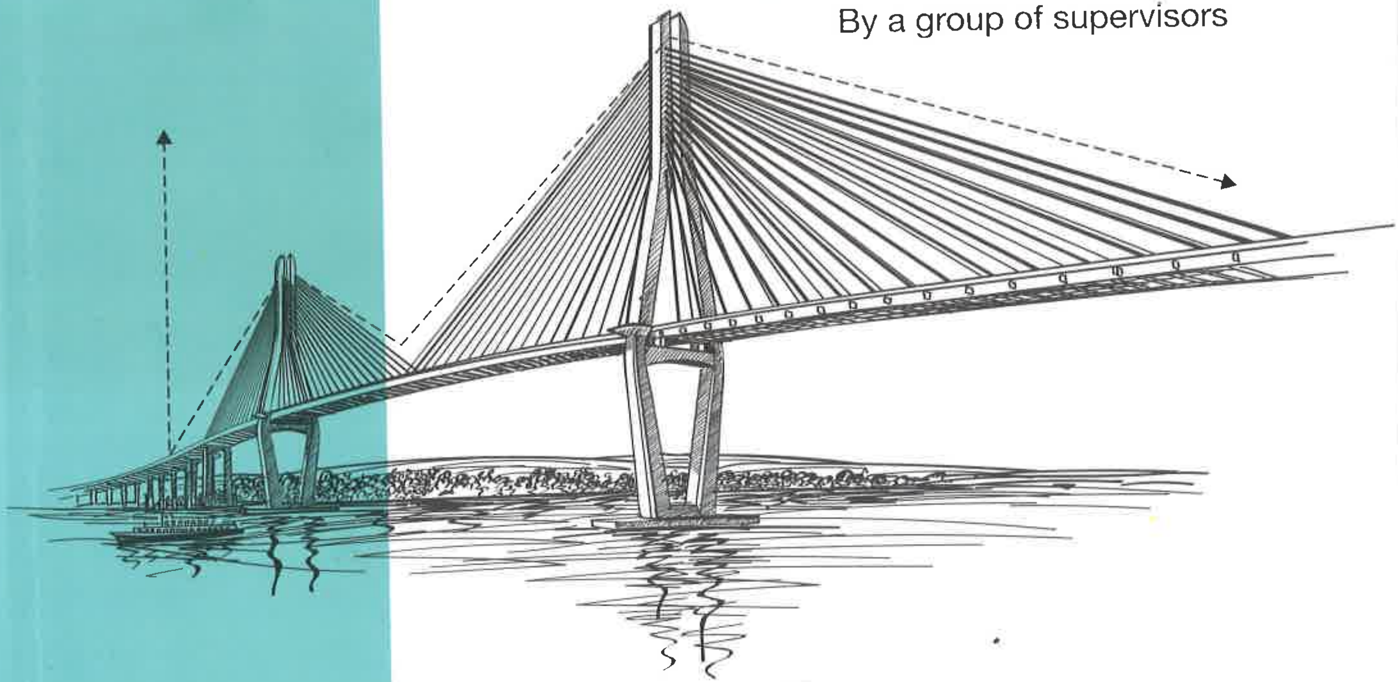
and in the other circle : $L = -5$, $K = -10$ and its equation is :

$x^2 + y^2 - 10x - 20y + 25 = 0$

EL-MOASSER

Mathematics
Applications

By a group of supervisors



Exercises

SCIENTIFIC SECTION

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2 SEC.

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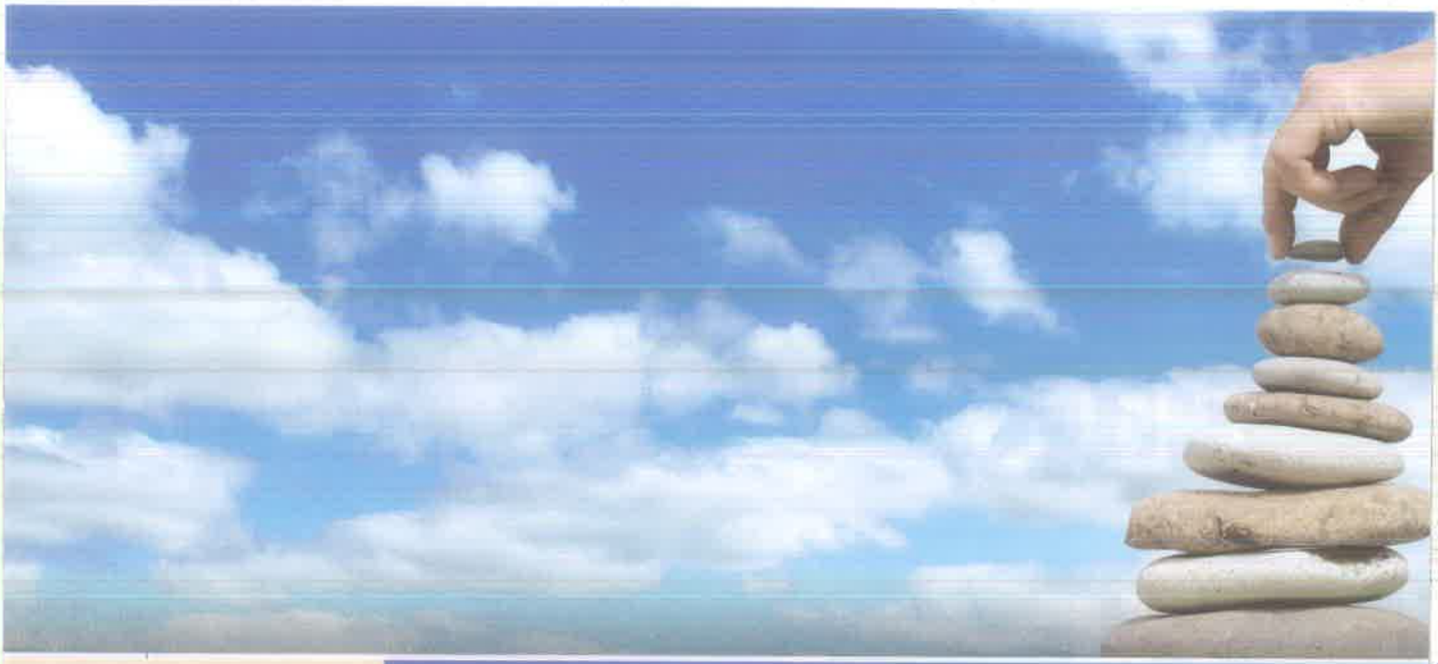


Accumulative exercise on vectors

From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

1 Choose the correct answer from the given ones :

(1) The norm of the vector $\overrightarrow{A} = -3\overrightarrow{i} + 4\overrightarrow{j}$ equals ……… length unit.

(a) 3 (b) 4 (c) 5 (d) 1

(2) The cartesian form of the vector $\overrightarrow{B} = \left(5\sqrt{2}, 225^\circ\right)$ is ………

(a) $(5, 5)$ (b) $(-5, -5)$ (c) $(5, -5)$ (d) $(-5, 5)$

(3) The measure of the polar angle of the vector $\overrightarrow{B} = -\overrightarrow{i} + \sqrt{3}\,\overrightarrow{j}$ equals ………

(a) 60° (b) 90° (c) 120° (d) 150°

(4) The polar form of the vector $\overrightarrow{A} = \sqrt{2}\,\overrightarrow{i} + \sqrt{2}\,\overrightarrow{j}$ is ………

(a) $(2, 135^\circ)$ (b) $(4, 45^\circ)$ (c) $(2, 45^\circ)$ (d) $(4, 135^\circ)$

(5) The polar form of the vector $\overrightarrow{m} = 5\overrightarrow{i} + 12\overrightarrow{j}$ is ………

(a) $(17, 67^\circ\, 22'\, 48'')$ (b) $(17, 22^\circ\, 37'\, 12'')$

(c) $(13, 67^\circ\, 22'\, 48'')$ (d) $(13, 22^\circ\, 37'\, 12'')$

(6) The vector that represents a force of magnitude 20 kg.wt. in the direction 30° South of East is written as ………

(a) $\left(10, -10\sqrt{3}\right)$ (b) $\left(10\sqrt{3}, -10\right)$ (c) $\left(-10, 10\sqrt{3}\right)$ (d) $\left(10\sqrt{3}, 10\right)$

(7) If $\overrightarrow{F} = k\,\overrightarrow{i} + 2\sqrt{2}\,\overrightarrow{j}$ and $\|\overrightarrow{F}\| = 2\sqrt{3}$ newton , then $|k| =$ ………

(a) $6\sqrt{2}$ (b) $2\sqrt{6}$ (c) -2 (d) 2

(8) If $\overrightarrow{F_1} = (5 , -3)$, $\overrightarrow{F_2} = (7 , 4)$, then the resultant of the two forces $\overrightarrow{R} =$ ....

(a) $\vec{i} + 12\vec{j}$ (b) $9\vec{i} + 4\vec{j}$ (c) $35\vec{i} - 12\vec{j}$ (d) $12\vec{i} + \vec{j}$

(9) If $\overrightarrow{F_1} = 5\vec{i}$, $\overrightarrow{F_2} = 7\vec{i} - 5\vec{j}$, then $\|\overrightarrow{R}\| =$ .... force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

(10) If $\overrightarrow{F_1} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{F_2} = \vec{i} + \vec{j}$, then the magnitude of their resultant equals .... force unit.

(a) 3 (b) 4 (c) 5 (d) 7

(11) Two forces of magnitudes 5 newtons and 7 newtons acting in the direction of East , then the magnitude of thier resultant equals ....

(a) 12 newton due East. (b) 2 newton due East.

(c) 12 newton due West. (d) 2 newton due West.

(12) If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three forces in equilibrium and meeting at one point where : $\overrightarrow{F_1} = (2 , -5)$, $\overrightarrow{F_2} = (-3 , 2)$, then $\overrightarrow{F_3} =$ ....

(a) (2 , 1) (b) (– 1 , – 3) (c) (1 , 3) (d) (3 , 1)

(13) If the set of forces $\overrightarrow{F_1} = a\vec{i} + 7\vec{j}$, $\overrightarrow{F_2} = -5\vec{i} - b\vec{j}$, $\overrightarrow{F_3} = \vec{i} + \vec{j}$ are in equilibrium , then (a , b) = ...

(a) (2 , 4) (b) (1 , 2) (c) (– 4 , – 8) (d) (4 , 8)

(14) If the set of forces $\overrightarrow{F_1} = 4\vec{i} - 5\vec{j}$, $\overrightarrow{F_2} = a\vec{i} + 3\vec{j}$, $\overrightarrow{F_3} = 7\vec{i} - b\vec{j}$ are in equilibrium , then a + b = ...

(a) 13 (b) – 13 (c) – 11 (d) – 2

(15) If the forces $\overrightarrow{F_1} = 4\vec{i} + 5\vec{j}$, $\overrightarrow{F_2} = a\vec{i} - 7\vec{j}$ and $\overrightarrow{F_3} = 3\vec{i} + b\vec{j}$ act at one point and the forces are in equilibrium , then a + 2 b = ...

(a) – 5 (b) 5 (c) 7 (d) – 3

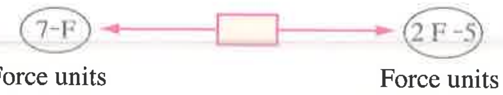
(16) If $\overrightarrow{F_1} = 2\vec{i} - 2\vec{j}$, $\overrightarrow{F_2} = 4\vec{i} - 8\vec{j}$, their resultant $\overrightarrow{R} = 2a\vec{i} - 3b\vec{j}$, then a + b = ...

(a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12

(17) If $\overrightarrow{F_1} = 5\vec{i} + 3\vec{j}$, $\overrightarrow{F_2} = a\vec{i} + 6\vec{j}$ and $\overrightarrow{F_3} = -14\vec{i} + b\vec{j}$ are three forces meeting at one point , $\overrightarrow{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$ then : (a , b) = ...

(a) (– 1 , 1) (b) (2 , 1) (c) (– 1 , 2) (d) (1 , – 1)

(18) In the opposite figure :

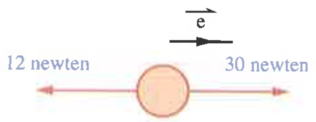


If the system is in equilibrium

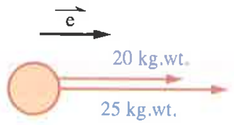
, then F = ... force units.

(a) 4 (b) 7 (c) 2.5 (d) 3.5

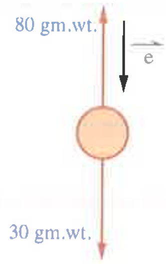
2 Write in terms of the unit vector $\vec{e}$ the resultant of the forces shown in each figure of the following figures :



The resultant is ....



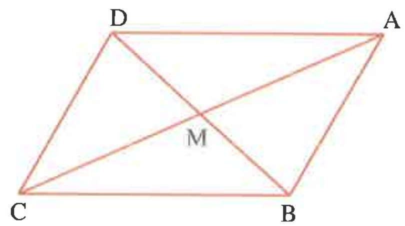
The resultant is ....



The resultant is ....

3 **In the opposite figure :**

ABCD is a parallelogram , M is the point of intersection of its diagonals , then :



- $\overrightarrow{AB} + \overrightarrow{BC} =$ ....
- $\overrightarrow{DA} + \overrightarrow{DC} =$ ....
- $\overrightarrow{AM} + \overrightarrow{CM} =$ ....
- $\overrightarrow{AB} + 2\,\overrightarrow{BM} =$ ....
- $\overrightarrow{AB} - \overrightarrow{AM} =$ ....

Exercise 1

Forces - Resultant of two forces meeting at a point

Interactive test

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The force is defined by ....

(a) its magnitude. (b) its direction.

(c) the point of action. (d) all the previous.

(2) Two forces act at a point. The magnitude of the two forces are 5 , 3 newton and the angle between them 60° , then the magnitude of their resultant = ... newton.

(a) 2 (b) 5 (c) 7 (d) 8

(3) Two forces act at a point the magnitude of the two forces $8\sqrt{3}$, 8 newton and the measure of the included angle between them 150° , then the magnitude of their resultant = ... newton.

(a) 64 (b) 32 (c) 16 (d) 8

(4) Two perpendicular forces act at a point. The magnitude of the two forces 12 , 5 newton , then the magnitude of their resultant = ... newton.

(a) 17 (b) 7 (c) 13 (d) 14

(5) The resultant of two forces 6 newton and 8 newton could be .... newton.

(a) 20 (b) 15 (c) 12 (d) 1

(6) The magnitude of two forces are 4 , 5 N. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant R = ... newtons.

(a) 15 (b) 5 (c) 20 (d) 25

(7) Two forces act at a point. The magnitude of the two forces are 6 , 3 newton and their resultant is perpendicular to one of them , then the magnitude of their resultant = ... newton.

(a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$

(8) Two forces enclosing between them an angle of measure θ , then the magnitude of their resultant ....

(a) increase as the value of θ increase.

(b) doubled as the value of θ doubled.

(c) increase as the value of θ decrease.

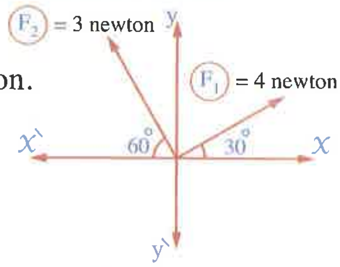
(d) don't change as change of the value of θ

(9) In the opposite figure :

The resultant of the two forces in the figure = ... newton.

(a) 7 (b) 5

(c) 1 (d) $\sqrt{7}$

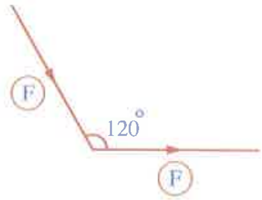


(10) In the opposite figure :

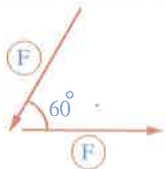
The magnitude of the resultant of the two forces = ... newton.

(a) 2 F (b) F

(c) $\sqrt{3}$ F (d) zero



(11) The magnitude of the resultant of the two forces shown in the opposite figure is ....



(a) $\frac{1}{2}$ F (b) F (c) $\sqrt{3}$ F (d) $\sqrt{5}$ F

(12) If the resultant of the two forces F_1 , F_2 bisects the angle between them. Which of the following statements is true ?

① $F_1 = F_2$ ② $\overrightarrow{F_1} = \overrightarrow{F_2}$ ③ $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

(a) only ① (b) only ① , ③

(c) only ② , ③ (d) All the previous.

(13) Two forces act at a point. The magnitude of the two forces are F , 2 newton and the measure of the angle between them is 60° , if their resultant equal $2\sqrt{3}$ newton , then F = ... newton.

(a) 2 (b) 4 (c) 8 (d) 12

(14) The magnitude of two forces F , 2 newton and the measure of their included angle = $\frac{2\pi}{3}$ and the magnitude of their resultant is F newton , then F = ... newton.

(a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$

(15) Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ If the magnitude of their resultant is 8 N. , then the value of each force measured in newton is ....

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

(16) Two equal forces in magnitude , the magnitude of their resultant = $7\sqrt{3}$ newton and the measure of the included angle is $\frac{\pi}{3}$, then the magnitude of each of them = ... newton.

(a) 3 (b) $5\sqrt{3}$ (c) 5 (d) 7

(17) The magnitude of two forces F , F kg.wt. , the magnitude of their resultant 24 newton and inclined to the first force by an angle of measure 30° , then F = ... kg.wt.

(a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12

(18) If F , $8\sqrt{3}$ are the magnitude of two forces act at a particle and the measure of the angle between them is 120° , their resultant bisects the included angle between the two forces , then F = ... newton.

(a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12

(19) Two forces of magnitudes 8 and F gm.wt. The measure of the angle between them is $\alpha \in]0 , \pi[$, their resultant bisects the included angle between them , then F = ... gm.wt.

(a) 4 (b) 16 (c) $2\sqrt{2}$ (d) 8

(20) Two forces of magnitudes 3 , F newton and the measure of the angle between them is 120°. If their resultant is perpendicular to the first force , so the value of F in newton is ....

(a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6

(21) The magnitude of two perpendicular forces are (2 F – 5) and (F + 2) newton and the magnitude of their resultant if $3\sqrt{5}$ newton , then F = ... newton.

(a) 7 (b) 4 (c) 6 (d) 3

(22) Two forces of magnitudes 6 N. and 10 N. , if the magnitude of their resultant is 14 N. , then the measure of the angle between the forces is ....

(a) 15° (b) 30° (c) 60° (d) 45°

(23) Two equal forces , the magnitude of each of them is 6 N. , the magnitude of their resultant is 6 N. , then the angle between them equals ....

(a) 30° (b) 60° (c) 120° (d) 150°

(24) Two forces of magnitudes 6 N. and 8 N. , if the magnitude of their resultant is 2 N. , then the measure of the angle between the two forces is ....

(a) 30° (b) 90° (c) 180° (d) 270°

(25) The resultant of two forces 6 , 2.5 newton is equal to 6.5 newton , then the angle between the two forces is ....

(a) an acute angle. (b) an obtuse angle.

(c) a right angle. (d) a straight angle.

(26) The magnitude of two forces are 2 F , 5 F newton and the measure of their included angle is θ and their resultant is 3 F , then θ = ...

(a) zero (b) 60° (c) 90° (d) 180°

(27) Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton , then the measure of the angle between them = ...

(a) 60° (b) 0° (c) 180° (d) 90°

(28) Two forces of magnitudes F and F act at a particle and their resultant is F , then the measure of the angle between the two forces = ...

(a) 120° (b) 60° (c) 45° (d) 90°

(29) The magnitude of two forces acting at a point F , $\sqrt{3}$ F newton. If the magnitude of their resultant is 2 F newton , then the measure of their included angle equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

(30) If $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ and $\| \overrightarrow{R} \| = \| \overrightarrow{F_1} \| - \| \overrightarrow{F_2} \|$, then the measure of the angle between $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ equals ....

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(31) If the magnitude of the resultant of two forces act at a point is maximum value , then the measure of the angle between the two forces equal ....

(a) 180° (b) 120° (c) zero (d) 60°

(32) The measure of the angle between $\overrightarrow{F_1}$ and the resultant of the two forces $(\overrightarrow{F_1} + \overrightarrow{F_2})$ and $(\overrightarrow{F_1} - \overrightarrow{F_2})$ is ....

(a) zero (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(33) If $\overrightarrow{R_1}$ is the resultant of the two forces $(\overrightarrow{F_1} , \overrightarrow{F_2})$ and $\overrightarrow{R_2}$ is the resultant of the two forces $(\overrightarrow{F_1} , -\overrightarrow{F_2})$, $\| \overrightarrow{F_1} \| = \| \overrightarrow{F_2} \|$, then ....

(a) $\overrightarrow{R_1} \perp \overrightarrow{R_2}$ (b) $\overrightarrow{R_1} = \overrightarrow{R_2}$

(c) $\| \overrightarrow{R_1} \| = \| \overrightarrow{R_2} \|$ (d) $\overrightarrow{R_1} \mathbin{//} \overrightarrow{R_2}$

(34) Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

(35) The magnitudes of two perpendicular forces are 6 , 8 newton then the measure of the angle between the resultant and the first force is ....

(a) $\sin^{-1} \frac{4}{3}$ (b) $\cos^{-1} \frac{4}{3}$ (c) $\tan^{-1} \frac{4}{3}$ (d) $\tan^{-1} \frac{3}{4}$

(36) Two forces of magnitudes F , 2 F newton act at a point , if the resultant of them is perpendicular to one of them , then R = ...

(a) $\sqrt{5}$ F (b) $\sqrt{3}$ F (c) 3 F (d) F

(37) Two forces of magnitudes $3\sqrt{2}$ and 6 newton and the measure of the angle between them is 135° , then the measure of the angle between their resultant and the second force is ....

(a) 30° (b) 45° (c) 60° (d) 90°

(38) Two forces of magnitudes 12 , 15 newton act at a particle and the measure of the enclosing angle between them is θ° , where $\cos \theta = \frac{-4}{5}$, then the measure of the included angle between the resultant and the first force = ...°

(a) zero (b) 30 (c) 90 (d) 36° 52̀

(39) The magnitude of two forces acting on a particle are 5 , 8 newton , then the smallest value of their resultant = ... newton.

(a) 2 (b) 3 (c) 7 (d) 13

(40) Two forces of magnitudes 9 , 6 newton , the maximum value of their resultant .... newton.

(a) 20 (b) 30 (c) 10 (d) 15

(41) The maximum value and the minimum value of the two forces 8 , 13 newton respectively are .... newton.

(a) 12 , 8 (b) 13 , 5 (c) 21 , 8 (d) 21 , 5

(42) Two forces of magnitudes 5 , F newton , if the smallest resultant of them is 10 newton , $F > 5$, then F = ... newton.

(a) 6 (b) 10 (c) 15 (d) 20

(43) Two forces act at a point. The magnitude of the two forces are 5 F , 3 F. If the maximum value of their resultant is 40 newton , then the minimum value of their resultant .... newton.

(a) 10 (b) 20 (c) 5 (d) zero

(44) Two forces act at a point. The magnitudes of the two forces are 5 , 3 newton , then the magnitude of their resultant measure by newton $\in$ ....

(a) [2 , 8] (b)]2 , 8[(c) [3 , 5] (d)]3 , 5[

(45) If θ is the angle between two forces of magnitudes 2 newton , 6 newton , $\theta \in]0 , \pi]$, then the magnitude of their resultant measured by newton $\in$ ....

(a)]4 , 8[(b) [4 , 8[(c)]4 , 8] (d) [4 , 8]

(46) Two forces of equal magnitude and the magnitude of their resultant equal 16 newton when the measure of the angle between the two forces is $\frac{\pi}{2}$, then the maximum value of their resultant equal .... newton.

(a) 32 (b) $8\sqrt{2}$ (c) $16\sqrt{2}$ (d) zero

(47) Two forces of magnitude F_1 , F_2 kg.wt. , where $F_1 > F_2$ and the magnitude of smallest and greatest resultant of them are 3 and 12 gm.wt. respectively , then $F_1^2 - F_2^2 =$ ....

(a) 12 (b) 3 (c) 9 (d) 36

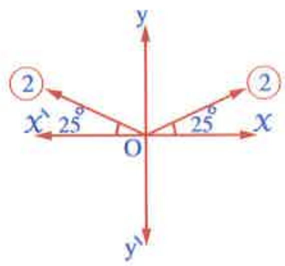
(48) The magnitude of two forces are 12 , 17 newton then the difference between the greatest and the smallest value of their resultant = ... newton.

(a) 29 (b) 5 (c) 14 (d) 24

(49) Two forces of magnitude F , $\sqrt{3}$ F newton meeting at a point and the magnitude of their resultant is R_1 when the measure of the angle between the two forces is 90° , and their resultant becomes R_2 when the measure of the angle between the two forces is 150° , then ....

(a) $R_1 = R_2$ (b) $R_1 = 2 R_2$ (c) $R_1 = \frac{3}{5} R_2$ (d) $R_1 = \frac{1}{2} R_2$

(50) The direction of the resultant of the forces which represented in the opposite figure is ....



(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$

(c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(51) Two forces act at a point and the magnitude of smallest and greatest resultant of them are 0 and 12 newton respectively , then ....

(a) mangitude of one force is three times magnitude of the other.

(b) magnitude of one force is twice magnitude of the other.

(c) the two forces are equal in magnitude.

(d) the two forces are perpendicular.

Second Essay questions

1 Find the magnitude and the direction of the resultant of two perpendicular forces of magnitudes 8 and 15 kg.wt. acting at a particle. « 17 kg.wt. , θ = 61° 55' 39'' »

2 Two forces act at a particle. If the maximum value of their resultant is 17 kg.wt. and the minimum value of the resultant is 7 kg.wt. Find the magnitude of each of the two forces. « 12 , 5 kg.wt. »

3 Two forces are equal in magnitude and the magnitude of their resultant is $4\sqrt{3}$ kg.wt. and the measure of the angle between the resultant and one of the two forces is 30°, find the magnitude of each of the two forces. « 4 kg.wt. »

4 The magnitude of the resultant of two perpendicular forces is 50 newton. If the resultant makes with the first force an angle of measure 30° , find the magnitude of each of these two forces. « $25\sqrt{3}$, 25 newton »

5 Two forces of magnitudes 30 and 16 newton act at a particle , if the magnitude of their resultant is 26 newton. Find the measure of the angle between these two forces. « 120° »

6 Two forces are of magnitude 8 and 16 gm.wt. acting at a particle. Find the measure of the angle included between the two directions of the forces if the resultant is perpendicular to the first force. « 120° »

7 Two forces are of magnitudes 9 and 6 kg.wt. act at a particle. The measure of the included angle is α , find α if the magnitude of the resultant is $3\sqrt{7}$ kg.wt. , find the measure of the angle between the resultant and the great force. « $\alpha = 120°$, $\theta \simeq 40° 53' 36''$ »

8 Two forces acted at a point. If the magnitude of the first is 15 kg.wt. towards East and the second is of magnitude 18 kg.wt. in the direction 30° West of the North. Calculate the magnitude and the direction of the resultant. « $3\sqrt{31}$ kg.wt. , $\theta \simeq 68° 56' 54''$ »

9 Two forces of magnitudes 12 , F kg.wt. act on a point. The first force acts in direction of East and the second force acts in direction 60° South of the West. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° South of the East. « 6 kg.wt. , $6\sqrt{3}$ kg.wt. »

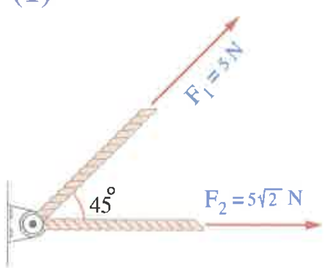
10 Two forces act at a particle and they include an angle of measure α where $\tan \alpha = \frac{-1}{\sqrt{3}}$

If the resultant is perpendicular to the small force and the magnitude of the great force equals 30 kg.wt. What is the magnitude of each of the small force and the resultant ?

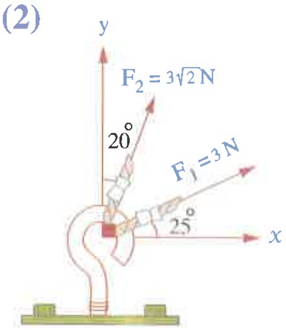
« $15\sqrt{3}$ kg.wt. , 15 kg.wt. »

11 **Find the magnitude and the direction of the resultant in each of the following figures :**

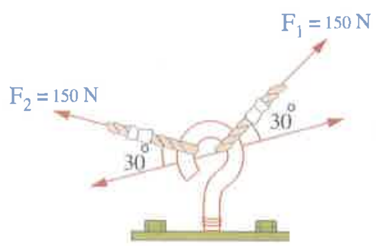
(1)



(2)



(3)



12 Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of $\vec{F}$ and the measure of the angle that $\vec{R}$ from with $\vec{F}$ « 8 newton , 30° »

13 Two forces of magnitudes $\sqrt{3}$ F and 2 F act at a point. Find the measure of the angle included between them if their resultant is perpendicular to the small force and if F = 15 Find the magnitude of the resultant. « 150° , 15 newton »

14 Two forces of magnitudes $2\sqrt{2}$ and F newton act at a particle and the magnitude of their resultant is $\sqrt{2}$ newton. If the resultant is perpendicular to the second force , find F and the measure of the angle between the two forces. « $\sqrt{6}$ newton , 150° »

15 Two forces of magnitudes 16 and F kg.wt. act on a particle and the measure of the angle between them is 120°. If their resultant is inclined to the force 16 kg.wt. by an angle whose measure is 30° , find the magnitude of F and the resultant.

« 8 kg.wt. , $8\sqrt{3}$ kg.wt. »

16 Three forces of magnitude 5 , 10 , $4\sqrt{7}$ N. act on a particle , if the measure of the angle between the first and the second forces equals 60° , find the magnitude of the maximum and the minimum resultant for the three forces. « $9\sqrt{7}$ newton , $\sqrt{7}$ newton »

17 **Two forces of magnitudes 2 F and 3 F newton. The angle between them is of measure θ , find the value of θ if the magnitude of their resultant is :**

(1) 3 F | (2) F

(3) 5 F | (4) $\sqrt{13}$ F

« 109° 28′ 16″ , 180° , zero , 90° »

18 **Two forces of magnitudes 2 , F newton , the angle between them is of measure 120° Find F in each of the two cases :**

(1) The direction of the resultant is perpendicular to the second force.

(2) The resultant inclines by 45° to the 2^{nd} force. « 1 , $\sqrt{3}+1$ newton »

19 F_1 and F_2 newton are magnitudes of two forces intersect at a point and their resultant equals R newton where $R \in [2 , 10]$, $F_1 > F_2$, find each of F_1 and F_2 , then find R when the measure of the angle between them is 120° « 6 , 4 , $2\sqrt{7}$ newton »

20 Two forces act at a point , the value of one is 3 N. more than the other. If the magnitude of their resultant is $3\sqrt{3}$ newton and is perpendicular to the smaller force. Find the magnitude of each force and the measure of the angle between them.

« 3 , 6 newton , α = 120° »

21 The resultant of two forces F_1 and F_2 is $\sqrt{10}$ newton when $F_1 \perp F_2$ and their resultant becomes $\sqrt{13}$ newton when the angle between F_1 and F_2 becomes 60° , find F_1 and F_2
« 1 , 3 newton »

22 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force. « $3\sqrt{5}$, $3\sqrt{5}$ kg.wt. »

23 Two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ meet at a point. Their resultant is R gm.wt. The angle between them is of measure 120°. If the direction of $\overrightarrow{F_2}$ is reversed , the resultant will be $R\sqrt{3}$ gm.wt., prove that $F_1 = F_2$ and the resultant in the first case is perpendicular to the second case.

24 4 , F are two forces acting at a point and their resultant is 10 newton and makes an angle of measure 60°with the force 4 newton. Find the value of F. « $2\sqrt{19}$ newton »

25 The difference between the magnitudes of two forces acting at a point is 15 newton. and their resultant = 35 newton in magnitude when the measure of the angle between the two forces = 120° , find the magnitude of each of the two forces. « 40 , 25 newton »

26 The sum of magnitudes of two forces is 4 newton when the measure of the angle between them is 60° , then the resultant becomes $\sqrt{13}$ newton. Find the magnitude of each of the two forces. « 1 , 3 newton »

27 The sum of magnitudes of two forces acting at a point is 40 kg.wt. the magnitude of their resultant is 20 kg.wt. and it is perpendicular to the smaller force. Find the magnitude of each of the two forces and the cosine of the angle between them. « 15 , 25 kg.wt. , $-\frac{3}{5}$ »

28 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two forces acting at a point where $F_1 > F_2$ and the measure of the angle between them is α , when $\alpha = 90°$, the resultant = 5 kg.wt. in magnitude and when $\alpha = 120°$, then the magnitude of the resultant becomes $\sqrt{13}$ kg.wt. Find each of F_1 and F_2
« 4 , 3 kg.wt. »

29 Two forces of same magnitude F kg.wt. enclose between them an angle of measure 120°. If the two forces are doubled and the measure of the angle between them became 60° , then the magnitude of their resultant increases by 11 kg.wt. , than the first case. Find the magnitude of F « $1 + 2\sqrt{3}$ »

29 F , 2 F are two forces act on a particle and enclose between them an angle of measure α The magnitude of their resultant equals $\sqrt{5}$ F (m + 1) and if the measure of the angle between them becomes $(90^\circ - \alpha)$, then the magnitude of the resultant will be $\sqrt{5}$ F (m − 1)

Prove that : $\tan \alpha = \frac{m-2}{m+2}$

Third Higher skills

1 Choose the correct answer from those given :

(1) If the ratio between the maximum and the minimum values of the resultant of two forces is 7 : 3 , then the ratio between the two forces = ...

(a) 7 : 4 (b) 7 : 3 (c) 5 : 3 (d) 5 : 2

(2) If the ratio among magnitudes of two forces and their resultant is $4 : 3 : \sqrt{13}$ respectively , then the measure of the angle between the two forces = ...

(a) 30° (b) 60° (c) 90° (d) 120°

(3) If the resultant of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ is perpendicular on $\overrightarrow{F_1}$, then the measure of the angle between the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ equals ....

(a) $\cos^{-1}\left(\frac{F_1}{F_2}\right)$ (b) $\cos^{-1}\left(\frac{-F_1}{F_2}\right)$ (c) $\sin^{-1}\left(\frac{F_1}{F_2}\right)$ (d) $\sin^{-1}\left(\frac{-F_1}{F_2}\right)$

(4) If the resultant of two perpendicular forces makes an angle of measure θ to the greater force which of the following values could be a value of θ ?

(a) 90° (b) 70° (c) 45° (d) 10°

(5) $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces acting at a point and their resultant is R. If $\overrightarrow{F_2}$ reveresed then their resultant rotates with angle of measure 90° , then ....

(a) $F_1 = F_2$ (b) $F_1 = 2 F_2$

(c) $F_1 = \frac{1}{2} F_2$ (d) nothing of the previous.

(6) The magnitudes of two forces acting at a point are 4 , F newton and the measure of their included angle is 120° , then F which makes the resultant minimum equals .... newton.

(a) 1 (b) 2 (c) 3 (d) 4

(7) If θ_1 is the measure of the angle between the resultant of two forces $\left(\overrightarrow{F_1}, \overrightarrow{F_2}\right)$ and the force $\overrightarrow{F_1}$ and θ_2 is the measure of the angle between the resultant of the two forces $\left(\overrightarrow{F_1}, 2\overrightarrow{F_2}\right)$ and the force $\overrightarrow{F_1}$, then ....

(a) $\theta_1 = \theta_2$ (b) $\theta_1 > \theta_2$ (c) $\theta_1 < \theta_2$ (d) $\theta_1 + \theta_2 = \frac{\pi}{2}$

(8) The magnitudes of two forces acting at a point are F, $\sqrt{3}$ F newton and the magnitude of their resultant is F newton and θ_1 is the measure of the angle between F, R and θ_2 is the measure between $\sqrt{3}$ F and R, then ....

(a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2}\theta_2$ (c) $\theta_1 = 3\theta$ (d) $\theta_1 = 4\theta_2$

(9) The magnitudes of two forces acting at a point are F_1, F_2 where : $3 \le F_1 \le 12$, $4 \le F_2 \le 16$ and the magnitude of their resultant is R and the measure of their included angle is 90°, then ....

(a) $5 \le R \le 20$ (b) $7 \le R \le 28$ (c) $0 \le R \le 18$ (d) $1 \le R \le 4$

(10) Two forces meet at a point, their magnitudes are F_1, F_2 where $1 \le F_1 \le 9$, $3 \le F_2 \le 7$ and the magnitude of their resultant R, then ....

(a) $2 \le R \le 16$ (b) $4 \le R \le 16$ (c) $6 \le R \le 16$ (d) $0 \le R \le 16$

(11) The magnitudes of two forces acting at a point are F_1, F_2 where $5 \le F_1 \le 20$, $12 \le F_2 \le 21$ and the magnitude of their resultant is R, the measure of the angle between them is θ where $0 \le \theta \le \frac{\pi}{2}$ then ....

(a) $13 \le R \le 29$ (b) $0 \le R \le 41$ (c) $13 \le R \le 41$ (d) $17 \le R \le 29$

2 One of two forces is half the other in magnitude, they have a certain resultant. If the small force increased by 4 kg.wt. and the great force becomes double, then their resultant stays in the same direction of the first case, find the magnitudes of the two forces and the ratio between the magnitudes of the two resultants in the two cases. « 4, 8 kg.wt., 1 : 2 »

3 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two forces meeting at a point and their resultant is R newton. If the direction of $\overrightarrow{F_2}$ becomes in the opposite direction, then the magnitude of the resultant becomes R$\sqrt{3}$ newton and the resultant becomes perpendicular to the first resultant. Find the measure of the angle between the two forces. « $\alpha = 120°$ »

Exercise 2 — Forces resolution into two components

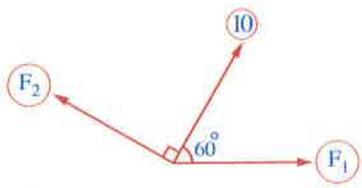
Interactive test

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

choose the correct answer from the given ones :

(1) In the opposite figure :

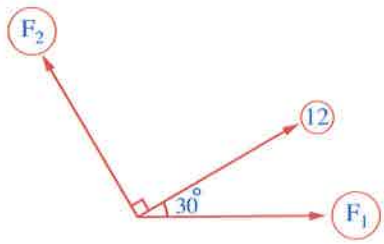


If the force of magnitude 10 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ inclined to the force by two angles of measures 60° and 90° respectively , then F_2 = ... N.

(a) $5\sqrt{3}$ (b) 10

(c) $10\sqrt{3}$ (d) 20

(2) In the opposite figure :



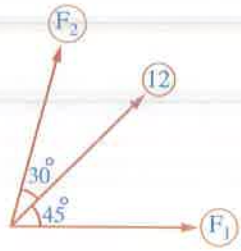
If the force of magnitude 12 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ inclined to the force by two angles of measures 30° and 90° respectively , then F_2 = ... N.

(a) 10 (b) $10\sqrt{3}$

(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

(3) In the opposite figure :

If the force of magnitude 12 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then F_1 = ... newton.

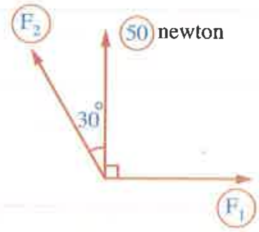


(a) 12 cos 75° (b) 12 cos 45°

(c) 6 csc 45° (d) 6 csc 75°

(4) In the opposite figure :

If the force of magnitude 50 newton is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then $F_1 + F_2$ = ... newton.

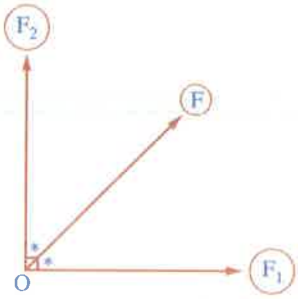


(a) 50 (b) 25

(c) $50\sqrt{2}$ (d) $50\sqrt{3}$

(5) In the opposite figure :

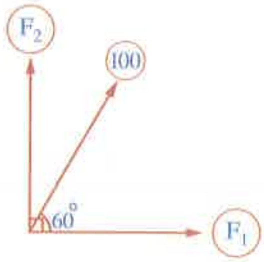
If the force $\vec{F}$ is resolved into the two perpendicular components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, the vector of the force $\vec{F}$ bisects the angle between the directions of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and $\|\overrightarrow{F_1}\| = 6\sqrt{2}$ newton , then $\|\vec{F}\|$ = ... newton.



(a) 6 (b) $6\sqrt{2}$ (c) 12 (d) $12\sqrt{2}$

(6) In the opposite figure :

If the force of magnitude 100 newton is resolved into two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and the force is measured by newton , then (F_1 , F_2) = ...

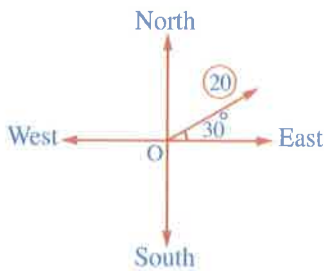


(a) $\left(50 , 50\sqrt{3}\right)$ (b) $\left(50\sqrt{3} , 10\right)$

(c) (50 , 50) (d) (10 , 10)

(7) In the opposite figure :

A force of magnitude 20 newton. acts in the direction 30° North of the East is resolved into two perpendicular components , then the magnitude of the component in North direction = ... newton.

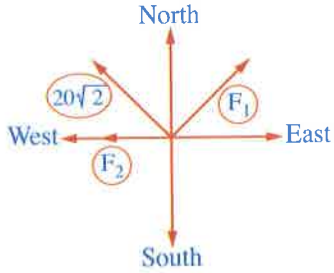


(a) $10\sqrt{3}$ (b) 20

(c) 10 (d) 5

(8) In the opposite figure :

A force of magnitude $20\sqrt{2}$ kg.wt. acts in the Western North direction , is resolved into two component. One of them of magnitude F_1 in the Eastern North direction and the other of magnitude F_2 in the direction of West , then F_2 = ... kg.wt.

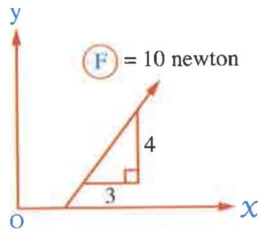


(a) 30 (b) 40

(c) 50 (d) $40\sqrt{2}$

(9) In the opposite figure :

If a force $\vec{F}$ is resolved into two components in the directions of the coordinate axes , then the magnitude of the component of this force in the direction of $\overrightarrow{Ox}$ equals .... newton.



(a) 10 (b) 6

(c) 8 (d) $\frac{40}{3}$

(10) A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction, is resolved into two perpendicular components , then the magnitude of the component in the South direction = ... gm.wt.

(a) 5 (b) 10 (c) $10\sqrt{2}$ (d) $5\sqrt{2}$

(11) A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components , so its component in direction of the East of magnitude .... newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

(12) A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components , so its component in the direction of Northern East of magnitude .... newton.

(a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

(13) The magnitude of a force is 6 newton and acts towards the North. It is resolved into two perpendicular components then its component in direction of Eastern North of magnitude .... newton.

(a) 6 (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) zero

(14) A force of magnitude $5\sqrt{3}$ newton acts in the direction 30° East of the North , is resolved into two perpendicular components , then the magnitude of its component in the East direction = ... newton.

(a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

(15) The magnitude of a force is 8 newton and acts in East direction. It is resolved into two components , the angle between the two components is 120° , then its component in South direction = ... newton.

(a) 16 (b) 8 (c) $8\sqrt{3}$ (d) $\frac{8\sqrt{3}}{3}$

(16) A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton , then the magnitude of the other = ... newton.

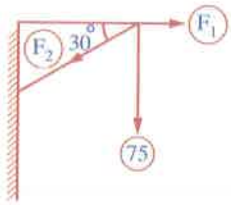
(a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$

(17) Force of magnitude F newton is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and they make angles of measure 60° , 90° respectively but on different sides from the line of action of $\overrightarrow{F}$, then F_1 = ...

(a) $2 F_2$ (b) $\frac{\sqrt{3}}{2} F_2$ (c) $\frac{2}{\sqrt{3}} F_2$ (d) $\frac{1}{2} F_2$

(18) In the opposite figure :

A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal of magnitude F_1 and the other is of magnitude F_2 , then F_2 = ... newton.

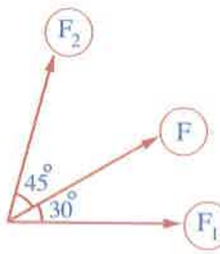


(a) 75 (b) $75\sqrt{3}$

(c) 150 (d) $150\sqrt{3}$

(19) In the opposite figure :

The force $\overrightarrow{F}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\frac{F_1 + F_2}{F}$ = ...



(a) sin 30° + sin 45° (b) $\frac{\sin 75° + \sin 30°}{\sin 75°}$

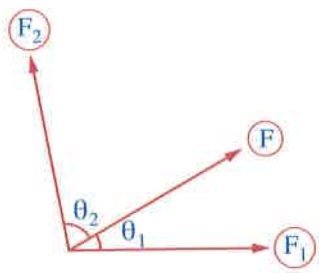
(c) $\frac{\sin 45° + \sin 30°}{\sin 75°}$ (d) $\frac{\sin 75°}{\sin 30°} + \frac{\sin 75°}{\sin 45°}$

(20) ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of $\overrightarrow{AD}$, then the magnitudes of the components of the force in direction of $\overrightarrow{AC}$, $\overrightarrow{AF}$ respectively are ....

(a) $10\sqrt{3}$, 10 (b) $5\sqrt{3}$, 10 (c) 10 , $10\sqrt{3}$ (d) $20\sqrt{3}$, 20

(21) In the opposite figure :

The force $\overrightarrow{F}$ has been resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\frac{F_1}{F_2}$ = ...



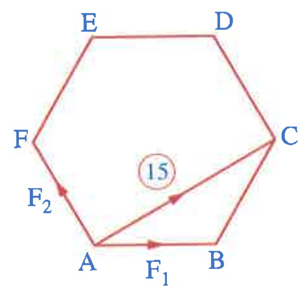
(a) $\frac{\sin\theta_2}{\sin\theta_1}$ (b) $\sin\left(\frac{\theta_2}{\theta_1}\right)$

(c) $\sin(\theta_1+\theta_2)$ (d) $\frac{\sin\theta_1}{\sin\theta_2}$

(22) In the opposite figure :

ABCDEF is a regular hexagon. Force of magnitude 15 N. acts along $\overrightarrow{AC}$ and it has been resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ as shown in the figure

$F_1 : F_2$ = ...

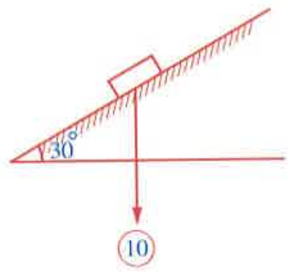


(a) $\sqrt{3} : 2$ (b) 2 : 1

(c) 1 : 2 (d) $1 : \sqrt{3}$

(23) In the opposite figure :

If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = ... N.



(a) $5\sqrt{2}$ (b) $5\sqrt{3}$

(c) 5 (d) $10\sqrt{3}$

(24) If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle (θ) , so the component of its weight in direction of the plane equals ....

(a) W (b) W sin θ (c) W cos θ (d) W tan θ

(25) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the horizontal , then its weight component in the perpendicular direction of the plane is ....

(a) W sin θ (b) W cos θ (c) W tan θ (d) W csc θ

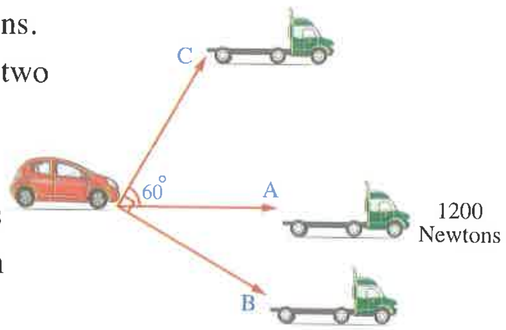
(26) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the vertical , then its weight component in direction of the plane is ....

(a) W sin θ (b) W cos θ (c) W (d) W tan θ

(27) A body of weight (W) newton is placed on an inclined plane makes an angle of measure (θ) with the horizontal , then the components of its weight in direction line of greatest slope and its perpendicular are 7 , 24 newton respectively , then the magnitude of the weight (W) = ... newton.

(a) 7 (b) 24 (c) 25 (d) 31

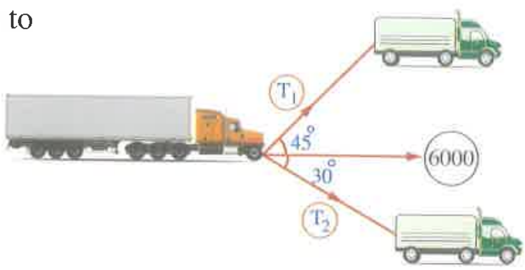
(28) A tractor drags a car with a force 1200 newtons. It's required to replace the tractor by another two tractors at B and C attached with two cables to the car and the angle between the two cables is 90°. If one of the two cables inclined to the tractor A at an angle 60° , then the tensions in the two cables B and C are .... newtons.



(a) 600 , 600 (b) 800 , 400

(c) $600\sqrt{3}$, 600 (d) 700 , 500

(29) A truck has broken down traffic officers try to pull the truck by using two draging cars. The resultant of their tensions is a horizontal tension of magnitude 6000 newtons as shown in the figure then T_2 = ... to the nearest newton.

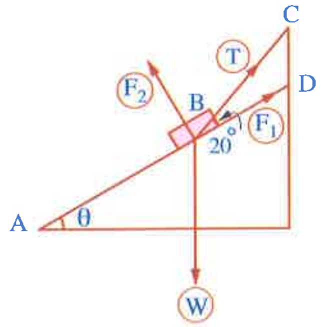


(a) 3105 (b) 3606

(c) 4392 (d) 4293

(30) In the opposite figure :

A body of weight (W) newtons is placed on a plane inclined to the horizontal at an angle of measure (θ). It is tied by a light string $\overline{BC}$ inclined to the plane at an angle of measure 20° above the plane. F_1 and F_2 are the components of the tension in direction of the plane and perpendicular to the plane then.........



(a) $F_2 = T \cos \theta$ (b) $F_1 = T \sin (20° + \theta)$

(c) $F_1 = T \cos (20° + \theta)$ (d) $T = F_1 \sec 20°$

Second Essay questions

1 A force of magnitude 600 kg. wt. acts on a particle. Find its two components in two directions making with the force two angles of measures 30° and 45° « 439.23 , 310.68 gm.wt. »

2 A force of magnitude 100 gm.wt. acts in the direction of Western North. Find its components in the North direction and in West direction. « $50\sqrt{2}$, $50\sqrt{2}$ gm. wt. »

3 A force of magnitude 12 kg. wt. acting in the direction of Eastern North was resolved into two components. One in the direction of East and the other in the direction of Western North. Find these two components. « $12\sqrt{2}$, 12 kg.wt. »

4 Resolve a horizontal force of magnitude 160 gm.wt. in two perpendicular directions. One of them inclined to the horizontal with an angle of measure 30° upwards. « $80\sqrt{3}$, 80 gm.wt. »

5 A force of magnitude 300 dyne. acts in the North direction. Find the magnitudes of the two perpendicular components if one of them acts in the direction 30° North of East. « 150 , $150\sqrt{3}$ dyne »

6 A force of magnitude 18 newton acts in the direction of South. Find its two components in the two directions 60° East of the South and the other direction towards 30° West of the South. « 9 , $9\sqrt{3}$ newton »

7 Resolve a force of magnitude 90 newton into two equal forces in magnitude and the measure of the angle between their lines of action is 60° « $30\sqrt{3}$ newton »

8 A body of weight 80 newton is placed on a horizontal plane. Find the two perpendicular components of the weight if one of them inclines to the horizontal with 30° downwards.

« 40 , $40\sqrt{3}$ newton »

9 Two forces act at a point. α is the angle between them and $\tan \alpha = -\frac{1}{\sqrt{3}}$,

If their resultant is perpendicular to the smaller force and the greater force 30 newton. Find the magnitude of the other force and the resultant.

« $15\sqrt{3}$, 15 newton »

10 Resolve a force of magnitude F newton in the North direction into two components , the first in the direction 30° North of East with magnitude 40 newton and the other is in the West direction. Find each of the magnitude of the force F and the magnitude of the other component.

« 20 , $20\sqrt{3}$ newton »

11 A rigid body of weight 42 netwon is placed on a plane inclined to the horizontal with an angle of measure 60°. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it.

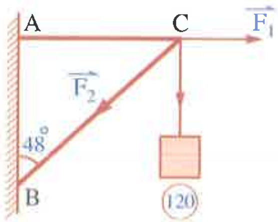
« $21\sqrt{3}$, 21 newton »

12 A body of weight 60 newton is placed on an inclined plane , at an angle of measure θ where $\tan \theta = \frac{3}{4}$, find the magnitudes of the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.

« 36 , 48 newton »

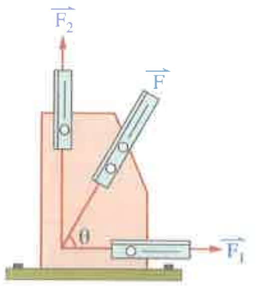
13 **In the opposite figure :**

Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force.



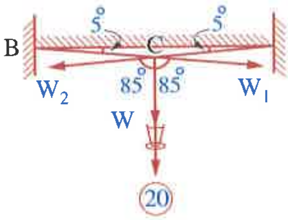
« 133.27 , 179.34 gm.wt. »

14 The opposite figure represents an angle of a bridge , the force $\vec{F}$ of magnitude 30 newton is resolved into two perpendicular components , the magnitude of one of them is $15\sqrt{3}$ newton Find the magnitude of the other component.



« 15 newton »

15 **In the opposite figure :**



A lamp of weight 20 newton suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles , the measure of each is 5° :

(1) Resolve the weight of the lamp into two components in the directions $\overrightarrow{AC}$, $\overrightarrow{BC}$ approximating the result to the nearest netwon.

(2) What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreased to be smaller than 5° ? And what do you expect to the components when the rods become horizontal ? Justify your answer.

« 114.74 , 114.74 newton »

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16 An inclined plane of length 130 cm. and height 50 cm. a rigid body of weight 390 gm.wt. is placed on it. Find the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.

« 150 , 360 gm.wt. »

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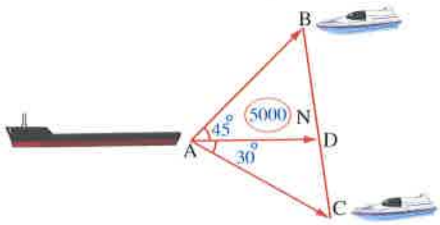
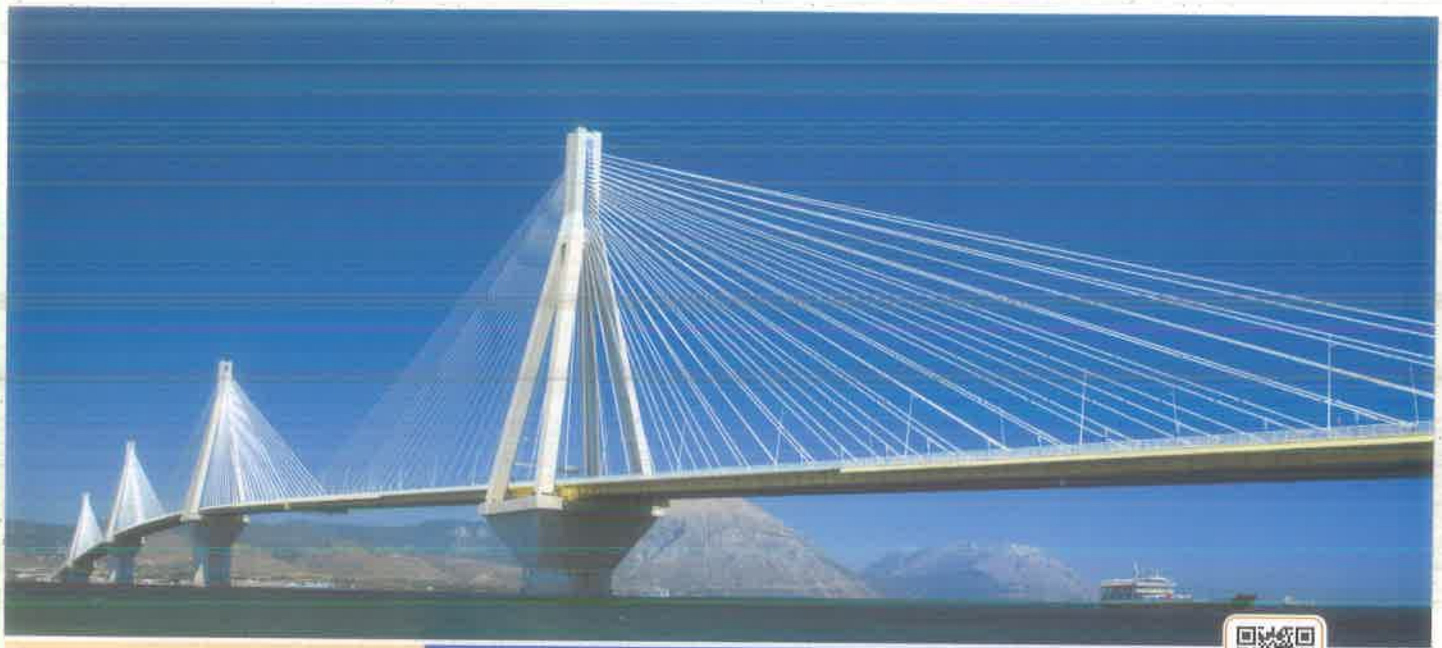
17 **In the opposite figure :**



A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser , the measure of the angle between the two strands equals 75° , if the measure of the angle between one of the strands and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 newton and acts on $\overrightarrow{AD}$

Find the tension in the two strands.

« 2588.2 , 3660.3 newton »



Exercise 3 The resultant of coplanar forces meeting at a point

Interactive test

From the school book · Remember · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(where $\vec{i}$ and $\vec{j}$ are the two fundamental unit vectors in two perpendicular directions)

(1) If $\vec{F_1} = \vec{i} - \vec{j}$, $\vec{F_2} = 2\vec{i} - 4\vec{j}$, $\vec{R} = 2a\vec{i} - 3b\vec{j}$, then $a + b =$ ....

(a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12

(2) If $\vec{F_1} = 3\vec{i} - 2\vec{j}$, $\vec{F_2} = a\vec{i} - \vec{j}$, $\vec{F_3} = 4\vec{i} - b\vec{j}$, $\vec{R} = 6\vec{i} - 4\vec{j}$, then $(a , b) =$ ....

(a) $(1 , -1)$ (b) $(-1 , 1)$ (c) $(-1 , -1)$ (d) $(1 , 1)$

(3) If $\vec{F_1} = 4\vec{i}$, $\vec{F_2} = 8\vec{i} - 5\vec{j}$, then $\|\vec{R}\| =$ .... force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

(4) If $\vec{F_1} = 3\vec{i} + 2\vec{j}$, $\vec{F_2} = a\vec{i} + 7\vec{j}$, $\vec{F_3} = -12\vec{i} + b\vec{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = \left(6\sqrt{2} , \frac{3}{4}\pi\right)$, then $a - b =$ ....

(a) -3 (b) 3 (c) zero (d) 6

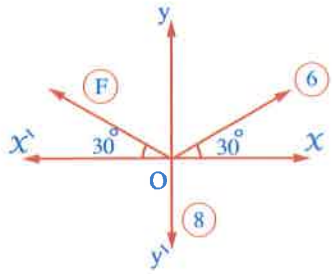
(5) Three coplanar forces $\vec{F_1} = 6\vec{i} + 7\vec{j}$, $\vec{F_2} = a\vec{i} - 9\vec{j}$, $\vec{F_3} = 5\vec{i} + b\vec{j}$ act at a particle and they are in equilibrium , then $a + 2b =$ ....

(a) -9 (b) 5 (c) 7 (d) -7

(6) If $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ are three coplanar equilibrium forces meeting at a point , and $\vec{F_1} = 2\vec{i} - 3\vec{j}$, $\vec{F_2} = 3\vec{i} + 5\vec{j}$, then $\vec{F_3} =$ ....

(a) $-5\vec{i} - 2\vec{j}$ (b) $-5\vec{i} + 2\vec{j}$ (c) $5\vec{i} + 2\vec{j}$ (d) $5\vec{i} - 2\vec{j}$

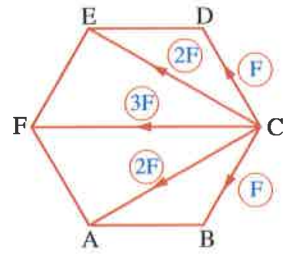
(7) If the resultant of the forces in the given figure acts in direction of y-axis , then F = ... force unit.



(a) 2 (b) 6

(c) 8 (d) 14

(8) The resultant of the forces in the opposite figure acts in direction of ....

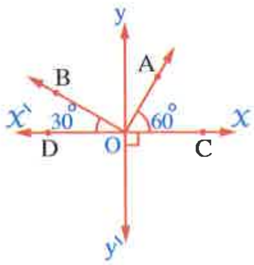


(a) $\overrightarrow{CD}$ (b) $\overrightarrow{CE}$

(c) $\overrightarrow{CF}$ (d) $\overrightarrow{CA}$

(9) In the opposite figure :

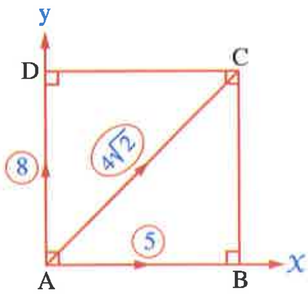
The magnitude of four coplanar forces are $1 , 2 , 4\sqrt{3} , 3\sqrt{3}$ newton act at point O in the direction of $\overrightarrow{OX} , \overrightarrow{OA} , \overrightarrow{OB}$ and $\overrightarrow{OY}$, $m(\angle AOC) = 60°$, $m(\angle BOD) = 30°$, then the magnitude and the direction of the resultant of the forces is ....



(a) $(4 , 180°)$ (b) $(4 , 0°)$

(c) $(3 , 0°)$ (d) $(5 , 90°)$

(10) ABCD is a square , the forces of magnitudes $5 , 8 , 4\sqrt{2}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AC}$ respectively , then the polar form of the resultant is ....

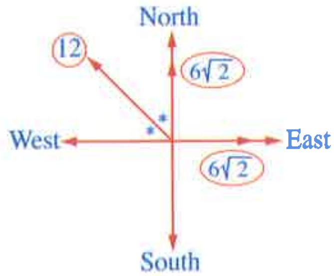


(a) $(5 , 54°)$ (b) $(15 , 60°)$

(c) $(15 , 53° 8')$ (d) $(13 , 90°)$

(11) In the opposite figure :

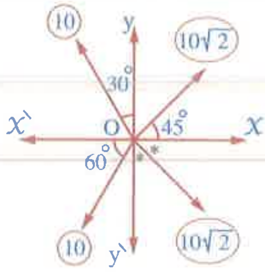
The direction of the resultant of the forces is ....



(a) South. (b) East.

(c) West. (d) North.

(12) In the opposite figure :

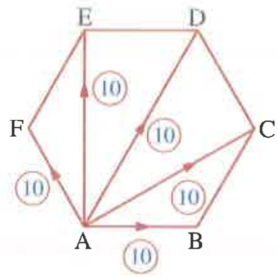


The resultant of the forces (R) = …………… newton.

(a) 20 (b) $10\sqrt{2}$

(c) 10 (d) zero

(13) In the opposite figure :

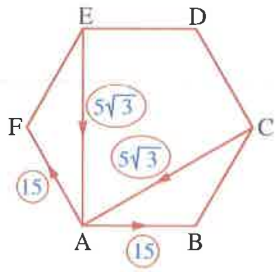


Five equal forces each of magnitude 10 newton act at one vertex of a regular hexagon and in direction of the other vertices of the hexagon , then the resultant of these forces = ……… newton.

(a) 50 (b) 20

(c) $30\sqrt{3}$ (d) $20 + 10\sqrt{3}$

(14) In the opposite figure :

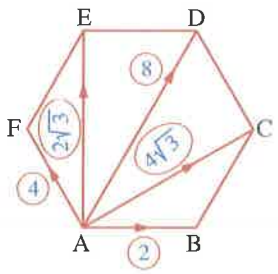


ABCDEF is a regular hexagon , the forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$, 15 newton act on $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$, $\overrightarrow{AF}$ respectively , then the magnitude of their resultant = ……… newton.

(a) 5 (b) 10

(c) 25 (d) zero

(15) In the opposite figure :



If ABCDEF is a regular hexagon , forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ respectively.

First : The magnitude of their resultant = ……… kg.wt.

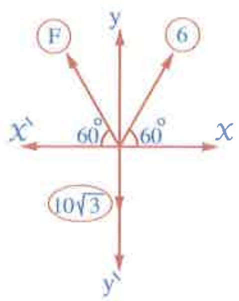
(a) $14 + 6\sqrt{3}$ (b) 20

(c) $20\sqrt{3}$ (d) $20 + \sqrt{3}$

Second : The direction of the resultant inclined by an angle of measure ……… with $\overrightarrow{AB}$

(a) 30° (b) 45° (c) 60° (d) 90°

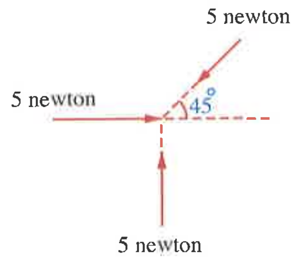
(16) If the resultant of the forces represented in the opposite figure acts in X-axis , then F = ……… newton.



(a) 10 (b) 14

(c) 18 (d) 6

(17) The opposite figure represents some of forces meeting at a point , then the magnitude of the resultant of these forces = ... newton.



(a) $15\sqrt{2}$ (b) 5

(c) $5\sqrt{2}-5$ (d) zero

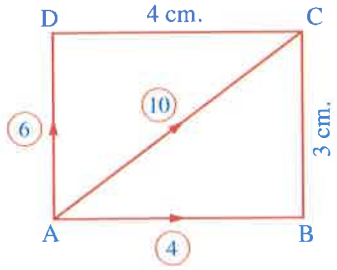
(18) Three coplanar forces meeting at a point , their magnitudes are 40 , 30 , 40 newton , the first is in direction 60° West of North , the second is towards West and the third in the direction 30° North of East , then the magnitude of their resultant equal .... newton.

(a) 30 (b) 110

(c) 60 (d) 50

(19) In the opposite figure :

ABCD is a rectangle AB = 4 cm. , BC = 3 cm. forces 4 N , 10 , 6 N acts along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ respectively. The resultant of these forces makes with $\overrightarrow{AB}$ an angle of measure ....



(a) 45° (b) 60°

(c) 30° (d) $\sin^{-1}\left(\frac{3}{5}\right)$

(20) ABCD is a right trapezium at A and D , in which AD = CD = 4 cm. , AB = 7 cm. , M $\in \overline{AB}$ where AM = 4 cm. , a set of forces their magnitudes 25 , F and $15\sqrt{2}$ gm.wt. act at $\overrightarrow{CB}$, $\overrightarrow{CM}$ and $\overrightarrow{CA}$ respectively and the norm of the resultant of these forces equals 45 gm.wt. , then the value of F = ... gm.wt.

(a) 10 (b) 50

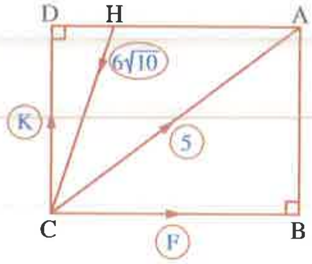
(c) 20 (d) 30

(21) The forces of magnitudes F , 12 , $8\sqrt{2}$, $10\sqrt{2}$, k newton act on a particle in the directions of East , North , Western North , Western South and South respectively. If the magnitude of the resultant = 4 newton due to North , then F – K = ... newton

(a) 24 (b) 27

(c) 12 (d) 6

(22) In the opposite figure :



The forces of magnitude F , 5 , K and $6\sqrt{10}$ N act in the rectangle ABCD in the directions $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{HC}$

Such that : AB = 6 cm. , BC = 8 cm. , AH = 6 cm.

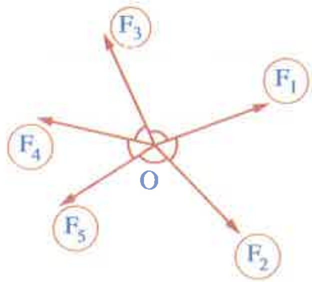
If these forces are in equilibrium , then K = ... newton.

(a) 12 (b) 15 (c) 18 (d) 20

(23) The coplanar forces of magnitudes 5 , 4 , F , 3 , k , 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60° , if the system is in equilibrium , then F + 2 K = ... kg.wt.

(a) 21 (b) 6 (c) 9 (d) 15

(24) The opposite figure represents a set of forces meeting at a point (O)



Mohamed took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_1}$

The magnitude of the resultant was R_1 and made angle of measure (θ_1) with the positive direction of X-axis and Ebrahim took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_2}$, the magnitude of the resultant was R_2 and made an angle of measure (θ_2) with the positive direction of X-axis , then ....

(a) $R_1 = R_2$, $\theta_1 = \theta_2$

(b) $R_1 = R_2$, $\theta_1 \neq \theta_2$

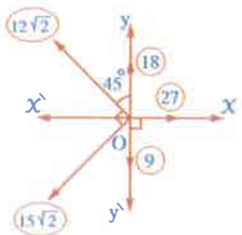
(c) $R_1 \neq R_2$, $\theta_1 = \theta_2$

(d) $R_1 \neq R_2$, $\theta_1 \neq \theta_2$

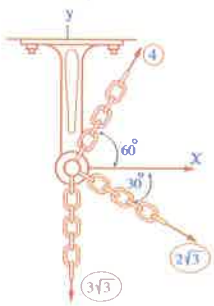
Second Essay questions

1 **Find the resultant (magnitude and direction) of the set of forces in each of the following figures (where each force magnitude is in newton) :**

(1)



(2)



2 Three coplanar forces of magnitudes 1 , 2 , $\sqrt{3}$ newton act at M , their directions are $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ respectively where m (∠ AMB) = 60° , m (∠ BMC) = 30° , m (∠ AMC) = 90° , find the resultant. « 4 newton , in direction of $\overrightarrow{MB}$ »

3 The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point , the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order.
Find the magnitude and direction of the resultant of these forces.

« 4 newton , in direction of 4th force »

4 The coplanar forces of magnitudes 2 , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act at a point.
If the measures between the first force and the second force is 45° , the measure between the second and the third is 105° and the measure between the third and the fourth is 120° taken in the same cyclic order , find the resultant of these forces.

« $\sqrt{13}$ newton , 11° 19ʹ with 2nd force»

5 Five coplanar forces meeting at a point , their magnitudes are 9 , 6 , $4\sqrt{2}$, $5\sqrt{2}$ and 5 newton act due to East , North , Western North , Western South and in the direction of South respectively. Prove that the set of forces are in equilibrium.

6 Three coplanar forces of magnitudes 60 , 88 and 60 gm.wt. act at a point , the 1st is towards North , the second is in the direction 30° South of West and the 3rd in the direction 30° South of East.
Find the magnitude of the resultant of these forces and its direction.

« 28 gm.wt. , 30° South of West »

7 Four coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction , the second of magnitude 2 newton , acts in direction 60° North of the East , the third of magnitude 5 newton , acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the South.
Find the magnitude and direction of their resultant. « 4 N. , 120° »

8 The forces of magnitudes 2 F , 3 F and 4 F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order.
Find the magnitude and the direction of the resultant of these forces.

« $\sqrt{3}$ F newton , perpendicular to the force 3 F »

9 ABC is an equilateral triangle. M is the point of intersection of its medians. the forces of magnitude 15 , 20 and 25 newton act on a particle at the point M in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$
Find the magnitude and the direction of the resultant of these forces.

« $5\sqrt{3}$ newton , 30° with $\overrightarrow{MA}$ »

10 Δ ABC is an isosceles triangle where m ($\angle$ BAC) = 120° , the forces of magnitudes 4 , $6\sqrt{3}$, 4 newton act at A in the directions $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CA}$ respectively.
Find the magnitude and the direction of the resultant of these forces.

« $10\sqrt{3}$ newton in the direction of $\overrightarrow{CB}$ »

11 Four coplanar forces of magnitude 2 , 1 , 4 and $3\sqrt{3}$ N. act at a point A in directions of $\overrightarrow{BC}$, $\overrightarrow{BA}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ where ABC is an equilateral triangle and D is the midpoint of $\overline{BC}$
Find the magnitude and direction of their resultant. « 1 newton in the direction of $\overrightarrow{AC}$ »

12 ABCD is a rectangle where AB = 4 cm. , BC = 3 cm. the forces of magnitudes 2 , 5 and 3 kg.wt. act at the point A in the directions $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.
Find the resultant of these forces and the measure of its angle of inclination on $\overrightarrow{AB}$

« $6\sqrt{2}$ kg.wt. , 45° »

13 ABCD is a rectangle in which AB = 8 cm. , BC = 6 cm. , E $\in \overline{CD}$ where ED = 6 cm. , a set of forces their magnitudes 12 , 40 , $26\sqrt{2}$ and 4 newton act at $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AE}$ and $\overrightarrow{AD}$ respectively.
Find the magnitude and the direction of the resultant of these forces.

« $6\sqrt{2}$ newton , 45° with $\overrightarrow{AB}$ »

14 ABCD is a rectangle in which : AB = 21 cm. , BC = 9 cm. The point O $\in \overline{AB}$ where AO = 9 cm. four forces of magnitudes 4 , 10 , 6 and $12\sqrt{2}$ kg.wt. act at the point O in the directions $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{BC}$ and $\overrightarrow{OD}$ respectively.
Find the magnitude of the resultant of these forces and prove that it is parallel to $\overrightarrow{BC}$

« 24 kg.wt. »

15 ABCDEF is a regular hexagon , the forces of magnitudes 8 , $6\sqrt{3}$, 5 , $4\sqrt{3}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant. « $\sqrt{651}$ newton , $40^\circ\ 9'$ with $\overrightarrow{AB}$ »

16 ABCDHE is a regular hexagon. Forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AH}$, $\overrightarrow{AE}$ respectively.
Find the magnitude and the direction of their resultant. « 20 kg.wt. , 60° with $\overrightarrow{AB}$»

17 ABCDEF is a regular hexagon. M is the point of intersection of its diagonals. the forces of magnitudes 4 , 1 , 4 , 5 , 2 and 3 gm.wt. act at M in the directions of $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$, $\overrightarrow{MD}$, $\overrightarrow{ME}$ and $\overrightarrow{MF}$
Find the resultant of these forces and prove that it is in the direction of $\overrightarrow{MD}$ « 2 gm.wt. »

18 ABC is a right-angled triangle at B where AB = 80 cm. , BC = 60 cm. , D $\in \overline{AC}$ where BD = DC
The four forces of magnitudes 8 , 12 , 15 and 10 newton act at the point B in the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{BD}$ respectively.
Find the resultant of these forces and prove that it acts in $\overrightarrow{BD}$ « 15 newton »

---

19 ABCD is a square of side length is 12 cm. H $\in \overline{BC}$ where BH = 5 cm. forces of magnitudes 2 , 13 , $4\sqrt{2}$, 9 gm.wt. act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ respectively.
Find the magnitude of the resultant of these forces. « $10\sqrt{2}$ gm.wt. in direction of $\overrightarrow{AC}$ »

---

20 ABCD is a square of side length 6 cm. The point E is the midpoint of $\overline{BC}$ and F is the midpoint of $\overline{DC}$, the five forces of magnitudes 2 , $12\sqrt{5}$, $6\sqrt{2}$, $4\sqrt{5}$ and 4 kg.wt. act at the point A in the directions of $\overrightarrow{AB}$, $\overrightarrow{AE}$, $\overrightarrow{CA}$, $\overrightarrow{AF}$ and $\overrightarrow{AD}$ respectively.
Find the magnitude and the direction of the resultant of these forces. « 30 kg.wt. , $36^\circ\ 52'\ 12''$ »

---

21 ABCD is a square , E $\in \overline{AD}$, four forces of magnitudes 4 , $4\sqrt{3}$, $10\sqrt{2}$, F kg.wt. act at point B in the directions $\overrightarrow{BA}$, $\overrightarrow{BE}$, $\overrightarrow{DB}$, $\overrightarrow{BC}$, if these forces are in equilibrium , find m (∠ ABE) and the value of F « 30° , $2\left(5-\sqrt{3}\right)$ kg.wt. »

---

22 The coplanar forces of magnitudes 5 , 4 , F , 3 , K and 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60°
Find the magnitude of F and K that makes the system in equilibrium. « 9 , 6 kg.wt. »

---

23 The forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K newton act on a particle in the directions of East , North , Western North , Western South and South respectively.
Find the values of F and K if the magnitude of the resultant = 2 newton due to North.
« 9 , 3 newton »

---

24 Forces of magnitudes F , $4\sqrt{3}$, $12\sqrt{3}$, 36 gm.wt. act at a particle. The last three forces are in the directions of North , 60° West of North , 60° South of East respectively. If the resultant of these four forces = 8 gm.wt. in magnitude in the direction of East.
Determine the value of F and its direction. « 16 gm.wt. , 60° North of East »

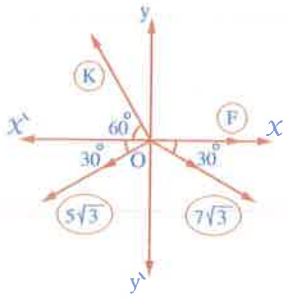
---

25 The forces of magnitudes F , 8 , K , 5 , $8\sqrt{3}$ newton act at a point in the directions of : East , 30° East of North , North , West and South respectively.
Find the values of F and K if the resultant is 4 newton in magnitude in the direction of 60° North of East. « 3 , $6\sqrt{3}$ newton »

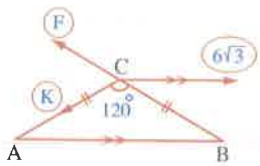
26 ABCD is a right trapezium at A and D , in which AD = CD = 40 cm. , AB = 70 cm. , $M \in \overline{AB}$ where AM = 40 cm. , a set of forces their magnitudes 25 , F , $10\sqrt{2}$ and 35 gm.wt. act at $\overrightarrow{CB}$, $\overrightarrow{CM}$, $\overrightarrow{CA}$ and $\overrightarrow{CD}$ respectively and the norm of the resultant of these forces equals 50 gm.wt. Find F « F = 10 gm.wt. »

27 **In each of the following figures find the magnitudes of F and K in newton that makes the system in equilibrium :**

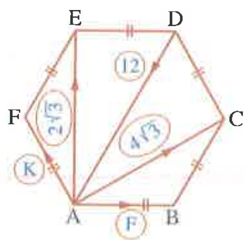
(1)



(2)



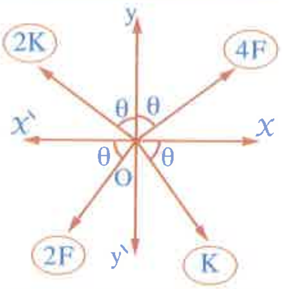
(3)



28 Coplanar forces of magnitudes F , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act on a particle. The first force acts in the east direction. The angle between the first and the second force is of measure 45° , the angle between the second and the third force is of measure 105° , the angle between the third and the fourth force is of measure 120°. If the magnitude of their resultant is $3\sqrt{2}$ newton , then find the value of F and measure of the angle between the resultant and the first force. « 3 newton , 45° »

29 ABCDEF is a regular hexagon.
Forces of magnitudes 4 , $2\sqrt{3}$, F , $2\sqrt{3}$ and K kg.wt. act in the directions of $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ respectively.
If the resultant of these forces is of magnitude 20 kg.wt. in the direction of $\overrightarrow{AD}$
Find the values of F , K « 10 , 4 kg.wt. »

30 **In the opposite figure :**



Four coplanar forces act at the point (O) in the directions shown in the figure where $\sin\theta = \frac{4}{5}$ and the resultant of these forces is $8\sqrt{2}$ N. and makes an angle of measure 135° with $\overrightarrow{OX}$, then find the values of F , K « 3 , 14 newton »

31 If $\overrightarrow{F_1} = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$, $\overrightarrow{F_3} = -14\,\overrightarrow{i} + b\,\overrightarrow{j}$ are three coplanar forces meeting at a point and their resultant is $\overrightarrow{R} = \left(10\sqrt{2}\ ,\ 135°\right)$, then find the values of a and b « a = -1 , b = 1 »

Exercise 4

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point (The triangle of forces rule - Lami's rule)

Interactive test

From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the .... of the included angle between the other two forces.

(a) cosine (b) sine (c) tangent (d) cotangent

(2) If a body is in equilibrium under action of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then ....

(a) $\overrightarrow{F_1} = \overrightarrow{F_2}$ (b) $F_1 = F_2$

(c) $\overrightarrow{F_1} + \overrightarrow{F_2} \neq 0$ (d) $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are not on the same line.

(3) If a body is kept in equilibrium under action of several forces , then the least number of forces could cause equilibrium equals ....

(a) 1 (b) 2 (c) 3 (d) 4

(4) The least number of coplanar unequal in magnitude forces could be in equilibrium is ....

(a) 1 (b) 2 (c) 3 (d) 14

(5) If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ = ...

(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

(6) Three equal forces in magnitude meeting at a point and they are in equilibrium , then the measure of the angle between each two forces = ...

(a) $60°$ (b) $90°$ (c) $120°$ (d) $150°$

(7) If $\vec{F}$ is in equilibrium with two perpendicular forces of magnitudes 8 newton and 15 newton , then F = ... newton.

(a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$

(8) If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is $60°$, then F = ... newton.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

(9) Which of the following sets of forces could be in equilibrium ?

① 8 newton , 8 newton , 8 newton.

② 8 newton , 8 newton , 16 newton.

③ 8 newton , 8 newton , 20 newton.

(a) ① only. (b) ② only. (c) ① , ② (d) ② , ③

(10) Which of the following systems of forces could not be in equilibrium ?

(a) 10 newton , 10 newton , 5 newton (b) 4 newton , 6 newton , 10 newton

(c) 11 newton , 7 newton , 8 newton (d) 8 newton , 4 newton , 14 newton

(11) Three coplanar forces not on the same straight line meeting at a point are in equilibrium , the magnitude of two forces of them are 7 and 3 newton , then the magnitude of the third could be .... newton.

(a) 10 (b) 4 (c) 5 (d) 3

(12) Three coplanar forces are in equilibrium act at a particle , the measure of the angle between the first two forces is $60°$, and between the second and third forces is $150°$ then the ratio between forces is ....

(a) $1 : 1 : \sqrt{3}$ (b) $1 : 2 : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$

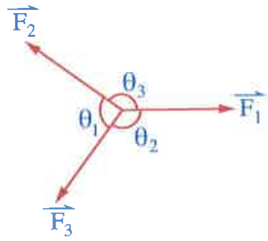
(13) The force which is in equilibrium with two perpendicular forces F , F newton makes with one of the two forces an angle of measure ....°

(a) 90 (b) 120 (c) 135 (d) 150

(14) Three coplanar forces of magnitudes 5 , 6 , 7 newton act at a particle. If the forces are in equilibrium , then the cosine of the angle between the second and the third force = ...

(a) $\frac{7}{5}$ (b) $\frac{-5}{7}$ (c) $\frac{15}{17}$ (d) $\frac{1}{2}$

(15) If a body is in equilibrium under action of three forces as shown in the figure.



Which of the following statements is true ?

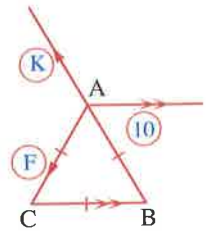
① $F_1 + F_2 + F_3 = \text{zero}$ ② $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \overrightarrow{\text{zero}}$

③ $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$

(a) ① , ② only. (b) ② , ③ only.

(c) ③ only. (d) ① , ② , ③

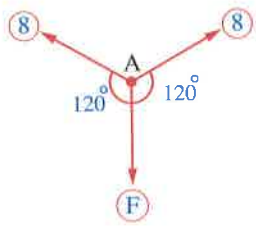
(16) In the opposite figure :



If the three coplanar forces meeting at a point and in equilibrium , then F = ... N.

(a) 10 (b) 12 (c) 20 (d) 13

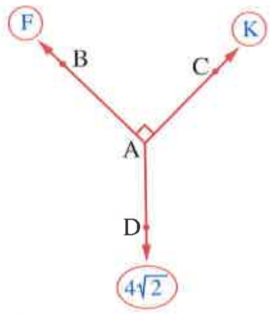
(17) Particle A is balanced under action of three forces as shown in the opposite figure where $\overrightarrow{F}$ is balanced with two forces the magnitude of each is 8 newton and makes an angle of measure 120° with each of them, then F = ... newton.



(a) zero (b) 8

(c) 16 (d) 8 sin 120°

(18) In the opposite figure :

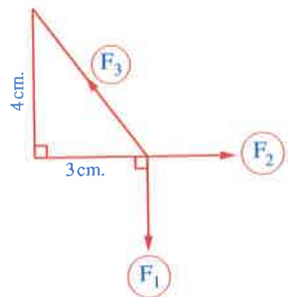


Three equilibrium forces of magnitudes F , k and $4\sqrt{2}$ newton , m (∠ BAC) = 90° , m (∠ BAD) = 135° , then (F , K) = ...

(a) (4 , 4) (b) $(4 , \sqrt{2})$

(c) $(\sqrt{2} , 4)$ (d) (2 , 2)

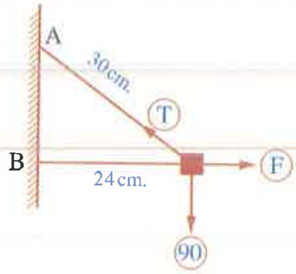
(19) In the opposite figure :



A body is in equilibrium under action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 =$ ....

(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

(20) In the opposite figure :

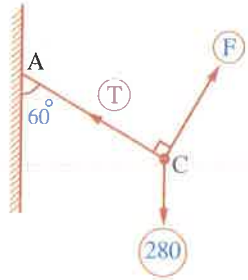


A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by a horizontal force. It comes to equilibrium when it is 24 cm. apart from the wall $\overline{AB}$ then T − F = ……… gm.wt.

(a) 150 (b) 120

(c) 50 (d) 30

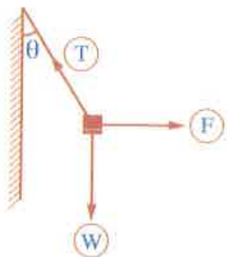
(21) In the opposite figure :



A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T}$ = ………

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

(22) In the opposite figure :

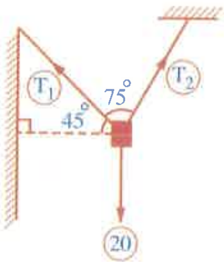


A body of weight (W) newton is suspended from the end of a string. The other end of the string is fixed to a vertical wall. The body is pulled by a horizontal force of magnitude (F) newton. The body is in equilibrium when the string makes an angle θ to the wall which of the following statements is false in case of equilibrium ?

(a) $F = W \tan \theta$ (b) $\overrightarrow{W} + \overrightarrow{F} + \overrightarrow{T} = \overrightarrow{Zero}$

(c) $T^2 = F^2 + W^2$ (d) $T = F + W$

(23) In the opposite figure :

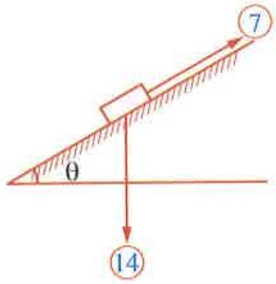


The weight of a body is 20 kg.wt , the body is in equilibrium then $\frac{T_1}{T_2}$ = …………

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{3}$ (d) $\frac{\sqrt{3}}{2}$

(24) In the opposite figure :

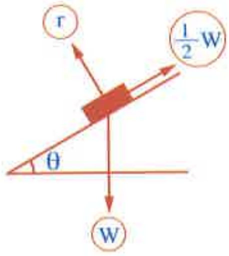


If the body is in equilibrium when it is placed on an inclined smooth plane , then m (∠ θ) = ...

(a) 60° (b) 45°

(c) 30° (d) 75°

(25) In the opposite figure :

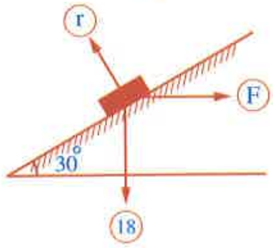


If the body is in equilibrium under action of forces shown , then m (∠ θ) = ...

(a) 30° (b) 60°

(c) 45° (d) 15°

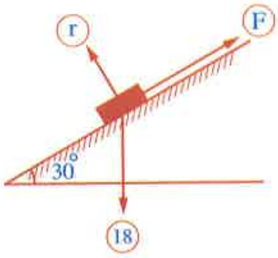
(26) In te opposite figure :



A body of weight 18 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force of magnitude F newton , then F + r = ... newton.

(a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$

(27) In the opposite figure :



A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward , then F + R = ... newton.

(a) $6\sqrt{3}$ (b) $9\sqrt{3}$ (c) $18\sqrt{3}$ (d) $9 + 9\sqrt{3}$

(28) The weight of a body is 6 kg.wt. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium by a horizontal force , then the magnitude of this horizontal force = ... kg.wt ,

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 6

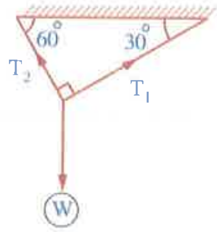
(29) The weight of a body is 6 newton. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium with a force of magnitude 49 newton which makes an angle of measure θ with the line of greatest slope of the plane , then cos θ = ...

(a) $\frac{3}{49}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

(30) The weight of a body is 20 kg.wt. It is placed on a smooth inclined plane makes an angle θ to the horizontal , where $\sin\theta = \frac{3}{5}$ and it prevent from sliding byl a horizontal force F , then F = ... kg.wt.

(a) 30 (b) 15 (c) 10 (d) $5\sqrt{3}$

(31) **In the opposite figure :**



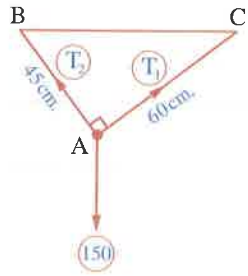
A body of weight (W) is hanged by two strings.

The two strings inclined to the horizontal as shown in the figure , then T_1 = ...

(a) $\frac{1}{3}$ W (b) $\frac{1}{2}$ W

(c) $\frac{\sqrt{3}}{3}$ W (d) $\frac{\sqrt{3}}{2}$ W

(32) **In the opposite figure :**



A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm. , and the other two ends C and B are on a horizontal line , then :

$T_2 - T_1$ = ... gm.wt.

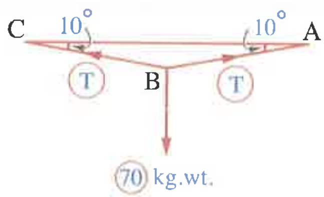
(a) 120 (b) 90

(c) 60 (d) 30

(33) A body of weight 28 kg.wt. is suspended by two perpendicular strings , if the measure of the angle between one strings and the line of the weight is 120° , then the magnitude of the tension of this strings equals .... kg.wt.

(a) 14 (b) 28 (c) $14\sqrt{3}$ (d) $28\sqrt{3}$

(34) **In the opposite figure :**



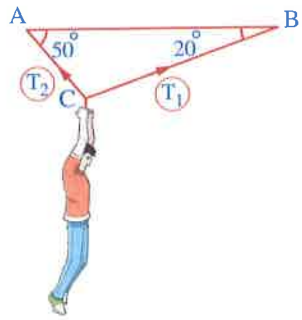
A man of weight 70 kg.wt. is walking on a rope.

If the rope lowered 10° from the horizontal when the man becomes at the middle of the rope , then the tension in the rope (T) = ... kg.wt.

(a) $\frac{70 \sin 20°}{\sin 100°}$ (b) $\frac{70 \sin 100°}{\sin 160°}$

(c) $\frac{70 \cos 100°}{\sin 160°}$ (d) $\frac{\sin 100°}{70 \sin 160°}$

(35) In the opposite figure :

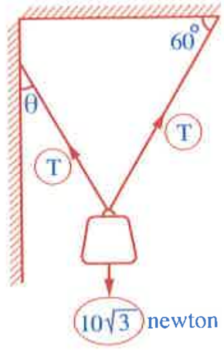


A man of weight (W) suspended vertically at C by two ropes $\overrightarrow{CB}$, $\overrightarrow{CA}$ as shown in the figure and $T_2 = 60$ kg.wt. then (W) = ... kg.wt.

(a) 87.7 (b) 70.6

(c) 60 (d) 49.8

(36) A body of weight $10\sqrt{3}$ newton is suspended by two strings as shown in the figure , then the value of θ which makes both tensions are equal is ....



(a) 15° (b) 30°

(c) 45° (d) 60°

Second Essay questions

1 Three forces of magnitudes F_1 , F_2 and 75 newton intersecting at one point they are represented by the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of Δ ABC respectively where : AB = 3 cm. , BC = 4 cm. and CA = 5 cm.
Find the value of each of F_1 and F_2 « 45 , 60 newton »

2 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. if the angle between the 1st and the 2nd force measures 120° and between the 2nd and the 3rd measures 90°
Find the value of each of F and K « 30 , $30\sqrt{3}$ newton »

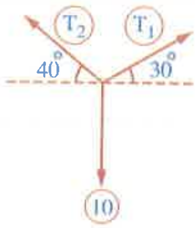
3 A body of weight 12 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force.
Find the magnitude of each of the force and the reaction of the plane. « $4\sqrt{3}$, $8\sqrt{3}$ kg.wt. »

4 A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. Find the magnitude of the weight W and the magnitude of the reaction of the plane. « 72 , $36\sqrt{3}$ newton »

5 The magnitudes of three coplanar concurrent forces are $F_1 = 8$ gm.wt. , $F_2 = 4\sqrt{3}$ gm.wt. and $F_3 = 4$ gm.wt. If these forces are in equilibrium , then find the measures of the angles between these forces. « 90° , 120° , 150° »

6 If M is the point of intersection of the two diagonals of a square ABCD , E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$ and F_1 , F_2 and 42 are the magnitudes of three forces in equilibrium act on $\overrightarrow{ME}$, $\overrightarrow{MF}$, $\overrightarrow{MD}$
Calculate the value of F_1 and F_2 « $21\sqrt{2}$, $21\sqrt{2}$ gm.wt. »

7 **In the opposite figure :**



A weight of magnitude 10 newton is suspended by two strings , the first is inclined by an angle of measure 30° to the horizontal and the second is inclined by an angle of measure 40° to the horizontal.

Find T_1 , T_2 in case of equilibrium. « 8.15 , 9.216 newton »

8 A string of length 40 cm. is fixed from its two ends at two points on a horizontal line where the distance between them is 32 cm. A body of weight 180 kg.wt. is suspended at the midpoint of the string. Find the values of the tensions in the two branches of the string. « 150 , 150 kg.wt. »

9 A body of weight 15 kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\sin^{-1}\frac{1}{2}$, a force inclined to the horizontal at an angle of measure 60° acted on the body to keep it in equilibrium. Find the magnitude of each of the force and the reaction of the plane. « $5\sqrt{3}$, $5\sqrt{3}$ kg.wt. »

10 A body of weight (W) kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\cos^{-1}\frac{1}{2}$, it is kept in equilibrium by means of a force inclined to the horizontal at an angle of measure 30° upwards. Find the magnitude of each of the force and the reaction of the plane in terms of (W) « F = r = W »

11 A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string in case of equilibrium. « 160 , 120 gm.wt. »

12 A body of weight 6.5 newton is suspended by two strings of lengths 0.5 and 1.2 m. the two other ends are fixed at two points on a horizontal line such that the strings are perpendicular to each other. Find the tension in each of the two strings in case of equilibrium. « 6 , 2.5 newton »

13 A weight of 50 gm.wt. is suspended by means of two perpendicular strings. If the tensions in the two strings are of magnitudes $25\sqrt{3}$, 25 gm.wt. Find the measures of the angles which the two strings are inclined to the vertical in case of equilibrium. « 30° , 60° »

14 A weight of 200 gm.wt. is suspended at the end of a light string , the other end of which is attached to the ceiling of a room. The weight is pulled by a horizontal force until the string is inclined to the vertical by an angle of measure 30°
Find the magnitude of each of the horizontal force and the tension in the string.

« $\frac{200\sqrt{3}}{3}$, $\frac{400\sqrt{3}}{3}$ gm.wt. »

---

15 A weight of 60 gm.wt. is suspended at the end of a string and the other end is fixed at a point of a vertical wall. A horizontal force of magnitude F acts on the weight in a perpendicular direction to the wall , the weight becomes in equilibrium when the string is inclined to the wall with an angle of measure θ where $\tan\theta = \frac{3}{4}$
Find the magnitude of each of F and the tension in the string. « 45 , 75 gm.wt. »

---

16 A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude F acts on the weight in a perpendicular direction of the string till it become in equilibrium when the string is inclined to the wall with an angle of measure 30°
Find the magnitude of the force F and the tension of the string. « 8 , $8\sqrt{3}$ newton »

---

17 The ball of a pendulum of weight 600 gm.wt. is displaced until the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string.
Find the magnitude of each of the force and the tension in the string. « 300 , $300\sqrt{3}$ gm.wt. »

---

18 A light string of length 170 cm. , its end A is fixed at a point of a ceiling of a room. From the other end B there is a lamp of weight 34 gm.wt. Find the magnitude of each of the tension and the required force to make the lamp in equilibrium at a distance 80 cm. down the ceiling in each of the following cases :
(1) If the force is horizontal. « 72.25 , 63.75 gm.wt. »
(2) If the force is perpendicular to $\overline{AB}$ « 16 , 30 gm.wt. »

---

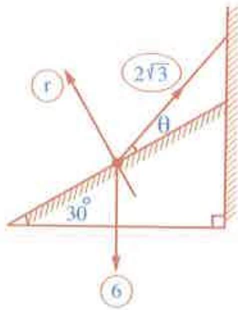
19 A body of weight 6 N. is placed on a smooth plane inclines to the horizontal by an angle θ. The body is kept in equilibrium by means of a force of magnitude $2\sqrt{3}$ N. inclines to the line of greatest slope of the plane by an angle of measure θ up.
Find the value of θ and the magnitude of the normal reaction of the plane.

« $\theta = 30°$, $r = 2\sqrt{3}$ N. »

20 A body is in equilibrium on a smooth plane inclines to the horizontal at an angle under the action of a force acting in the direction of the plane upwards. Its magnitude equals half the magnitude of weight of the body. Find the measure of the angle of inclination of the plane and the magnitude of the reaction of the plane.
« $30°$, $\frac{\sqrt{3}}{2}$ W »

21 **In the opposite figure :**

A body of weight 6 kg.wt. is placed on a smooth plane inclines to the horizontal by an angle of measure $30°$ The body is kept in equilibrium by a tension force (T) of magnitude $2\sqrt{3}$ kg.wt. The tension force acts along one end of the string of which is fixed by the body and the other end at a point on a vertical wall.
Find the measure of the angle of inclination of the string to the plane and the magnitude of the reaction of the plane on the body.



« $30°$, $2\sqrt{3}$ kg.wt. »

22 A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ The body is prevented from sliding by a force form with the line of the greatest slope an angle of measure $30°$ upwards.
Find the magnitude of the force and the reaction of the plane. « $100\sqrt{3}$, $100\sqrt{3}$ gm.wt.»

23 A body of weight 800 gm.wt. is placed on a smooth plane inclines to the horizontal by an angle θ , where $\sin\theta = 0.6$ the body is kept in equilibrium by a horizontal force.
Find the magnitude of this force and the reaction of the plane on the body.
« 600 , 1000 gm.wt. »

24 A smooth string of length 30 cm. is attached by its end in the two points A , B such that $\overline{AB}$ is horizontally , AB = 18 cm. if a smooth ring of weight 150 gm.wt. slides on the string. Prove that in the case of equilibrium the lengths of the two parts of the strings are equal , then find the tension in each part. « 93.75 gm.wt. »

25 A body of weight 24 newton is suspended at one end of a string of length 130 cm. , the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. Find the magnitudes of the force and the tension in the string.

(1) When the body is at a distance = 50 cm. from the wall. « 10 , 26 newton »

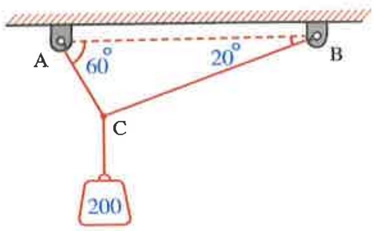
(2) When the string is inclined to the wall with an angle of measure $30°$ « $8\sqrt{3}$, $16\sqrt{3}$ newton »

26 A body of weight 72 gm.wt. is suspended at one end of a string. The other end of the string is fixed at a point A on a vertical wall. Another string is attached to the first one at a point B 25 cm. a part from A and pulled horizontally until the point B becomes 7 cm. apart the wall.
Find the tension in the horizontal string and in each part of the first string.

« 21 , 75 , 72 gm.wt. »

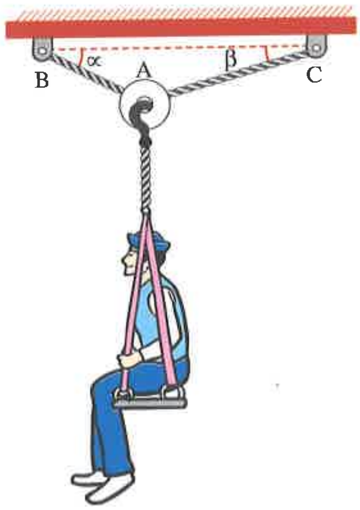
27 A particle of weight 200 gm.wt. is suspended by two light strings. One of them is inclined to the vertical by an angle of measure θ and the other inclined to the vertical by an angle of measure 30°. If the magnitude of the tension in the first string is 100 gm.wt. , then find θ and the magnitude of the tension in the second string. « 60° , $100\sqrt{3}$ gm.wt. »

28 The opposite figure represents a weight of magnitude 200 newton hanged vertically at a point C by two strings $\overline{BC}$ and $\overline{AC}$ which make with the horizontal the angles of measures 20° , 60° respectively find in the state of equilibrium the tension in the two strings to the nearest newton.



« 102 , 191 newton »

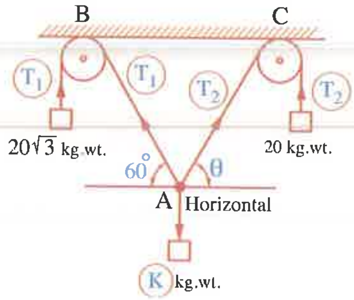
29 **Join with navigation :** The operation of saving a nautilus is done by using the captain chair which is hanged in a bully. Two ropes $\overline{AB}$ and $\overline{AC}$ are passing over the bully making two angles α , β with the horizontal whose measures are 25° , 15° respectively. If the tension in the rope $\overline{AB}$ equals 80 newton , find the weight of the nautilus and the chair together and the tension in the rope $\overline{AC}$ in the state of equilibrium.



« 53 , 75 newton »

30 **In the opposite figure :**

A weight of magnitude K is suspended by an end of a string , the other end is suspended by two strings passing over two smooth pulleys at B , C and carries two weights of magnitudes $20\sqrt{3}$ kg.wt. and 20 kg.wt.
Find the value of the weight K and the measure of angle θ in state of equilibrium.



« 40 kg.wt. , 30° »

Third Higher skills

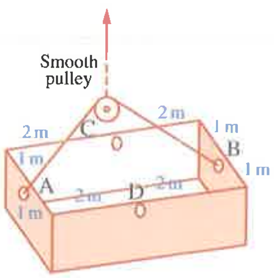
1 A body of weight 400 gm.wt. is suspended at point A by a string. From a point B on the string another string is attached and pulled horizontally by a second string $\overline{BC}$ passing over a smooth fixed pulley and carries at its other end a body of weight 300 gm.wt. Find the inclination of $\overline{AB}$ with the vertical and the tension in each of the two strings $\overline{AB}$, $\overline{BC}$

« 36° 52̀ , 500 , 300 gm.wt. »

2 $\overline{AB}$ is a light string , its two ends are fixed at two points on a horizontal line. C and D are two points of the string. Two weights K and 20 gm.wt. are suspended from C and D respectively. If the set of forces are in equilibrium when $\overline{CD}$ is horizontal and the two parts $\overline{AC}$ and $\overline{BD}$ of the string incline to the vertical by angles of measures 30° and 60° respectively. Find the magnitudes of tensions in the three parts of the string and the value of K

« $40\sqrt{3}$, $20\sqrt{3}$, 40 kg.wt. , K = 60 gm.wt. »

3 A box of weight 20 newton is suspended by a string as in the opposite figure. If the box can be fixed with the string through two methods one of them is A , B and the other is C , D which of these two methods can produce the less tension in the string to become the set in equilibrium ?



Exercise 5

Follow : The equilibrium

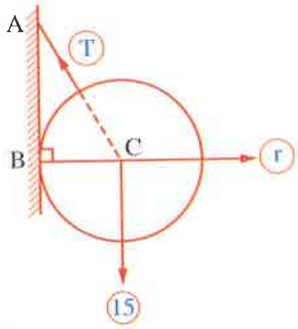
(Meeting lines of action of three equilibrium forces)

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In the opposite figure :

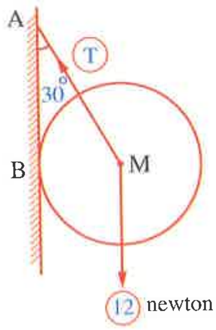


A solid uniform sphere of weight 15 gm.wt. and radius length 10 cm. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then (r , T) = ...

(a) $\left(4\sqrt{3} , 8\sqrt{3}\right)$ (b) $\left(5\sqrt{3} , 10\sqrt{3}\right)$

(c) (5 , 10) (d) $\left(5\sqrt{3} , 8\sqrt{3}\right)$

(2) In the opposite figure :



If the sphere is in equilibrium

, then T – r = ... newton

(Where r is the magnitude of the wall reaction on the sphere)

(a) $8\sqrt{3}$ (b) $4\sqrt{3}$

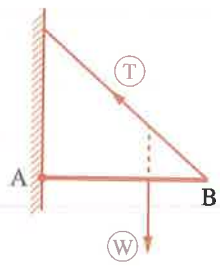
(c) 4 (d) 8

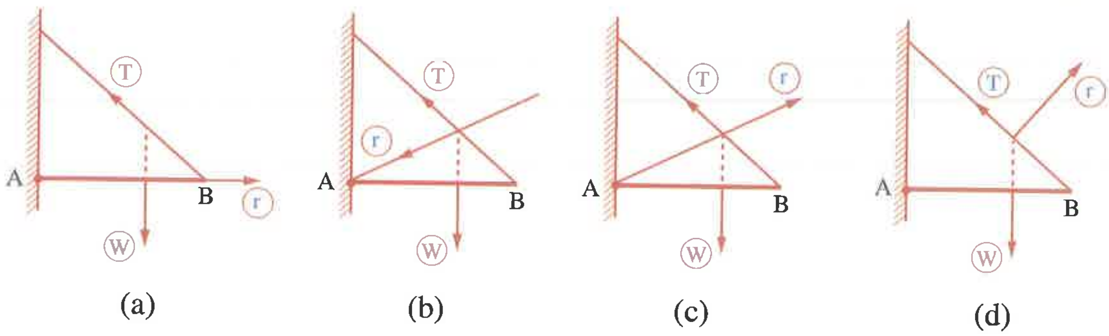
(3) A solid uniform sphere of weight 20 kg.wt. and its radius length 5 cm. If it is in equilibrium by a string of length 5 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then the reaction of the vertical plane r = ... kg.wt.

(a) $\frac{20}{\sqrt{3}}$ (b) 20 (c) $\frac{20}{\sqrt{5}}$ (d) zero

(4) In the opposite figure :

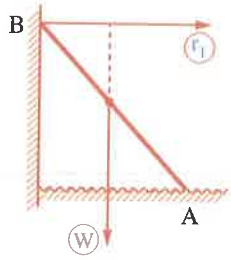
$\overline{AB}$ is a rod. The end A is attached to a hinge fixed on a vertical smooth wall , if the rod is in equilibrium , then which of the following figures represent the correct direction of the reaction of the hinge ?

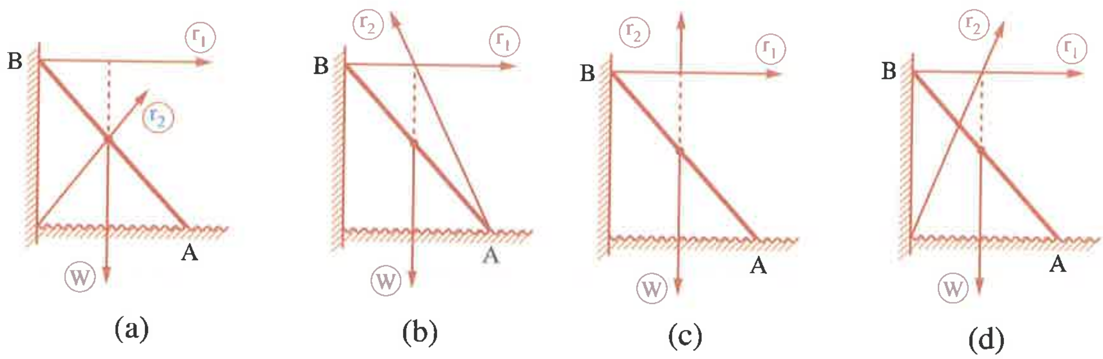




(5) In the opposite figure :

$\overline{AB}$ is a uniform rod and its weight is wrests at the end A against a horizontal rough ground , and the end B on a vertical smooth wall. Then which of the following figures represent the correct direction of the reaction of the ground ?





(6) In the opposite figure :

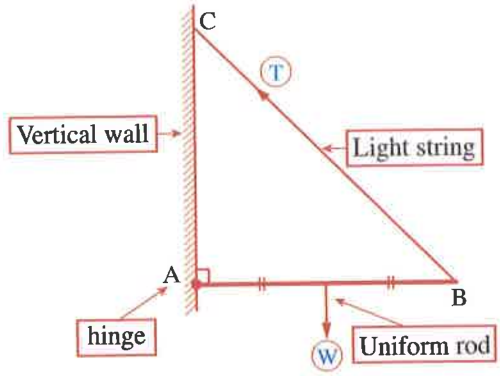
The direction of the reaction of the hinge on the rod at A ………

(a) in the direction of $\overrightarrow{AB}$

(b) in the direction of $\overrightarrow{AC}$

(c) bisects $\overline{BC}$

(d) perpendicular to $\overline{BC}$



(7) In the opposite figure :

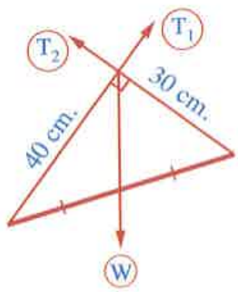
$T_1 : T_2 : W =$ ………

(a) 5 : 3 : 4

(b) 8 : 5 : 4

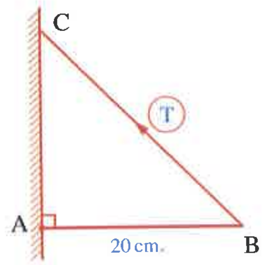
(c) 4 : 3 : 5

(d) 5 : 4 : 3



(8) In the opposite figure :

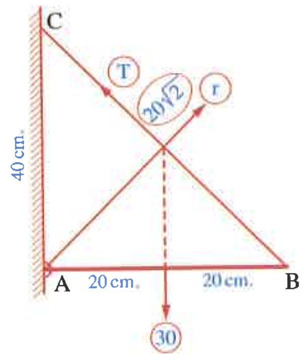
$\overline{AB}$ is uniform rod with length 20 cm. and weight 30 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B of length $20\sqrt{2}$ cm. fixed at a point C on the wall just above A , then magnitude of the reaction of the hinge = ……… newton.



(a) $10\sqrt{2}$ (b) 10 (c) 15 (d) $15\sqrt{2}$

(9) In the opposite figure :

$\overline{AB}$ is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A , AC = 40 cm.



First : The reaction of the hinge r = ……… newton.

(a) 30 (b) 20

(c) $40\sqrt{2}$ (d) $15\sqrt{2}$

Second : The tension of the string T = ……… newton.

(a) $15\sqrt{2}$ (b) 30 (c) 20 (d) $40\sqrt{2}$

(10) A uniform rod of weight 20 newton which is movable around a hinge at on its ends is pulled a side by a horizontal force of magnitude 10 newton acting on the other end , then the measure of the angle of inclination of the rod to the vertical when it is in equilibrium = ………

(a) 60° (b) 45° (c) 30° (d) 90°

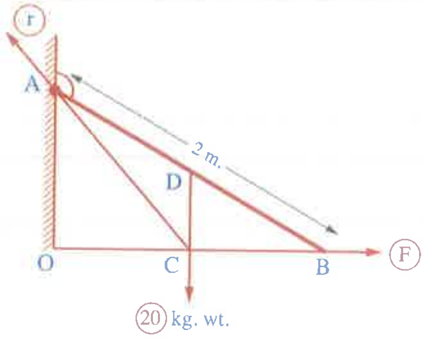
(11) A uniform rod of weight 24 newton is placed on two smooth planes inclined at two angles of measures 60° and 30° to the horizontal , then the magnitude of the pressure on each plane ……… newton.

(a) 12 , 15 (b) 12 , $12\sqrt{3}$

(c) $12\sqrt{3}$, 10 (d) 15 , 13

(12) **In the opposite figure :**

$\overline{AB}$ is a uniform rod of length 2 m. and weight 20 kg.wt. It is connected to a hinge fixed to a vertical wall at A. A horizontal force acts at B. If the rod is kept in equilibrium when it is inclined to the vertical at an angle of measure 60° , then the reaction of the hinge on the rod = ……… kg.wt.



(a) $10\sqrt{3}$ (b) $10\sqrt{5}$ (c) $10\sqrt{7}$ (d) $20\sqrt{2}$

Second Essay questions

1 A smooth sphere of radius length 30 cm. and of weight 200 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 20 cm. , one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the touch point of the sphere and the wall.
Find the magnitudes of the tension in the string and the reaction of the wall in case of equilibrium.

« 250 , 150 gm.wt. »

2 A smooth sphere of weight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertical an angle of measure 30°
Find the tension in the string and the reaction of the wall in case of equilibrium.

« 20 , 10 gm.wt. »

3 A smooth sphere of weight 15 newton is on a smooth vertical wall and suspended by a light string from a point on its surface. The other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere. If the length of the string equals the radius length of the sphere. Find the pressure on the wall and the tension in the string in case of equilibrium. « $5\sqrt{3}$, $10\sqrt{3}$ newton »

4 A metallic sphere of weight 15 kg.wt. is put such that it touches two smooth planes , one of them is vertical and the other inclines to the vertical by an angle of measure 30° Find the reaction of each of the two planes. « $15\sqrt{3}$, 30 kg. wt. »

5 $\overline{AB}$ is a unifrom rod of length 100 cm. and weight 30 kg.wt. is suspended from its two ends A and B by means of two strings , their other ends are fixed at a pin in the ceiling at the point C , if the two strings are perpendicular and AC = 50 cm.
Find the tension in each of the two strings. « 15 , $15\sqrt{3}$ kg.wt. »

6 A uniform rod of length 130 cm. and weight 26 newton is suspended at its ends by two strings tied at one point. If the length of one of them is 50 cm. and the length of the other one is 120 cm. What is the position in which the rod is in equilibrium and what is the tension in each of the two strings ? « 24 , 10 newton »

7 $\overline{AB}$ is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod keept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall just above A and at a distance 60 cm. from A. Find the tension on the string and the reaction on the hinge at A.
« $20\sqrt{2}$, $20\sqrt{2}$ newton »

8 $\overline{AB}$ is a uniform rod of length 80 cm. and weight 24 kg.wt. The end A is attached to a hinge fixed on a vertical wall , and the end B is tied by a light string of length $80\sqrt{3}$ cm. fixed at a point C on the wall which lies directly above A and at a distance 80 cm. If the rod is in equilibrium, find the magnitude of the tension and the reaction of the hinge.
« $12\sqrt{3}$, 12 kg.wt. »

9 A homogeneous sphere rests on two parallel rods lie on the same horizontal plane. The distance between them equals the radius length of the sphere. Find the pressure on each rod if the weight of the sphere is 60 newton in case of equilibrium. « $20\sqrt{3}$, $20\sqrt{3}$ newton »

10 A sphere in which M is its centre and its radius length is 12 cm. and its weight is (W) newton rests at B against a smooth vertical wall , from a point C on its surface , it is tied by a string , its other end is fixed at A of the wall lies directly above B
If the tension in the string is 50 newton. Find the length of the string and the weight of the sphere when the reaction of the wall to the sphere equals 25 newton.

« 12 cm. , $25\sqrt{3}$ newton »

11 A uniform rod whose length is 80 cm. and its weight is 12 newton , the rod is freely suspended from its ends by means of two strings, and the other ends are attached to a fixed nail in the ceiling. If the two strings are perpendicular and one of them is of length 48 cm. Find in equilibrium the magnitude of the tension in each of the two strings.

« 7.2 , 9.6 newton »

12 $\overline{AB}$ is a uniform ladder of weight 36 kg.wt. rests at the end A against a vertical smooth wall , and the other end B on a horizontal rough ground. If the ladder is in equilibrium when its end A is at a distance 3 metres from the ground and the end B is at a distance 2.5 metres from the wall. Find the reaction of each of the ground and the wall on the ladder.

«15 , 39 kg.wt. »

13 $\overline{AB}$ is not a uniform rod of length 60 cm. and weight 16 kg.wt. acts at a point D on the rod where AD = 20 cm. The rod is attached to a hinge at A and the hinge is fixed on a vertical wall. The end B of the rod is tied by a light string its other end is fixed at a point C on the wall lying directly above A and at a distance 80 cm. from it , then the rod becomes in equilibrium such that it is perpendicular to the wall.
Find the tension in the string and the reaction of the hinge.

« $6\frac{2}{3}$, $\frac{4\sqrt{73}}{3}$ kg.wt. »

14 $\overline{AB}$ is a uniform rod of length 2 L cm. and weight 8 kg.wt. acting at its midpoint. its end A is hinged at a point in a vertical wall where its end B is attached to a light string and the other end of the string is fixed to a point C on the wall situated vertically above A
If AB = AC = BC and the rod is in equilibrium. Find the tension in the string and the reaction of the hinge at A

« 4 kg.wt. , $4\sqrt{3}$ kg.wt. »

15 $\overline{AB}$ is a uniform rod of length 60 cm. and weight (W) kg.wt. The end A is attached to a hinge fixed on a vertical wall and the end B is tied by a string of length 80 cm. , its other end is fixed to a point on the wall vertically above A directly and at a distance 100 cm. of it , then the rod became in equilibrium. Find the tension in the string and the reaction of the hinge , also find the measure of the angle of inclination of the reaction of the hinge to the rod.

« $\frac{2}{5}$ W , $\frac{\sqrt{13}}{5}$ W kg.wt. , 33° 41̀ »

16 $\overline{AB}$ is a uniform rod of length 90 cm. , and weight (W) kg.wt. Its end A is fixed to a vertical wall by a hinge and the rod is kept in equilibrium horizontally by means of a string of length 50 cm. , one of its ends is tied at the point C on the rod at a distance 30 cm. from A , the other end of the string is fixed at a point D on the vertical wall above A directly, calculate the tension in the string and the reaction of the hinge on the rod.

« $\frac{15}{8}$ W , $\frac{\sqrt{97}}{8}$ W kg.wt. »

17 $\overline{AB}$ is a uniform rod , its end A is attached by a hinge fixed in a vertical wall. A horizontal force acts at the end B to keep the rod in equilibrium while it is inclined to the wall by an angle of measure 45° , if the weight of the rod is 4 kg.wt. acts at its midpoint , then find the magnitude of the force and the reaction of the hinge.

« 2 , $2\sqrt{5}$ kg.wt. »

18 A uniform rod which is movable around one of its ends is pulled a side by a horizontal force acting on the other end and equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the vertical when it is in equilibrium and also the reaction at the first end.

« 45° , $\frac{\sqrt{5}}{2}$ of the weight of the rod »

19 A uniform rod of weight 4 newton is placed on two smooth planes inclined at 30° and 60° to the horizontal. Find the magnitude of the pressure on each plane and the measure of the angle of inclination of the rod to the horizontal in state of equilibrium.

« $2\sqrt{3}$, 2 newton , 30° »

20 A smooth iron sphere of weight (W) kg.wt. rests against a vertical smooth wall and a smooth plane inclines to the horizontal at an angle θ where $\cos\theta = \frac{3}{5}$, if the sphere is in equilibrium. Find the pressure on each of the wall and the inclined plane.

« $\frac{4}{3}$ W , $\frac{5}{3}$ W kg.wt. »

21 A uniform rod of weight 20 kg.wt. rests at one of its ends against a smooth vertical plane and at the other end on a smooth plane inclined to the vertical at an angle of measure 60°, in the state of equilibrium. Find the magnitude of each of the two reactions of the two planes , also find the measure of the angle at which the rod inclines to the vertical.

« $\frac{20\sqrt{3}}{3}$, $\frac{40\sqrt{3}}{3}$ kg.wt. , 49° 6 »

22 A uniform rod $\overline{AB}$ of weight 8 newton acting at its midpoint is placed on two smooth perpendicular planes that are inclined to the horizontal. Such that the vertical plane of the rod and the two lines of greatest slope of the two inclined planes is perpendicular to the intersection line of the two planes. If the magnitude of the pressure on the plane at the end B is 4 newton.

Find the magnitude of the pressure on the other plane and measures of the two inclination angles of the planes to the horizontal , in the state of equilibrium.

« $4\sqrt{3}$ newton , 30° , 60° »

23 A uniform hollow sphere of radius length (r) and weight $12\sqrt{3}$ kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30°, it is prevented from motion on the plane by means of a string fixed at a point on its surface , the length of the string equals the radius length of the sphere. The other end of the string is fixed at a point on the inclined plane. In the state of equilibrium , prove that the string is horizontal , then find the tension in the string and the reaction of the inclined plane upon the sphere.

« 12 , 24 kg.wt. »

24 $\overline{AB}$ is a uniform rod can move in a vertical plane freely around a hinge at A, the other end B is tied to a string passes over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the horizontal in state of equilibrium given that AC = AB « 30° »

25 $\overline{AB}$ is a uniform rod which is 40 cm. long and weight 12 N. The rod rests with its end A on a vertical smooth wall. It is kept in equilibrium by means of a light inextensible string , one of its ends is attached to point C where $C \in \overline{AB}$ and BC = 10 cm.
, and the other end is fixed to a point D on the wall directly vertical above A.
If the rod is inclined by an angle whose measure is 60° to the vertical , then find the magnitudes of the tension in the string and the reaction of the wall. « $8\sqrt{3}$, $4\sqrt{3}$ N. »

26 A uniform rod $\overline{AB}$ of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A. The rod is kept horizontally by attaching it at a point C on the rod (where AC = 4 metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge in case of equilibrium.

« $6\sqrt{2}$, $2\sqrt{10}$ kg.wt. »

Use the resolution to solve

27 A body of weight 100 newton is placed on a smooth plane inclined to the horizontal at an angle of measure θ where $\sin\theta = \frac{3}{5}$, the body is kept in equilibrium by means of a force inclines to the line of the greatest slope at an angle of measure α where $\cos\alpha = \frac{12}{13}$

Find F and the reaction of the plane. « 65 , 55 newton »

28 A body is placed on a smooth plane inclined to the horizontal at an angle of measure 30°, it is kept in equilibrium by means of two forces , one of them in the direction of the line of the greatest slope upwards , its magnitude = 50 newton and the second inclines to the line of the greatest slope upwards with an angle of measure 30° and its magnitude is $20\sqrt{3}$ newton. Find each of the weight of the body and the reaction of the plane.

« 160 , $70\sqrt{3}$ newton »

29 A smooth ring and a string passes through it. The length of the string is 40 cm. its two ends are fixed at the two points A and B on the same horizontal line , the distance between them is 20 cm. A horizontal force $\vec{F}$ acts on the ring to be in equilibrium vertically down B and the string is in tension. Find the value of F and the magnitude of tension in the string given that the weight of the ring = 400 gm.wt.

« 200 , 250 gm.wt »

UNIT 2

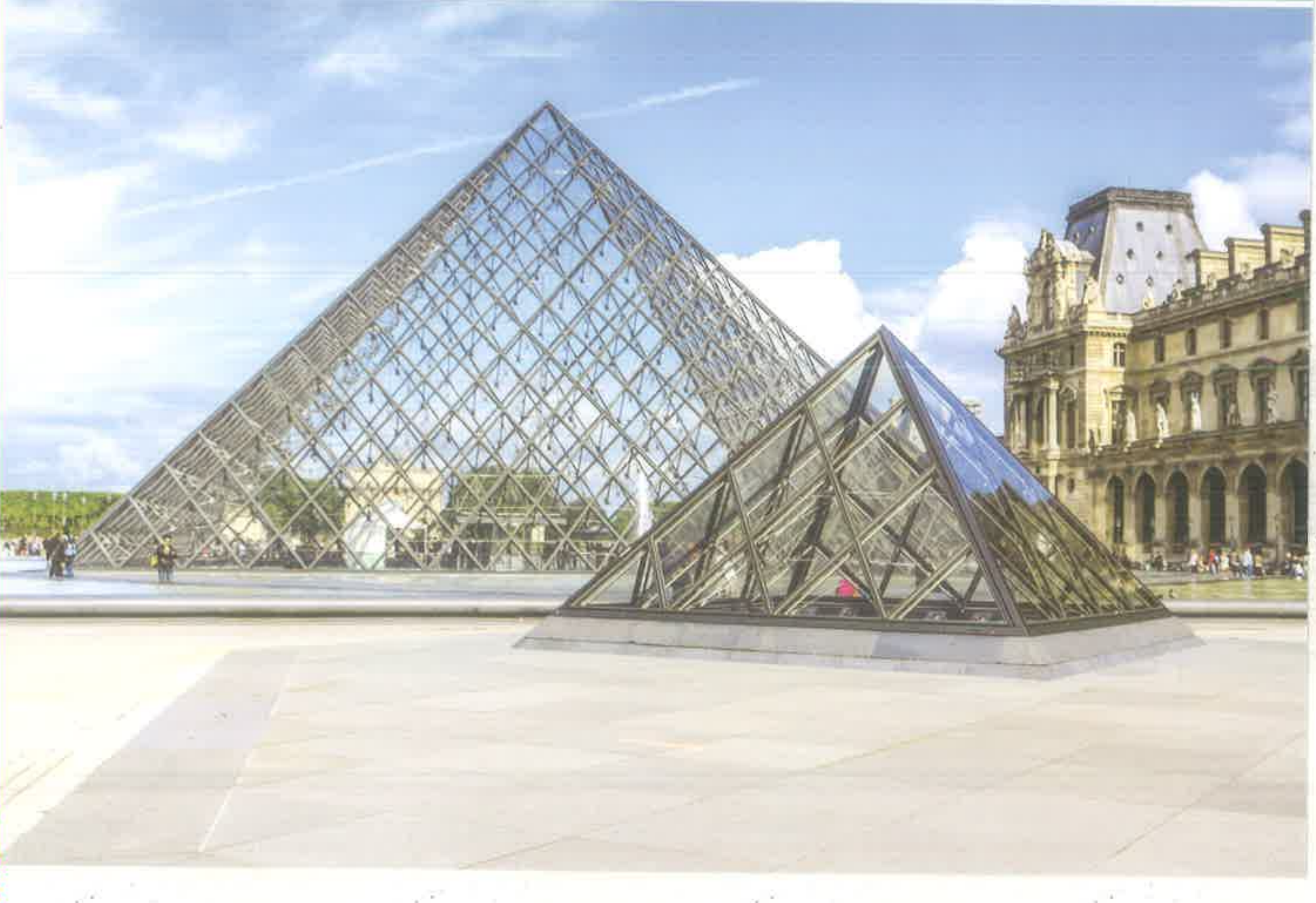
Geometry and measurement

Exercise 6 The straight lines and the planes in the space.

Exercise 7 The pyramid.

Exercise 8 The cone.

Exercise 9 The circle.



Exercise 6

The straight lines and the planes in the space

Interactive test

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) Number of straight lines that are passing through a given point is ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(2) Number of straight lines that are passing through two given points is ....

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(3) Number of planes that are passing through two given points is ....

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(4) Number of planes that are passing through three non-collinear points is ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(5) Number of planes that are passing through three collinear points is ....

(a) Zero (b) 1 (c) 3 (d) an infinite number.

(6) All of the following cases determine a plane except ....

(a) a straight line and a point doesn't belong to it.

(b) two parallel and not coincident straight lines.

(c) two intersecting straight lines.

(d) two skew straight lines.

(7) All of the following cases determine a plane except ....

(a) two intersecting straight lines.

(b) two different parallel straight lines.

(c) a straight line and a point belong to it.

(d) three non-collinear points.

(8) Number of planes that are passing through two different parallel straight lines = ...

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(9) Two skew straight lines are two straight lines which are ....

(a) not intersect. (b) not perpendicular.

(c) not parallel. (d) not intersect and not parallel.

(10) The two straight lines are skew if they are ....

(a) not parallel. (b) not intersecting.

(c) not coincident. (d) not contained in the same plane.

(11) If the straight line L // the plane X and $A \in X$, then $L \cap X =$ ....

(a) $\varnothing$ (b) L (c) X (d) $\{A\}$

(12) If the straight line $L \subset$ the plane X and $A \in X$, then $L \cap X =$ ....

(a) $\varnothing$ (b) L (c) X (d) $\{A\}$

(13) If the two straight lines L_1 and L_2 are skew, then $L_1 \cap L_2 =$ ....

(a) $\varnothing$ (b) L_1 (c) L_2 (d) the plane contains L_1 and L_2

(14) The two not parallel planes are intersecting in ....

(a) a point. (b) a straight line.

(c) a plane. (d) ray.

(15) If X, Y are two planes where $X \cap Y = \varnothing$, then X .... Y

(a) $\perp$ (b) // (c) = (d) $\subset$

(16) The two planes are coincident if they have .... in common.

(a) only one point. (b) two points.

(c) three collinear points. (d) three non-collinear points.

(17) If a straight line and a plane have two point in common, then the straight line ....

(a) is parallel to the plane.

(b) intersects the plane in only one point.

(c) lies completely inside the plane.

(d) intersects the plane in only two points.

(18) If A , B and C are three points determine a plane , then ....

(a) AB = BC = AC (b) AB + BC = AC

(c) AB + BC > AC (d) AB + BC < AC

(19) All different vertical straight lines in the space are ....

(a) parallel. (b) skew.

(c) contained in the same plane. (d) intersecting.

(20) Relative position of two straight lines in one plane could be each of the following except ....

(a) parallel. (b) intersecting. (c) skew. (d) coincident.

(21) If X , Y and Z are planes in the space where $X \cap Y \cap Z = \{A\}$ and $X \cap Y$ = the straight line L , then which of the following is not true ?

(a) $A \in L$ (b) $L \cap Z = \{A\}$

(c) $L // Z$ (d) $A \in Z$

(22) If M is a point outside the plane that contains the three points A , B and C , then $\overleftrightarrow{MA}$ ....

(a) lies completely inside the plane. (b) intersects the plane at a point.

(c) intersects the plane at two points. (d) is parallel to the plane.

(23) If $\overleftrightarrow{AB} \subset$ plane X , $\overleftrightarrow{CD}$ // plane X , then $\overleftrightarrow{CD}$, $\overleftrightarrow{AB}$ are ....

(a) parallel only. (b) skew only.

(c) parallel or skew. (d) intersecting.

(24) X and Y are two parallel planes and straight line $L_1 \subset X$ and straight line $L_2 \subset Y$, then which of the following can not be happen ?

(a) $L_1 // L_2$ (b) L_1 and L_2 are skew.

(c) $L_1 // Y$ and $L_2 // X$ (d) L_1 and L_2 are intersecting.

(25) The least number of planes that determine a solid is ....

(a) 1 (b) 2 (c) 3 (d) 4

(26) ABCD $A'B'C'D'$ is a cuboid , how many straight lines carry edge from the edges of the cuboid and skew to $\overleftrightarrow{AB}$?

(a) not exist (b) one (c) two (d) four

(27) Which of the following statements is not true ?

(a) For any two points in the space , there is only one plane is passing through them.

(b) Any three non-collinear points in the space determine a plane.

(c) Vertices of the triangle determine a plane.

(d) For each two intersecting straight lines there is only one plane contains them.

(28) Which of the following statements is not true ?

(a) Any two different parallel straight lines determine a plane.

(b) For any two different intersecting straight lines there is only one point in common.

(c) The two skew straight lines can't be contain in the same plane.

(d) For any three non-collinear points there is one plane passing through them at least.

(29) Which of the following statements is not true ? (where L_1 and L_2 are two straight lines , X and Y are two planes) ?

(a) If $L_1 \cap L_2 = \varnothing$, then $L_1 // L_2$ or L_1 and L_2 are skew.

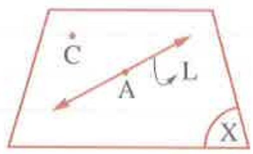
(b) If $L_1 \cap X = \varnothing$, then $L_1 // X$

(c) If $L_2 \cap X = L_2$, then $L_2 \subset X$

(d) If $L_2 \subset Y$, then $L_2 \cap Y = \varnothing$

(30) Using the opposite figure ,

which of the following statements is not true ?

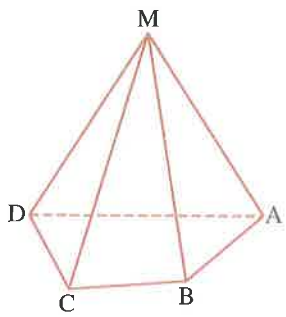


(a) $L \subset X$ (b) $A \in L , A \notin X$

(c) $C \in X , C \notin L$ (d) $\overline{AC} \cap L = \{A\}$

(31) **In the opposite figure :**

The plane ABD $\cap$ The plane MCD = ...



(a) $\overleftrightarrow{AM}$

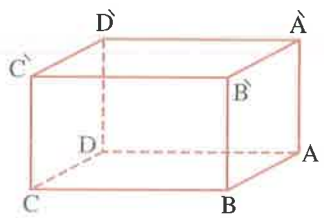
(b) $\overleftrightarrow{CD}$

(c) $\{D\}$

(d) $\overleftrightarrow{MC}$

(32) **In the opposite figure :**

The plane $AA'B'$ $\cap$ the plane $A'CC'$ = ...

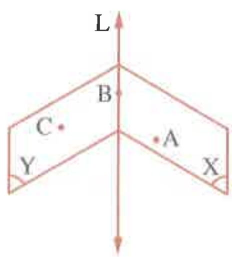


(a) $\overleftrightarrow{AA'}$ (b) $\overleftrightarrow{BB'}$

(c) $\overleftrightarrow{CC'}$ (d) $\overleftrightarrow{A'C}$

(33) **In the opposite figure :**

The plane X $\cap$ the plane Y = ...



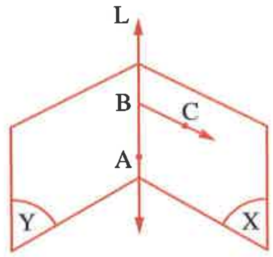
(a) $\{B\}$

(b) $\{A , B , C\}$

(c) the straight line L

(d) $\varnothing$

(34) In the opposite figure :



First : L ·········· X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

Second : A ·········· X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

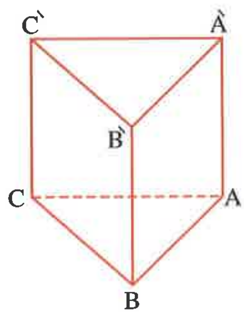
Third : C ·········· Y

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

Fourth : $\overleftrightarrow{BC}$ ·········· Y

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

(35) In the opposite figure :



First : The plane $ABB'A'$ $\cap$ the plane $BCC'B'$ = ··········

(a) $\overleftrightarrow{BB'}$ (b) $\varnothing$ (c) $\{B'\}$ (d) $\overleftrightarrow{AC}$

Second : The plane ABC $\cap$ the plane $A'B'C'$ = ··········

(a) $\overleftrightarrow{BB'}$ (b) $\varnothing$ (c) $\overleftrightarrow{AA'}$ (d) $\overleftrightarrow{AB}$

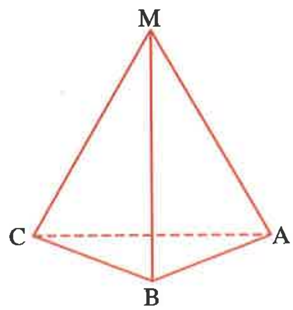
Third : $\overleftrightarrow{AC} \cap \overleftrightarrow{A'C'}$ = ··········

(a) $\{A\}$ (b) $\{C'\}$ (c) $\overleftrightarrow{AA'} \cap \overleftrightarrow{BB'}$ (d) $\overleftrightarrow{AC}$

Fourth : $\overleftrightarrow{BB'}$ $\cap$ the plane ABC = ··········

(a) $\overleftrightarrow{BB'}$ (b) $\{B'\}$ (c) $\{B\}$ (d) $\varnothing$

(36) In the opposite figure :



First : The plane MAB $\cap$ the plane MBC = ··········

(a) $\overleftrightarrow{AB}$ (b) $\overleftrightarrow{MB}$ (c) $\varnothing$ (d) $\{M\}$

Second : The plane MBC $\cap$ the plane ABC = ··········

(a) $\{B\}$ (b) $\varnothing$ (c) $\overleftrightarrow{AB}$ (d) $\overleftrightarrow{BC}$

Third : $\overleftrightarrow{MB}$ $\cap$ the plane ABC = ··········

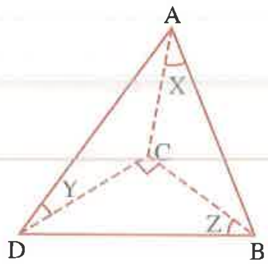
(a) $\overleftrightarrow{MB}$ (b) $\varnothing$ (c) $\{B\}$ (d) $\{M\}$

Fourth : The plane MAB $\cap$ the plane MBC $\cap$ the plane MAC = ··········

(a) $\overleftrightarrow{MB}$ (b) $\overleftrightarrow{MC}$

(c) the solid MABC (d) $\{M\}$

(37) In the opposite figure :



If $A \notin$ the plane BCD , then :

First : $X \cap Y =$ ....

(a) $\overleftrightarrow{AC}$ (b) $\varnothing$ (c) $\{A\}$ (d) $\{C\}$

Second : $X \cap Z =$ ....

(a) $\varnothing$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{AC}$ (d) $\{C\}$

Third : $Y \cap Z =$ ....

(a) $\{C\}$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{CD}$ (d) $\varnothing$

Fourth : $\overleftrightarrow{AB} \cap X =$ ....

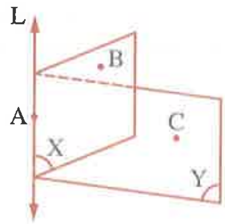
(a) $\overleftrightarrow{AB}$ (b) $\varnothing$ (c) $\overleftrightarrow{AC}$ (d) $\{B\}$

Fifth : Let m ($\angle$ BCD) = 90° , BC = 3 cm. , CD = 4 cm.

, then BD = ... cm.

(a) 6 (b) 5 (c) 4 (d) 7

(38) In the opposite figure :



X and Y are two intersecting planes at the straight line L

, $A \in L$, $B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$:

First : The plane X $\cap$ the plane ABC = ...

(a) $\overleftrightarrow{AB}$ (b) $\overleftrightarrow{AC}$ (c) $\overleftrightarrow{BC}$ (d) L

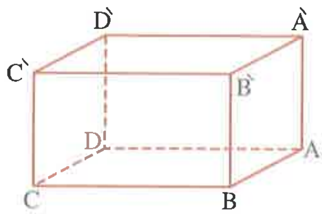
Second : The plane Y $\cap$ the plane ABC = ...

(a) $\overleftrightarrow{AB}$ (b) $\{A\}$ (c) $\overleftrightarrow{CB}$ (d) $\overleftrightarrow{AC}$

Third : The plane X $\cap$ the plane Y $\cap$ the plane ABC = ...

(a) $\varnothing$ (b) L (c) $\{A\}$ (d) $\{B\}$

(39) In the opposite figure :



First : The plane ABCD // the plane ....

(a) ABC' (b) $A'B'D'$

(c) ABB' (d) $A'B'C$

Second : The plane $BCC'B'$ // the plane ....

(a) ABC (b) $A'B'C'$ (c) ABD (d) $AA'D$

Third : The plane $ABB'A'$ $\cap$ the plane ABCD = ...

(a) $\{B\}$ (b) $\{A , B\}$ (c) $\overline{AB}$ (d) $\overleftrightarrow{AB}$

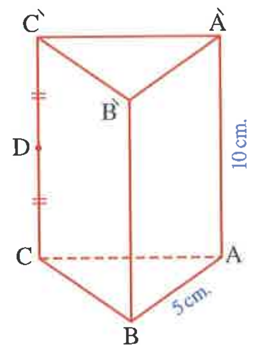
Fourth : The plane $ABB'A'$ $\cap$ the plane $DCC'D'$ = ...

(a) $\overleftrightarrow{BC}$ (b) $\overleftrightarrow{AD}$ (c) $\varnothing$ (d) $\{C\}$

Fifth : The plane $DCC'D'$ $\cap$ the plane $ABCD$ $\cap$ the plane $A'ADD'$ = ...

(a) $\varnothing$ (b) $\overleftrightarrow{AB}$ (c) $\{C\}$ (d) $\{D\}$

(40) In the opposite figure :



$ABB'A'$, $BB'C'C$, $ACC'A'$ are three congruent rectangles, each pairs are intersecting, D is the midpoint of $\overline{CC'}$, if AB = 5 cm., AA' = 10 cm.

First : The plane ADA' $\cap$ the plane BDB' = ...

(a) $\overleftrightarrow{CC'}$ (b) $\overleftrightarrow{BB'}$ (c) $\{D\}$ (d) $\overleftrightarrow{AC'}$

Second : The plane ADB $\cap$ the plane ABC = ...

(a) $\varnothing$ (b) $\overleftrightarrow{AB}$ (c) $\{B\}$ (d) $\overleftrightarrow{AC}$

Third : The plane ADB $\cap$ the plane $BCC'B'$ = ...

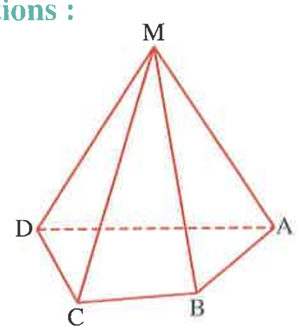
(a) $\{B\}$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{BD}$ (d) $\varnothing$

Fourth : $m(\angle BDB')$ = ...°

(a) 60 (b) 120 (c) 90 (d) 100

Second Essay questions

1 Meditate the opposite figure and answer the following questions :



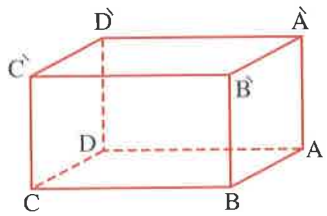
(1) How many lines which carry edges in the figure ?

(2) State the names of the straight lines which carry edges and passing through point A

(3) How many planes which carry faces in the figure ?

(4) State the names of three planes passing through point A

2 Meditate the opposite figure and answer the following questions :



(1) Write three straight lines passing through point A

(2) Write the straight lines passing through points A and B together.

(3) Write three planes passing through point A

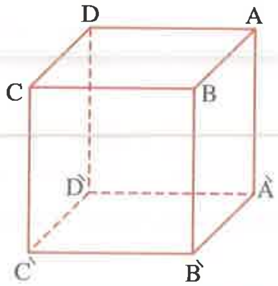
(4) Write three planes passing through points A and B together.

3 The opposite figure represents a classroom , find :

(1) The lines which carry edges and intersect with $\overleftrightarrow{AB}$

(2) The lines which carry edges and parallel to $\overleftrightarrow{AB}$

(3) The lines which carry edges and skew to $\overleftrightarrow{AB}$



4 Write the number of planes which passing through :

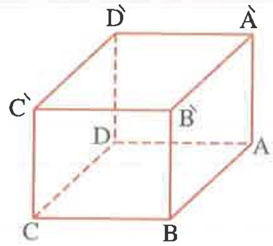
(1) One given point.

(2) Two different points.

(3) Three collinear points.

(4) Three non-collinear points.

5 In the opposite figure , $ABCDA'B'C'D'$ is a cube of edge length 6 cm.



(1) Identify the relative positions for each pair of the following straight lines :

① $\overleftrightarrow{AB}$, $\overleftrightarrow{DD'}$

② $\overleftrightarrow{AB}$, $\overleftrightarrow{D'C'}$

③ $\overleftrightarrow{AB}$, $\overleftrightarrow{A'D}$

④ $\overleftrightarrow{AC}$, $\overleftrightarrow{A'C'}$

⑤ $\overleftrightarrow{AB}$, $\overleftrightarrow{DC'}$

⑥ $\overleftrightarrow{AC}$, $\overleftrightarrow{AA'}$

(2) Identify the relative positions for each pair of the following planes :

① $ABB'A'$, $DCC'D'$

② $A'B'BA$, $A'B'CD$

③ ABC , $D'B'D$

(3) If $\overleftrightarrow{AB} \perp \overleftrightarrow{AD'}$, Find the length of $\overline{BD'}$

6 Draw the figures which represent the plane (X) , the straight line (L) and the point (A) in the following cases :

(1) $A \in L$

(2) $L \subset X$

(3) $L \cap X = \{A\}$

(4) $L \parallel X$

(5) $A \in X$, $A \notin L$, $L \subset X$

Exercise 7 The pyramid

Interactive test

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The line segment joining the vertex of the pyramid and any vertex of its base vertices is called ....

(a) the height of the pyramid. (b) the slant height of the pyramid.

(c) the lateral edge of the pyramid. (d) the side of its base.

(2) If MABCD is a regular quadrilateral pyramid , then this pyramid must be ....

(1) regular faces (2) its base is a square (3) right

(a) (1) , (2) (b) (2) , (3) (c) (1) only (d) (1) , (2) , (3)

(3) Which of the following statements is true ?

(a) The lateral faces of the right pyramid are congruent.

(b) The regular pyramid is a right pyramid.

(c) The heights of the lateral faces of the right pyramid are equal.

(d) The least number of planes that can determine a solid = 3 planes.

(4) Which of the following statements is not true ?

(a) The base of the right pyramid can be a surface of a rhombus.

(b) The triangular pyramid has three faces.

(c) The pentagonal pyramid has six faces.

(d) All lateral faces of the quadrilateral pyramid are surfaces of triangles.

(5) In the regular pyramid , which of the following is the right ascendingly order of the lengths ?

(a) The length of the lateral edge , the height , the slant height.

(b) The height , the slant height , the length of the lateral edge.

(c) The slant height , the height , the length of the lateral edge.

(d) The length of the lateral edge , the slant height , the height.

(6) The shape of the base of a regular pyramid must be ....

(a) parallelogram. (b) rhombus.

(c) rectangle. (d) square.

(7) If MABCD is a regular quadrilateral pyramid , then all lateral edges are ....

(a) parallel. (b) congruent.

(c) perpendicular to the base. (d) mutually perpendicular.

(8) If MABC is a right triangular pyramid , N is the projection of the point M on the plane ABC , E is midpoint of $\overline{BC}$, then all the following triangles are right except ....

(a) Δ MNC (b) Δ MNE (c) Δ MBC (d) Δ MNA

(9) If MABC is a regular faces pyramid , N is the projection of the point M on the plane ABC , E is the midpoint of $\overline{AB}$, which of the following is an equilateral triangle ?

(a) Δ MNE (b) Δ MBE (c) Δ ACE (d) Δ MBC

(10) The number of all faces of a regular pentagonal pyramid is ....

(a) 5 (b) 6 (c) 7 (d) 10

(11) If the number of the faces of a pyramid = m and the number of its vertices = n , then the number of its edges = ...

(a) m + n (b) m + n – 1 (c) m + n – 2 (d) m + n + 2

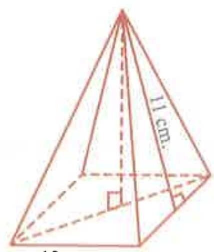
(12) In the hexagonal pyramid :

number of faces + number of vertices – number of edges = ...

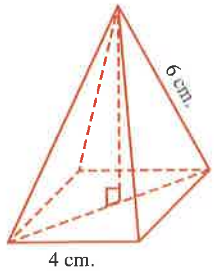
(a) 1 (b) 2 (c) 3 (d) 4

(13) The opposite figure represents a regular quadrilateral pyramid of height = ... cm.

(a) $\sqrt{85}$ (b) $\sqrt{157}$ (c) 10 (d) 8



(14) The opposite figure represents a regular quadrilateral pyramid of height = ... cm.



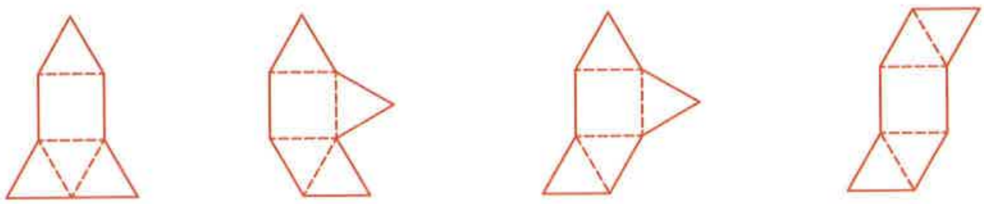
(a) $7\sqrt{2}$ (b) $2\sqrt{7}$

(c) $4\sqrt{2}$ (d) $2\sqrt{5}$

(15) A regular quadrilateral pyramid , if the length of its base side is 6 cm. , the length of its lateral edge is 8 cm. , then the length of its height = ... cm.

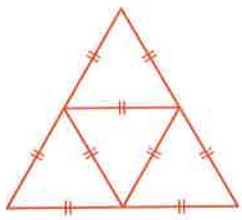
(a) $5\sqrt{2}$ (b) $\sqrt{46}$ (c) $\sqrt{85}$ (d) $\sqrt{48}$

(16) Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



(a) (b) (c) (d)

(17) Which solid represents the opposite net ?



(a) Quadrilateral pyramid.

(b) Regular quadrilateral pyramid.

(c) Triangular regular faces pyramid.

(d) Otherwise.

(18) The ratio between the edge length of the triangular regular faces pyramid : its height = ...

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

(19) The ratio between the length of the edge of the regular faces pyramid : its slant height = ...

(a) $2\sqrt{2} : \sqrt{3}$ (b) $2\sqrt{3} : 3$ (c) $\sqrt{6} : 2$ (d) $\sqrt{6} : 3$

(20) If we cut a regular quadrilateral pyramid by a plane parallel to its base , then the resulted section is ....

(a) triangle. (b) square. (c) rectangle. (d) circle.

(21) A right quadrilateral pyramid of height 10 cm. , its base is a rhombus whose diagonal lengths are 12 cm. and 8 cm. , then its volume = ... cm^3

(a) 40 (b) 80 (c) 160 (d) 200

(22) A regular quadrilateral pyramid whose base perimeter is 36 cm. , and its height is 10 cm. , then its volume = ... cm^3

(a) 810 (b) 180 (c) 360 (d) 270

(23) A regular hexagonal pyramid , the side length of its base = 8 cm. and its height = 10 cm. , then its volume equal .... cm^3

(a) $320\sqrt{3}$ (b) $960\sqrt{3}$ (c) $\frac{320\sqrt{3}}{3}$ (d) 160

(24) A regular pyramid whose volume is 12 cm^3 , and its base area is 4 cm^2 , then its height = ... cm.

(a) 3 (b) 6 (c) 9 (d) 2

(25) A regular quadrilateral pyramid whose volume 64 cm^3 , and its height is 6 cm. , then its base perimeter = ... cm.

(a) 8 (b) $8\sqrt{2}$ (c) 16 (d) $16\sqrt{2}$

(26) A regular quadrilateral pyramid whose volume is 480 cm^3 , and its base length is 12 cm. , then the length of its height = ... cm.

(a) 10 (b) 20 (c) 30 (d) 15

(27) If the volume of a regular hexagonal pyramid equal $8\sqrt{3}$ cm^3 , and its height length equal 4 cm. , then the perimeter of its base = ... cm.

(a) 2 (b) 12 (c) 6 (d) $6\sqrt{3}$

(28) In a regular quadrilateral pyramid , the length of its base is 10 cm. , the length of its slant height is 13 cm. , then its lateral area equal .... cm^2

(a) 260 (b) 360 (c) 130 (d) 520

(29) A regular quadrilateral pyramid , the area of its base = 100 cm^2 and its height is 12 cm. , then its lateral area equal .... cm^2

(a) 260 (b) 520 (c) 130 (d) 360

(30) The total area of a right quadrilateral pyramid , its base is a regular polygon and its diagonal length = $10\sqrt{2}$ cm. and its height = $5\sqrt{3}$ cm. equals .... cm^2

(a) 40 (b) 100 (c) 200 (d) 300

(31) A regular quadrilateral pyramid whose lateral area = 30 cm^2 , and its slant height = 5 cm. , then its base perimeter = ... cm.

(a) 6 (b) 12 (c) 24 (d) 36

(32) A triangular regular faces pyramid , its edge length 10 cm. , then its total area equal .... cm^2

(a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$

(33) If the sum of edge lengths of a triangular regular faces pyramid equals 18 cm. , then its total area = ... cm^2.

(a) $\frac{27\sqrt{2}}{4}$ (b) $\frac{27\sqrt{3}}{4}$ (c) $9\sqrt{3}$ (d) $\frac{27\sqrt{3}}{2}$

(34) If the total area of a regular faces pyramid = $36\sqrt{3}$ cm^2 , then the sum of its edges lengths = ... cm.

(a) 6 (b) 12 (c) 18 (d) 36

(35) If the total area of a triangular regular faces is $9\sqrt{3}$ cm^2 , then the length of its edge .... cm.

(a) 3 (b) 9 (c) 27 (d) $\sqrt{3}$

(36) A regular triangular pyramid , its base length is 6ℓ cm. and its height ℓ cm. , then its lateral area = ... cm^2.

(a) $27\sqrt{3}\,\ell^2$ (b) $18\,\ell^2$ (c) $9\sqrt{3}\,\ell^2$ (d) $36\,\ell^2$

(37) A triangular regular faces pyramid , its edge length 6 cm. , then its volume = ... cm^3.

(a) $27\sqrt{3}$ (b) $36\sqrt{3}$ (c) $54\sqrt{2}$ (d) $18\sqrt{2}$

(38) A triangular regular faces pyramid , if the sum of the lengths of its edges equal 18 cm. , then its volume = ... cm^3.

(a) $9\sqrt{2}$ (b) $\frac{9\sqrt{2}}{4}$ (c) $\frac{27\sqrt{2}}{5}$ (d) $9\sqrt{3}$

(39) If the slant height of a triangular regular faces pyramid equals $5\sqrt{3}$ cm. , then the sum of areas of its faces = ... cm^2.

(a) $\frac{50\sqrt{3}}{3}$ (b) $25\sqrt{3}$ (c) $100\sqrt{3}$ (d) $50\sqrt{3}$

(40) A right quadrilateral pyramid whose base is a rhombus of side length equals to one of the diagonals of the rhombus equals 6 cm. , if the height of the pyramid = 12 cm. , then its volume = ... cm^3.

(a) $72\sqrt{3}$ (b) $216\sqrt{3}$ (c) 144 (d) 72

(41) $ABCDA'B'C'D'$ is a cube of edge length = 6 cm. , then volume of the pyramid $B'ABC$ = ... cm^3.

(a) 36 (b) 72 (c) $36\sqrt{3}$ (d) $18\sqrt{3}$

(42) A regular quadrilateral pyramid whose total area = 70 cm^2 and its lateral area = 45 cm^2 , then its height = ... cm.

(a) 2.5 (b) 5 (c) $\sqrt{14}$ (d) 4.5

(43) In a regular quadrilateral pyramid the length of its base side 10 cm. and the area of one of its lateral faces 60 cm^2, then its total area equal .... cm^2

(a) 600 (b) 340 (c) 160 (d) 240

(44) The ratio between the lateral surface area of a triangular pyramid of regular faces to the area of its total surface area = ...

(a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2

(45) A quadrilateral regular pyramid , the length of its base side = its slant height , then the ratio between its lateral surface area to its total surface area = ...

(a) 2 : 3 (b) 3 : 4 (c) 1 : 2 (d) 3 : 5

(46) A quadrilateral regular pyramid , the area of any face from its lateral faces equals the area of its base , if the side length of the base of the pyramid is 6 cm. , then volume of the pyramid = ... cm^3

(a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$

(47) If the side length of the base of quadrilateral regular pyramid is doubled but its height remains constant , then its volume ....

(a) is doubled. (b) will not change.

(c) become four times its first volume.

(d) become six times its first volume.

(48) In a regular quadrilateral pyramid , the side length of its base = 18 cm. , if its volume = 1296 cm^3, then its lateral area = ... cm^2

(a) 270 (b) 360 (c) 450 (d) 540

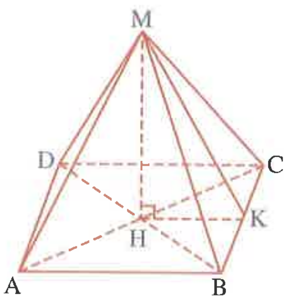
(49) A right pyramid whose base is a square , and all its eight edges are equal in length and each one = a cm. , then its lateral area = ...

(a) $3 a^2$ (b) $4 a^2$ (c) $\sqrt{3} a^2$ (d) $4\sqrt{3} a^2$

(50) In the triangular pyramid MABC , the vertex (M) is at a distance 15 cm. from its base ABC and the sides lengths of its base 5 , 6 , 7 cm.
, then its volume = ... cm^3

(a) $15\sqrt{3}$ (b) $10\sqrt{6}$ (c) $30\sqrt{6}$ (d) 90

(51) **In the opposite figure :**

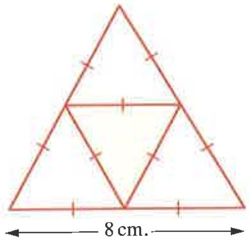


MABCD is a regular quadrilateral pyramid
, its volume 48 cm^3, its height 4 cm.
, KC = KB , $\overline{AC} \cap \overline{BD} = \{H\}$,
m (∠ MHK) = m (∠ HKB) = m (∠ MKB) = 90°
, then its lateral area = ... cm^2

(a) 18 (b) 24 (c) 36 (d) 60

(52) By using the opposite solid net :

The lateral area of the resulted solid = ... cm^2



(a) $8\sqrt{3}$ (b) $12\sqrt{3}$

(c) $16\sqrt{3}$ (d) 24

Second Essay questions

1 **In the regular pentagonal pyramid :**

(1) What the number of its lateral faces ?

(2) What the number of its faces ?

(3) What the number of its lateral edges ?

(4) What the number of its edges ?

(5) The pyramid has one vertix regardless of the vertices of the base. What is the number of all vertices of pentagonal pyramid ? Is your answer prove Euler's rule for any solid , its base is a polygon.

2 MABCD is a regular quadrilateral pyramid , the length of its base side is 10 cm. , and its height is 12 cm. , find its slant height. « 13 cm. »

3 MABCD is a regular quadrilateral pyramid of height 20 cm. and slant height 25 cm. Find the length of its base side. « 30 cm. »

4 MABCD is a regular quadrilateral pyramid , its base as a square ABCD , if its height equals $4\sqrt{3}$ cm. , and its lateral edge length $MA = 4\sqrt{5}$ cm. , find the length of its base side. « 8 cm. »

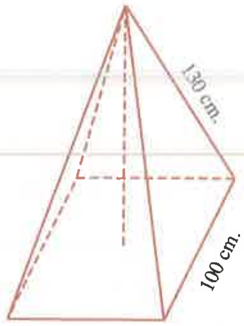
5 MABC is a regular triangular pyramid whose base is the equilateral triangle ABC whose side length 12 cm. , if the height of the pyramid is 6 cm. Find the length of its lateral edge. « $2\sqrt{21}$ cm. »

6 MABC is a regular triangular pyramid whose base ABC as an equilateral triangle of side length 3 cm. , if the length of its lateral edge is $\sqrt{7}$ cm. Find the height of the pyramid. « 2 cm. »

7 MABC is a triangular pyramid with regular faces , the length of its edge is 12 cm. Find its height and its slant height. « $4\sqrt{6}$ cm. , $6\sqrt{3}$ cm. »

8 A regular hexagonal pyramid whose height 8 cm. , its base as a regular hexagon of perimeter $24\sqrt{3}$ cm. Find the length of its lateral edge and its slant height. « $4\sqrt{7}$ cm. , 10 cm. »

9 The opposite figure represents a water tank as a regular quadrilateral pyramid , use the given data to find the height of the lateral face and the height of the tank.



« 120 cm. , $10\sqrt{119}$ cm. »

10 Each of the following figures represents a solid net. Describe the solid and find its height :

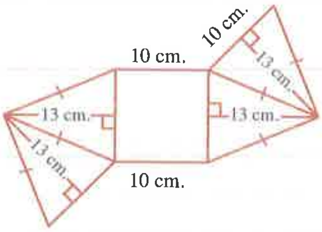


Fig. (1)

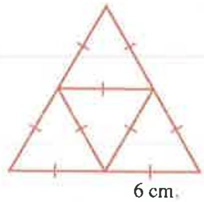


Fig. (2)

« 12 cm. , $2\sqrt{6}$ cm. »

11 The great pyramid of Giza (Khopho pyramid) is a regular quadrilateral pyramid the side length of its base is 232 metres and its slant height is 186 metres. Find height of the pyramid. « 145.4 m. »

12 MABC is a right triangular pyramid , the length of its edge MA = 25 cm. , and its base ABC as a right-angled triangle at A. If BA = 16.8 cm. , CA = 12.6 cm.
Find the height of the pyramid. « 24 cm. »

13 MABCD is a right quadrilateral pyramid , whose base is the rhombus ABCD where AC = 16 cm. , BD = 12 cm. , N is the point of intersection of its diagonals.
If the height of the pyramid MN = 10 cm. Find the lengths of its lateral edges.

« $2\sqrt{34}$ cm. , $2\sqrt{41}$ cm. »

14 A regular triangular pyramid whose height 12 cm. , and the side length of its base is 18 cm.
Find its volume. « $324\sqrt{3}$ cm.^{3} »

15 MABCD is a regular quadrilateral pyramid , its base ABCD where AB = 10 cm. , and the height of the pyramid = 12 cm.

Find : (1) The length of any slant height.
(2) The volume of the pyramid.
(3) The total area of the pyramid.

« 13 cm. , 400 cm.^{3} , 360 cm.^{2} »

16 A regular quadrilateral pyramid the length of its base is 20 cm. , and its height is $10\sqrt{3}$ cm.

Find : (1) Its lateral surface area.

(2) Its volume. « 800 cm^2 , $\frac{4000}{3}\sqrt{3}$ cm^3 »

---

17 A regular quadrilateral pyramid , the length of its base diagonal is $24\sqrt{2}$ cm. , and its slant height = 20 cm. , find its total area and its volume. « 1536 cm^2 , 3072 cm^3 »

---

18 MABCD is right quadrilateral pyramid , its base is the square ABCD whose side length $8\sqrt{2}$ cm. , and the length of its lateral edge is $4\sqrt{6}$ cm.

Find : (1) The lateral surface area of the pyramid.

(2) The volume of the pyramid. « $128\sqrt{2}$ cm^2 , $\frac{512}{3}\sqrt{2}$ cm^3 »

---

19 MABCD is a regular quadrilateral pyramid , the side length of its base = 20 cm. and the length of its lateral edge is 26 cm.

Find : (1) The slant height of the pyramid.

(2) The height of the pyramid.

(3) The lateral area of the pyramid.

(4) The volume of the pyramid. « 24 cm. , $2\sqrt{119}$ cm. , 960 cm^2 , $\frac{800}{3}\sqrt{119}$ cm^3 »

---

20 A triangular regular faces pyramid , its edge length = 12 cm. , find its height , volume and total area. « $4\sqrt{6}$ cm. , $144\sqrt{2}$ cm^3 , $144\sqrt{3}$ cm^2 »

---

21 MABCD is a right pyramid whose base ABCD as a square of side length = 18 cm. , MA = MB = MC = MD = 15 cm.

Find : (1) The total area. **(2)** The volume. « 756 cm^2 , $324\sqrt{7}$ cm^3 »

---

22 Calculate to the nearest tenth the volume of a regular pentagonal pyramid whose side length of its base = 16 cm. and its height = 12 cm. « 1761.8 cm^3 »

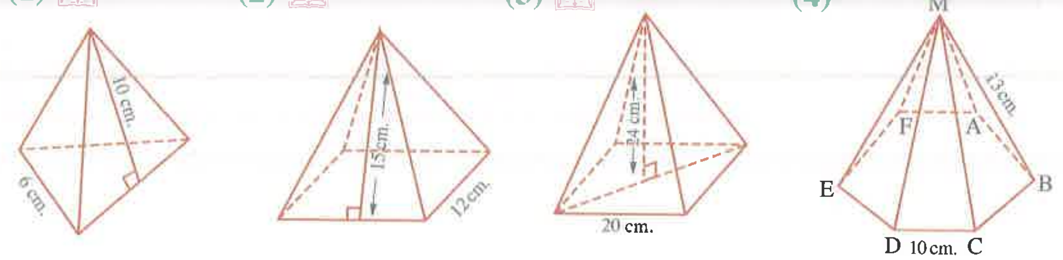
---

23 A regular hexagonal pyramid , the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm. ,

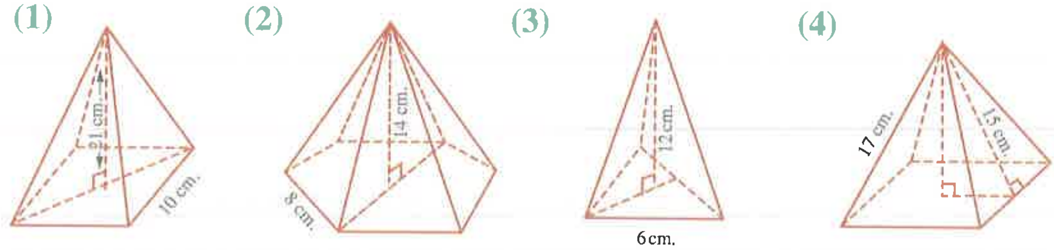
Find : (1) Its lateral area.

(2) Its total area. « $360\sqrt{3}$ cm^2 , $576\sqrt{3}$ cm^2 »

24 Find the lateral area and the total area of each regular pyramid of the following :



25 Find the volume of each of the following regular pyramids :



26 A quadrilateral pyramid whose height is 12 cm. , and its base as a rhombus of diagonals length 4 cm. and 8 cm. , prove that its volume equals the volume of a cube of edge length 4 cm.

27 MABC is a triangular pyramid whose vertex M at distance 15 cm. form its base ABC , and the side lengths of its triangular base are 5 , 6 and 7 cm. Find its volume « $30\sqrt{6}$ cm.^{3} »

28 A regular quadrilateral pyramid whose base area is 700 cm.^{2} and its slant height is 20 cm. Find its volume. « 3500 cm.^{3} »

29 A regular quadrilateral pyramid whose base area is 9 cm.^{2} and the length of its lateral edge is 5 cm. Find its volume. « 13.6 cm.^{3} »

30 A regular quadrilateral pyramid whose volume is 400 cm.^{3} and its height is 12 cm. Find its lateral area. « 260 cm.^{2} »

31 A regular quadrilateral pyramid , the side length of its base is 18 cm. , and its volume is 1296 cm.^{3}
Find its slant height and its lateral surface area. « 15 cm. , 540 cm.^{2} »

32 A regular quadrilateral pyramid , the side length of its base = 12 cm. , and its total area = 384 cm.^{2} Find its volume. « 384 cm.^{3} »

33 A right pyramid whose base is as a square of diagonal length = $10\sqrt{2}$ cm.

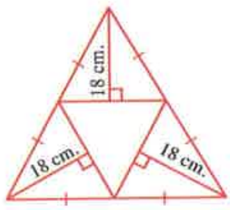
If its lateral area = 260 cm^2. Find the volume of the pyramid. « 400 cm^3. »

34 MABC is a regular triangular pyramid , the side length of its base is 3 cm. , and the length of its lateral edge = $\sqrt{7}$ cm. Find its volume and lateral area. « $\frac{3\sqrt{3}}{2}$ cm^3. , $\frac{9}{4}\sqrt{19}$ cm^2. »

35 A regular hexagonal pyramid whose height is 8 cm. and its base perimeter is $24\sqrt{3}$ cm. , Find its lateral and total area. « $120\sqrt{3}$ cm^2. , $192\sqrt{3}$ cm^2. »

36 Find the volume of the right pyramid whose slant height is 10 cm. , and its base as an equilateral triangle drawn inside a circle of radius length 12 cm. « $288\sqrt{3}$ cm^3. »

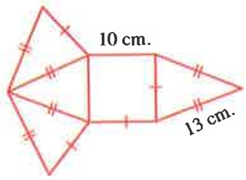
37 Use the opposite net to discribe the solid , then find its total area.



« $432\sqrt{3}$ cm^2. »

38 **Connecting to industry :**

Products containers of a factory manufactured from cardboard by folding the net of the opposite figure.



(1) Find the area of the used cardboard to produce 1000 containers.

(2) Calculate the costs of the used cardboard if each square metre costs 15 pounds.

« 34 m^2. , 510 pounds »

39 MABCD is a right quadrilateral pyramid whose base is the square ABCD , if the length of each lateral edge equals $6\sqrt{5}$ cm. and height of the pyramid = $6\sqrt{3}$ cm.

Find : (1) The total area of the pyramid.

(2) The volume of the pyramid. « 432 cm^2. , $288\sqrt{3}$ cm^3. »

40 **Connecting to tourism :**

A model of the great pyramid (regular quadrilateral pyramid) is made of metallic alloy its density is 3.2 gm./cm^3. If the length of the model base side 11.5 cm. and its height 7 cm. , then calculate its mass to the nearest one decimal place. « 987. 5 gm. »

41 France cared about the ancient Egyptian monuments. So it transported some of them to Paris to be shown in their museums. It also set up a pyramid its side faces of transparent glass similar to the great pyramid (regular quadrilateral pyramid) to be a main entrance to the Louvre in Paris. If you know its height 21.6 metres , and the length of its base side 35 metre , then find the area of the glass used in its building to the nearest square cm. « 1946 m^2. »

Third Higher skills

1 A regular hexagonal pyramid , the side length of its base = 2ℓ , and its height = 3ℓ ,
Prove that : The lateral area of the pyramid equals twice of its base area.

2 MABCD is a regular quadrilateral pyramid , if the length of its lateral edge = length of the diagonal of its base = ℓ , Prove that : the total area of the pyramid = $\frac{\ell^2}{2}\left(1+\sqrt{7}\right)$

3 A hollow circular cylinder , put inside it a triangular pyramid MABC whose base ABC is an equilateral triangle whose vertices lies on perimeter of the lower base of the cylinder , M (vertex of the pyramid) is the centre of the upper base of the cylinder.
Find the ratio between volume of the pyramid and volume of the cylinder. « $\frac{\sqrt{3}}{4\pi}$ »

4 A right pyramid whose base as a square , and all its eight edges are equal in length , if its total area = $\left(\sqrt{3}+1\right)$ A , Find the length of its edge in terms of (A) « $\sqrt{A}$ »

Exercise 8 — The cone

Interactive test

From the school book | ● Remember | ● Understand | ● Apply | Higher Order Thinking Skills

First — Multiple choice questions

Choose the correct answer from the given ones :

(1) The right circular cone is generated by folding a paper in the shape of ....

(a) an equilateral triangle. (b) a right-angled triangle.

(c) a circular segment. (d) a circular sector.

(2) The measure of the smallest rotation angle of an isosceles triangle around its axis of symmetry to form a right circular cone is ....

(a) 90° (b) 180° (c) 270° (d) 60°

(3) The right circular cone is formed from rotation of a right-angled triangle a complete rotation about ....

(a) its hypotenuse.

(b) one of its right sides.

(c) any straight line in the plane of the triangle.

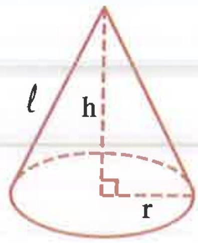
(d) any straight line passes through one of its vertices and parallel to the opposite side of this vertex.

(4) If a right circular cone intersected by a plane parallel to its base , then the resulted sector is ....

(a) an isosceles triangle. (b) an equilateral triangle.

(c) a circle. (d) a trapezoid.

(5) The total area for the opposite right cone equals ....



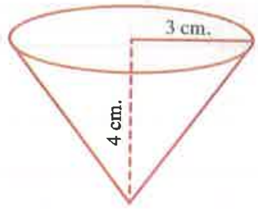
(a) $\pi r \ell$

(b) $\frac{\pi}{3} \pi^2 h$

(c) $\pi r (r + \ell)$

(d) $\frac{\pi}{3} r (r h + 3 \ell)$

(6) In the opposite figure :

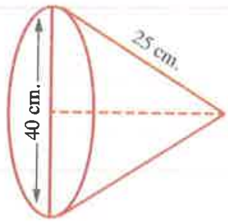


The length of the drawer = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

(7) In the opposite figure :



The height of the cone = ... cm.

(a) 15 (b) 20

(c) 25 (d) 40

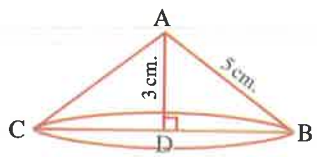
(8) In a right circular cone , if the length of its height 15 cm. , and the length of its drawer 17 cm. , then its radius length equal .... cm.

(a) 10 (b) 8 (c) 7 (d) 9

(9) In a right circular cone , the radius length of its base = 15 cm. and its height = 20 cm. , then its lateral area = ... cm^2.

(a) 375π (b) 600π (c) 1500π (d) 1875π

(10) In the opposite figure :



If AD = 3 cm. , AB = 5 cm.

, then the total area of the cone = ... cm^2.

(a) 8π (b) 24π

(c) 48π (d) 36π

(11) If the length of the diameter of the base of a right circular cone is 12 cm. and its height 8 cm. , then its lateral area equal .... cm^2.

(a) 60π (b) 28π (c) 10π (d) 48π

(12) The height of a right circular cone is 6 cm. and the circumference of its base is 16π cm. , then its lateral area = ... cm.^{2}

(a) 144π (b) 64π (c) 60π (d) 80π

(13) A right circular cone , the length of its drawer equal the length of the diameter of its base , then its total area = ... cm^2.

(a) $3\pi r^2$ (b) $3\pi r^3$ (c) $4\pi r^2$ (d) $4\pi r^3$

(14) A right circular cone , its height 24 cm. , and the length of its drawer 26 cm. , then the area of its base .... cm^2.

(a) 25π (b) 100π (c) 20π (d) 50π

(15) The radius length of the base of a right circular cone where its total area = 616π cm^2 , and the length of its drawer is 30 cm. is .... cm.

(a) 44 (b) 14 (c) 30 (d) 34

(16) Lamp cover is in the form of a right circular cone , the circumference of its base circle = 88 cm. , its height = 20 cm. , then its lateral area ≃ .... cm^2 $\left(\pi = \frac{22}{7}\right)$

(a) 88 (b) 596 (c) 1074 (d) 1047

(17) A right circular cone , the radius length of its base = 6 cm. and the length of its drawer = 10 cm. , then its volume = ... cm^3.

(a) 32π (b) 64π (c) 96π (d) 228π

(18) A right circular cone where its height 4 cm. , the length of its drawer 5 cm. , then its volume .... cm^3.

(a) 36π (b) 15π (c) 24π (d) 12π

(19) ABC is an equilateral triangle. Its side length (ℓ) it turned around $\overline{BC}$ as a rotation axis a complete turn , then the volume of the generated solid in terms of π and ℓ is ....

(a) $\frac{\pi \ell}{4}$ (b) $\frac{\pi}{4}\ell^2$ (c) $\frac{\pi \ell^3}{4}$ (d) $\sqrt{3}\pi \ell^2$

(20) A right circular cone where its volume 27π cm^3 , and the circumference of its base 6π cm. , then its height equal .... cm.

(a) 27 (b) 18 (c) 9 (d) 6

(21) A right circular cone , the radius length of its base 5 cm. and its total area = 90π cm^2 , then its volume = ... cm^3.

(a) 105π (b) 95π (c) 100π (d) 120π

(22) The volume of a right circular cone , if the length of its drawer = 15 cm. and its total area = 216 π cm^2. equal .... cm^3.

(a) 205 π (b) 220 π (c) 280 π (d) 324 π

(23) A right circular cone where the length of its drawer 25 cm. and its lateral area 550 cm^2. , then its volume = ... cm^3. where $\left(\pi = \frac{22}{7}\right)$

(a) 1223 (b) 1232 (c) 1322 (d) 3122

(24) If the volume of a right circular cone is 9 π cm^3. and the length of its base radius equal the length of its height , then its base area = ... cm^2.

(a) 9 π (b) 3 π (c) 27 π (d) 12 π

(25) The volume of a right circular cone 100 cm^3. , then its volume when the radius length of its base is doubled = ... cm^3.

(a) 100 (b) 200 (c) 300 (d) 400

(26) In a right circular cone , if the length of its radius base increased to its double , and the length of the height is decreased to its half , then its volume ....

(a) don't change. (b) increased to its double.

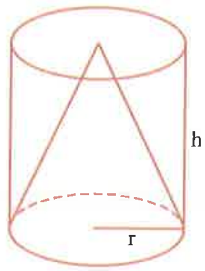
(c) decreased to its half. (d) increased to its four times.

(27) The radius of the base of a right circular cone = twice its height = 6 cm. and uniform quadrilateral pyramid its base length = its height = 6 cm. , then the ratio between the volume of the cone : the volume of the pyramid = ... : ....

(a) 3 : π (b) π : 3 (c) π : 2 (d) 2 : π

(28) **In the opposite figure :**

$\frac{\text{the volume of the cone}}{\text{the volume of the cylinder}} = \text{..............}$

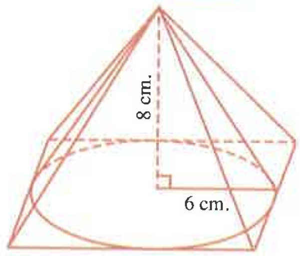


(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{1}$

(29) **In the opposite figure :**

A right regular pyramid and right circular cone have common vertex and the circle of base of the cone touches sides of the base of the pyramid internally :



First : The lateral area of the right circular cone = ... cm^2.

(a) 60 (b) 60 π (c) 48 (d) 48 π

Second : The total area of the regular pyramid equals .... cm^2.

(a) 360 (b) 240 (c) 384 (d) 432

Third : The volume of the pyramid equals .... cm^3.

(a) 64 (b) 96 (c) 480 (d) 384

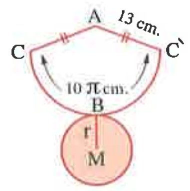
Fourth : The ratio between the volume of the pyramid and the volume of the cone equals ....

(a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$

Fifth : The ratio between the lateral area of the pyramid and the lateral area of the cone equals ....

(a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$

(30) The opposite net describes a solid its volume = ... cm^3.

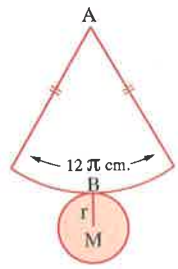


(a) 25π (b) 50π

(c) 75π (d) 100π

(31) The opposite net describes a solid its volume = 96π cm^3.

, then its total area = ... cm^2.



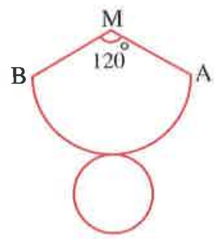
(a) 16π (b) 32π

(c) 48π (d) 96π

(32) The opposite figure represents the net of a solid

, MB = 3π cm. , m ($\angle$ AMB) = 120°

, then the volume of the solid = ... cm^3.

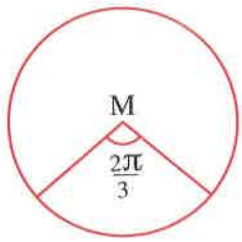


(a) $2\sqrt{2}\,\pi^2$ (b) $\frac{2\sqrt{2}}{3}\pi^4$

(c) $2\sqrt{2}\,\pi$ (d) $\frac{2\sqrt{2}}{3}\pi^3$

(33) In the opposite figure a circle is divided into two circular sectors such that they form two right cone nets

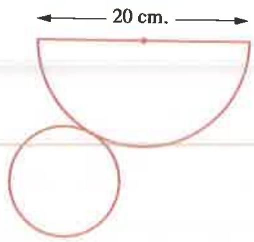
, then $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}}$ = ...



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

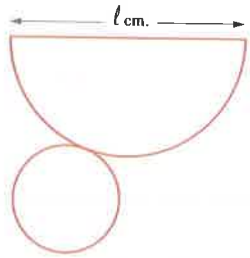
(34) If we fold the opposite net it becomes a cone its base radius length is .... cm.



(a) 10 (b) 8

(c) 5 (d) 2.5

(35) **In the opposite figure :**



If we folded this net it becomes a cone its base radius is .... cm.

(a) $\frac{\ell}{2}$ (b) $\frac{\ell}{3}$

(c) $\frac{\ell}{4}$ (d) $\frac{\ell}{5}$

(36) The central angle of the sector if be folded it becomes the opposite cone is ....

(a) acute. (b) obtuse.

(c) straight. (d) reflex.

(37) If we folded the circular sector it becomes a right circular cone , its drawer length 10 cm. and the radius length of its base 5 cm. , then the central angle of this sector is ....

(a) acute. (b) obtuse. (c) straight. (d) reflex.

(38) If we have a quarter circle , its radius length 16 cm. , then the radius length of the base of the cone which can be formed from the arc of the quarter circle = ... cm.

(a) 16 (b) 8 (c) 4 (d) 2

(39) The area of a circular sector : the total area of the circular solid cone which can be formed from folding this sector ....

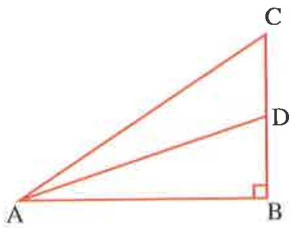
(a) > 1 (b) < 1 (c) $= 1$ (d) ≥ 1

(40) The ratio between the volume of a regular quadrilateral pyramid and the volume of the smallest circular cone contains the pyramid equals ....

(a) $2 : \pi$ (b) $4 : \pi$ (c) $6 : \pi$ (d) $8 : \pi$

(41) In the opposite figure :

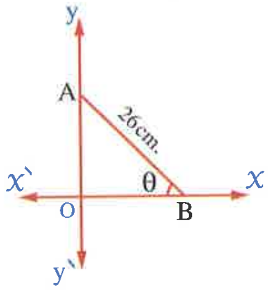
If AB = 3 cm. , BD = CD = 1 cm. , $m(\angle ABC) = 90°$, then the volume of the solid generated by turning the shaded part around $\overleftrightarrow{AB}$ as a rotation axis a complete turn = ...



(a) π (b) 2π (c) 3π (d) 4π

(42) In the opposite figure :

If $\tan\theta = \frac{5}{12}$, AB = 26 cm. , then the lateral area of the solid generated by rotating the triangle ABO a complete revolution about the x-axis = ... π cm.^{2}



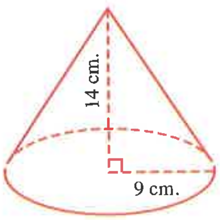
(a) 360 (b) 260π

(c) 260 (d) 360π

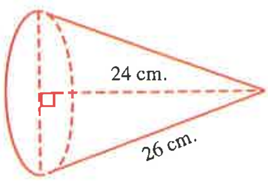
Second Essay questions

1 **Find the volume of the right circular cone shown in each figure using the given data :**

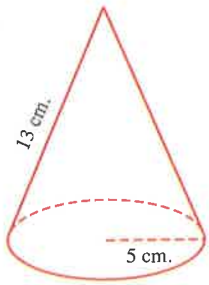
(1)



(2)

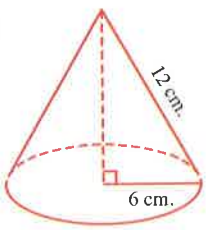


(3)

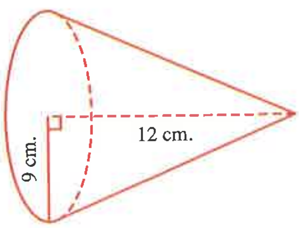


2 **Find the lateral and the total areas of each right circular cone due to the given data :**

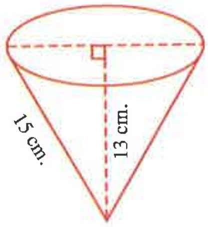
(1)



(2)



(3)



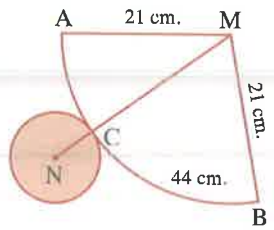
3 A right circular cone , its drawer length = 17 cm. its height = 15 cm. **Find :**

(1) Its lateral area. (2) Its total area.

(3) Its volume. « 136π cm.^{2} , 200π cm.^{2} , 320π cm.^{3} »

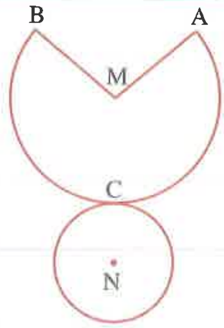
4 Find in terms of π the circumference and the area of the base of a right circular cone whose height is 24 cm. , and the length of its drawer is 26 cm. « 20π cm. , 100π cm.^{2} »

5 The opposite figure shows a net of a right cone , use the given data to find its height $\left(\pi = \frac{22}{7}\right)$



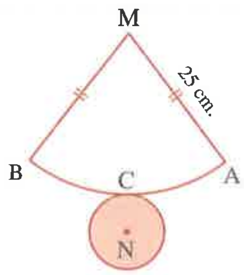
« $14\sqrt{2}$ cm. »

6 The opposite figure represents a right cone net form from a circular sector whose area is 20π cm^2 , the length of its arc $\overset{\frown}{ACB} = 8\pi$ cm. Find the height of the solid.



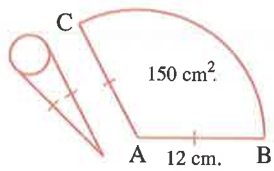
« 3 cm. »

7 The opposite figure represents a solid net. Describe the resulting solid of the folding process and find its height if MA = MB = 25 cm. , area of the circle N = 49π cm^2.



« 24 cm. »

8 The frozen milk is encapsulated (kept) on a right circular cone by folding a piece of healthy - insulated paper in the form of circular sector the length of its radius is 12 cm. and its area is 150 cm^2 , where the two radii of the circle $\overline{AB}$, $\overline{AC}$ become in contact. Find the height of the cone to the nearest one decimal.



« 11.3 cm. »

9 Find to the nearest tenth , the total area of the right circular cone in which the diameter length of its base is 10 cm. and its height is 12 cm. « 282.7 cm^2 »

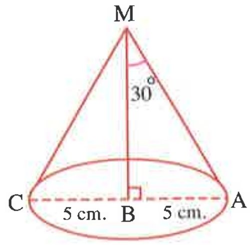
10 Find the volume of the right circular cone where the circumference of its base is 44 cm. and its height is 25 cm. « 1283.8 cm^3 »

11 **In the opposite figure :**

A right circular cone in which m (∠ AMB) = 30° ,

the radius length of the base = 5 cm.

Calculate its lateral area and also the total area.



« 50π cm.^{2} , 75π cm.^{2} »

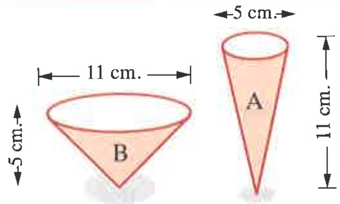
12 A right circular cone , the radius length of its base is 8 cm. and its lateral area = 96π cm.^{2}
Find to the nearest one decimal the volume of this cone. « 599.5 cm.^{3} »

13 **In the opposite figure :**

A , B are two cups for drinking

which of them has greater capacity ?

Find the difference between their capacities.

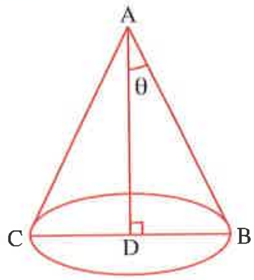


« The capacity of B is the greater. , $\frac{55}{2}\pi$ cm.^{3} »

14 **In the opposite figure :**

If $\sin\theta = \frac{3}{5}$, and the height of the cone = 12 cm. ,

Find the total area of the cone.



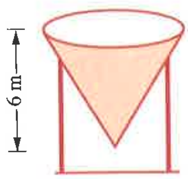
« 216π cm.^{2} »

15 **Civil engineering :**

The opposite figure shows a water tank in the shape of a right circular cone ,

its volume = 32π m.^{3} and its height = 6 m.

Find the radius length of its base and its total area.



« 4 m. , $\left(16 + 8\sqrt{13}\right)\pi$ m.^{2} »

16 Which is greater in volume ?
A right circular cone in which the radius length of its base is 15 cm. and its height is 20 cm.
or a regular quadrilateral pyramid whose height is 40 cm. and its base perimeter = 48 cm.

17 A right circular cone , its height = h and its volume = πh^3. Prove that its lateral area equals the lateral area of a right circular cylinder which is common with the cone in the base and the height.

18 **Connecting to physics :**

A cylindrical shaped vessel contain water , a metal body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. and is completely immersed in it raising the surface of the water in the vessel with the value 1 cm.

Find the length of base diameter of the vessel. « 8 cm. »

19 A cube made of wax , its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. Find the radius length of the base of the cone given that 12% from wax had been lost during melting and reforming. $\left(\pi = \frac{22}{7}\right)$ « $8\sqrt{5}$ cm. »

20 A container in the shape of a right cone of capacity 2.2 litre and its height = 21 cm. Find the radius length of its base. $\left(\pi = \frac{22}{7}\right)$ « 10 cm. »

21 A circular sector MAB , the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone. « 167.3 cm^3. »

22 AMB is a quadrant of a circle of centre M and its radius length = 20 cm. It is converted to the surface of a right circular cone where M is its vertex such that $\overline{MA}$ coincide $\overline{MB}$

Find the radius length of the base of the cone also find its volume in π

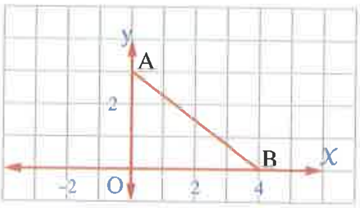
« 5 cm. , $\frac{125\sqrt{15}}{3}\pi$ cm^3. »

23 ABC is a right-angled triangle at B in which AB = 6 cm. , BC = 8 cm.

Find the volume of the solid generated by turning Δ ABC a complete turn around :

(1) $\overleftrightarrow{BC}$ (2) $\overleftrightarrow{AC}$ « 96 π cm^3. , 76.8 π cm^3. »

24 The opposite figure shows a coordinate perpendicular plane. Calculate in terms of π the volume of solid generated when revolving triangle ABO one complete revolution around :



(1) The x-axis

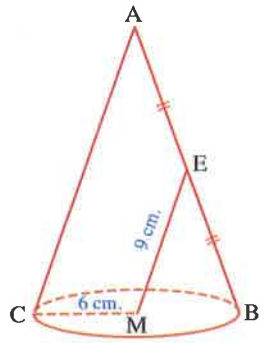
(2) The y-axis

« 12 π cubic units , 16 π cubic units »

25 ABC is an isosceles triangle in which AB = AC = 10 cm. and BC = 12 cm. It turned around the base $\overline{BC}$ a complete turn. Calculate the volume of the generated solid. « 256 π cm^3. »

26 **In the opposite figure :**

Find the lateral area and the total area and the volume of the right circular cone.

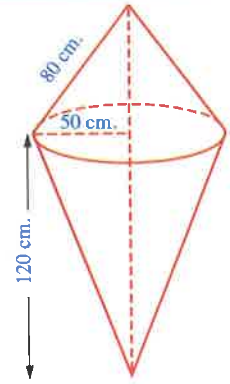


« 108π cm^2 , 144π cm^2 , $144\sqrt{2}\pi$ cm^3 »

27 **Marine navigation :**

The opposite figure shows a guide sign (Shamandora) (Buoy) to determine the waterway , and it is in the form of two right cones have a common base.

Find the costs of its painting with a material which resists erosion factor , note that each square metre of its costs 300 pound.



« 990 pounds »

28 **Connecting with industry :**

A regular pentagon pyramid made of copper , the side length of its base = 10 cm. and its height = 42 cm. it is melted and converted to a right circular cone the radius length of its base = 15 cm. given that 10% of copper has been lost during melting and converting it. Find the height of the cone to the nearest one decimal. « 9.2 cm. »

29 **Critical thinking :**

A right circular cone of volume 100 cm^3.

Find its volume when :

(1) Its height is doubled.

(2) The length of its radius is doubled.

(3) Its height is doubled and the length of its radius is doubled.

What you conclude ? Explain your answer. « 200 cm^3 , 400 cm^3 , 800 cm^3 »

Remember Understand Apply Higher Order Thinking Skills

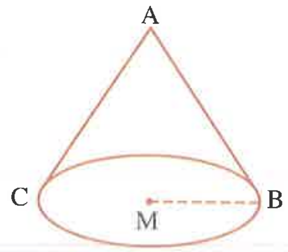
Third Higher skills

1 Choose the correct answer from those given :

(1) If the volume of hemisphere with raduis (r) equals the volume of cone with basc raduis length (r) and height (h) , then ....

(a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

(2) In the opposite figure :



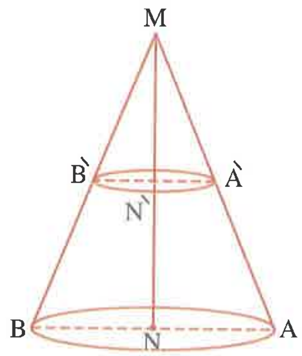
The volume of a right circular cone is 96π cm^3 and $\frac{MB}{AB} = \frac{3}{5}$, then its total surface area = ... cm^2

(a) 24π (b) 48π (c) 96π (d) 192π

(3) The arc length of a circular sector that if it is folded it becomes a right circular cone whose volume is 49π cm^3 and height 3 cm. equals .... cm.

(a) 2π (b) 4π (c) 8π (d) 14π

(4) In the opposite figure :



If a plane is drawn perpendicular to the cone axis and intersects it at midpoint of $\overline{MN}$, then

First : $\frac{\text{The volume of the smaller cone}}{\text{The volume of the greater cone}} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Second : $\frac{\text{The lateral area of the smaller cone}}{\text{The lateral area of the greater cone}} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(5) The ratio between the volume of a regular triangular pyramid and the volume of the greatest right circular cone can fit inside of the pyramid equals ....

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

(6) The ratio between the volume of a regular triangular pyramid and the volume of the smallest right circular cone can contain it equals ....

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

(7) The volume of a right circular cone is (v). If its base radius length is inscreased 50 % and its height is increased 50 % and its volume after increase is ($\grave{v}$) , then ....

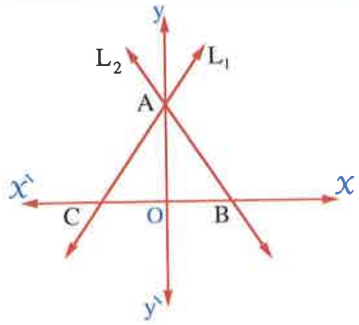
(a) $\grave{v} = 150\ \%\ v$ (b) $\grave{v} = 225\ \%\ v$

(c) $\grave{v} = 337.5\ \%\ v$ (d) $\grave{v} = 450\ \%\ v$

2 **In the opposite figure :**

The equation of the straight line L_1 is $3x - \sqrt{3}\,y + 6 = 0$ and the equation of the straight line L_2 is $\sqrt{3}\,x + y - 2\sqrt{3} = 0$

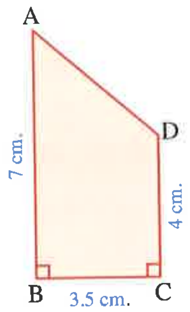
Find the volume of the body generated from turning ΔABC a complete turn around x-axis.



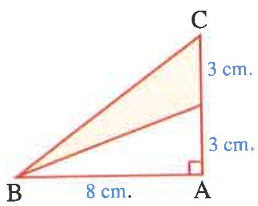
« 16 π cube units »

3 **Find the volume of the solid generated by turning the shaded part a complete turn around $\overleftrightarrow{AB}$ as an axis of rotation in each of the following figures :**

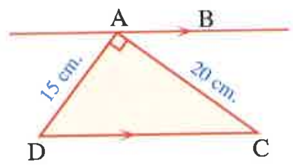
(1)



(2)



(3)



« 192.4 , 226.2 , 7539.8 cm^3. »

Exercise 9 — The circle

Interactive test

From the school book · Remember · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The centre of the circle in which its diameter is $\overline{AB}$ where $A = (-1 , 3)$, $B = (5 , -3)$ is ....

(a) $(4 , 0)$ (b) $(2 , 0)$ (c) $(-6 , -6)$ (d) $(0 , 4)$

(2) The radius length of the circle whose equation $x^2 + y^2 - 4x + 2y - 4 = 0$ is .... length unit.

(a) 2 (b) 4 (c) 3 (d) 9

(3) The radius length of the circle whose equation $(x + 2)^2 + y^2 + 2y = 0$ is .... length unit.

(a) zero (b) 1 (c) 2 (d) 4

(4) The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals .... length unit.

(a) 3 (b) 6 (c) 12 (d) 24

(5) If the two straight lines $y = -6$, $y = 8$ are two tangents to the circle M, then its radius length = ... length unit.

(a) 1 (b) 2 (c) 7 (d) 14

(6) If the straight line $y = 2$ touches the circle M whose center is $(6 , 9)$, then its diameter length = ... length unit.

(a) 6 (b) 7 (c) 14 (d) 15

(7) The radius length of the circle $(n+3)x^2+y^2-4y+(m-2)xy+(m-n)x-8=0$ is ········· length unit.

(a) 2 (b) 4 (c) 6 (d) $2\sqrt{2}$

(8) The area of the circle whose equation is $(x-5)^2+(y+4)^2=7$ equals ········· square unit.

(a) 3.5π (b) 7π (c) 12.25π (d) 49π

(9) If the equation $2x^2+ay^2+bxy-5=0$ represents a circle, then its area = ········· square unit.

(a) 5π (b) $\sqrt{5}\pi$ (c) $\frac{5}{2}\pi$ (d) $5\sqrt{2}\pi$

(10) The circumference of the circle whose equation is $(x-3)^2+(y+2)^2=25$ equal ········· length unit.

(a) 2π (b) 3π (c) 10π (d) 25π

(11) The circumference of the circle whose equation $x^2+y^2+2x-2y-2=0$ is ········· length unit.

(a) π (b) 2π (c) 4π (d) 8π

(12) The circumference of the circle whose equation is $x^2+y^2=8$ is ········· length units.

(a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

(13) If the two straight lines : $x=-3$, $x=4$ touch the circle M, then its circumference = ········· length units. where $\left(\pi=\frac{22}{7}\right)$

(a) 22 (b) 44 (c) 12 (d) 14

(14) If $\begin{pmatrix} x & y & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ -2 \end{pmatrix}=\boxed{}$, then the obtained equation represents a circle with diameter length = ········· length unit. (Where $\boxed{}$ is the zero matrix)

(a) 2 (b) 4 (c) 6 (d) 8

(15) The equation : $\begin{vmatrix} x & yi \\ yi & x \end{vmatrix}-49=0$ represents the equation of a circle with radius length ········· length unit.

(a) 49 (b) 14 (c) 9 (d) 7

(16) Which of the following equations represent a circle ?

(a) $x^2-y^2+x-y=6$ (b) $2x^2+y^2-x+y=5$

(c) $x^2+y^2-x=6$ (d) $x^2+y^2-xy=6$

(17) If the equation : $2x^2 + (a-1)y^2 + 5x - 3y = 7$ represent a circle , then a = ...

(a) 1 (b) 2 (c) 3 (d) 4

(18) If $x^2 + y^2 + 2(\cos\theta)x - 2(\sin\theta)y - 8 = 0$ represent an equation of a circle , then r = ... length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) 8

(19) The center of the circle whose equation $(x-2)^2 + (y+3)^2 = 16$ is ....

(a) (2 , 3) (b) (2 , − 3) (c) (13 , 16) (d) (4 , 9)

(20) The centre of the circle whose equation $2x^2 + 2y^2 - 32 = 0$ is ....

(a) (0 , 0) (b) (2 , 2) (c) (1 , 1) (d) (− 1 , − 1)

(21) The centre of the circle whose equation $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) (3 , − 4) (b) (4 , − 3) (c) (− 3 , 4) (d) (− 4 , 3)

(22) The centre of the circle whose equation $2x^2 + 2y^2 + 12x - 16y = 0$ is ....

(a) (3 , − 4) (b) (− 6 , 8) (c) (− 3 , 4) (d) (6 , − 8)

(23) The circle $(x+2)^2 + y^2 + 2y = 0$ its centre is the point ....

(a) (2 , 2) (b) (− 2 , − 1) (c) (2 , − 1) (d) (− 2 , 0)

(24) Centre of the circle passes through the origin and the two points A (− 6 , 0) , B (0 , 8) is ....

(a) (4 , − 3) (b) (− 5 , 5) (c) (5 , 5) (d) (− 3 , 4)

(25) If any circle touches the two coordinate axes and it is drawn in the first quadrant , then its centre lies on the straight line ....

(a) $y = x$ (b) $y = -x$ (c) $y = x + 1$ (d) $y = x - 1$

(26) How many circles whose centre (3 , − 5) and touches one of the two axes ?

(a) 1 (b) 2 (c) 3 (d) 4

(27) The point (2 , 2) lies .... the circle whose equation $x^2 + y^2 = 9$

(a) on (b) inside (c) outside (d) in the centre of

(28) The point (2 , 0) lies on ....

(a) x-axis. (b) y-axis.

(c) the straight line $y = 2x$ (d) the circle $x^2 + y^2 = 9$

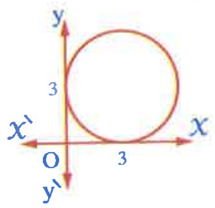
(29) The point which lies on the circle : $(x-2)^2 + y^2 = 13$ is ....

(a) (2 , 3) (b) (3 , − 2) (c) (2 , 5) (d) (4 , 3)

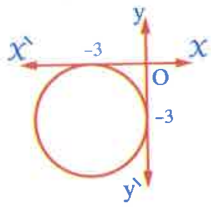
(30) The circle whose equation : $(x-1)^2 + (y+2)^2 = 5$ passes through the point ....

(a) $(0 , 0)$ (b) $(3 , -1)$ (c) $(2 , -4)$ (d) All the previous.

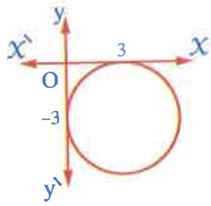
(31) The circle C : $(x+3)^2 + (y-3)^2 = 9$ is represented by the figure ....



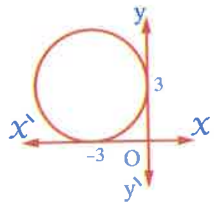
(a)



(b)



(c)



(d)

(32) The general form of the equation of a circle if its centre is $(2 , -1)$ and its radius length = 3 cm. is ....

(a) $x^2 + y^2 - 4x + 2y - 9 = 0$ (b) $x^2 + y^2 - 4x + 2y - 4 = 0$

(c) $x^2 + y^2 + 2x - y + 3 = 0$ (d) $x^2 + y^2 + 2x - y + 9 = 0$

(33) The equation of the circle whose centre $(4 , 3)$ and touches x-axis is ....

(a) $(x-3)^2 + (y-4)^2 = 16$ (b) $(x-4)^2 + (y-3)^2 = 9$

(c) $(x+3)^2 + (y+4)^2 = 9$ (d) $(x+3)^2 + (y-4)^2 = 16$

(34) The equation of the circle whose centre $(-4 , 4)$ and touches the two coordinate axes is ....

(a) $x^2 + y^2 + 8x - 8y + 16 = 0$ (b) $x^2 + y^2 = 16$

(c) $x^2 + y^2 - 8x + 8y + 16 = 0$ (d) $x^2 + y^2 = 8$

(35) The equation of the circle which is the image of the circle $x^2 + y^2 - 12x + 6y + 20 = 0$ by translation $(x+2 , y-2)$ is ....

(a) $(x+8)^2 + (y+5)^2 = 25$ (b) $(x-8)^2 + (y+5)^2 = 25$

(c) $(x-8)^2 + (y-5)^2 = 25$ (d) $(x+5)^2 + (y-8)^2 = 25$

(36) The equation of the circle whose centre $(-4 , 3)$ and passes through the origin point is ....

(a) $(x+4)^2 + (y-3)^2 = 5$ (b) $(x-4)^2 + (y+3)^2 = 25$

(c) $(x+4)^2 + (y-3)^2 = 625$ (d) $(x+4)^2 + (y-3)^2 = 25$

(37) The equation of the circle whose centre $(1 , 2)$ and touches the line : $3x + 4y + 9 = 0$ is ....

(a) $x^2 + y^2 - 2x - 4y = 16$ (b) $x^2 + y^2 + 2x + 4y - 11 = 0$

(c) $x^2 + y^2 + 2x + 4y - 16 = 0$ (d) $x^2 + y^2 - 2x - 4y = 11$

(38) The equation of the circle which touches the straight line $x + y = 2$ and its centre $(3, 5)$ is ....

(a) $(x-3)^2 + (y-5)^2 = 18$ (b) $(x+3)^2 + (y+5)^2 = 18$

(c) $(x+3)^2 + (y+5)^2 = 36$ (d) $(x-3)^2 + (y-5)^2 = 9$

(39) The circle equation whose centre lies on the straight line $y = \frac{1}{2}x$ and touches x-axis could be ....

(a) $(x-2)^2 + (y-1)^2 = 4$ (b) $(x-4)^2 + (y-2)^2 = 16$

(c) $(x-2)^2 + (y-4)^2 = 16$ (d) $(x-4)^2 + (y-2)^2 = 4$

(40) The equation of the circle which concentric with the circle whose equation $x^2 + y^2 - 6x + 2y - 6 = 0$ and passes through the point $(-3, 4)$ is ....

(a) $(x+3)^2 + y^2 = 16$ (b) $(x-3)^2 + (y+1)^2 = 25$

(c) $(x-3)^2 + (y+1)^2 = 16$ (d) $(x-3)^2 + (y+1)^2 = 61$

(41) In the following equations : The circle whose centre lies on the y-axis and does not intersect the x-axis is ....

(a) $x^2 + (y-1)^2 = 4$ (b) $x^2 + (y-5)^2 = 25$

(c) $x^2 + (y+5)^2 = 9$ (d) $(x+5)^2 + y^2 = 16$

(42) The equation of circle whose centre $(-4, -3)$ and its surface area is 25π cm^2 is ....

(a) $x^2 + y^2 - 8x + 6y - 25 = 0$ (b) $x^2 + y^2 + 8x + 6y = 0$

(c) $x^2 + y^2 + 4x + 3y + 25 = 0$ (d) $x^2 + y^2 + 8x - 6y = 0$

(43) ABCD is a rectangle in which $A = (-1, 4)$, $B = (7, 8)$, $C = (9, 4)$, $D = (1, 0)$, then the equation of the circumcircle of the rectangle is ....

(a) $(x-4)^2 + (y-4)^2 = 25$ (b) $(x-4)^2 + (y-4)^2 = 16$

(c) $(x+4)^2 + (y+4)^2 = 25$ (d) $(x-4)^2 + (y+4)^2 = 16$

(44) The geometrical centre of square ABCD is the origin and its side length is $2\sqrt{3}$, then the equation of the circle that touches its sides is ....

(a) $x^2 + y^2 = 3$ (b) $x^2 + y^2 = 12$

(c) $x^2 + y^2 = 6$ (d) $\left(x-\sqrt{3}\right)^2 + \left(y-\sqrt{3}\right)^2 = 3$

(45) The equation of the circle passes through the vertices of a regular hexagon that has area $6\sqrt{3}$ cm^2 and the centre of the circle is the origin is ....

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 9$ (d) $x^2 + y^2 = 16$

(46) The circle whose equation is $(x-a)^2+(y-b)^2=a^2$ where $(a \neq b)$ ....

(a) touches x-axis. (b) touches y-axis.

(c) touches the two coordinates axes. (d) does not touch any of the two axes.

(47) If y-axis is a tangent to the circle $x^2+y^2+4x+my+4=0$, then m = ...

(a) 4 (b) -4 (c) 0 (d) ± 4

(48) If the circle whose equation $x^2+y^2-6x+8y+c=0$ touches x-axis ,

then c = ...

(a) -9 (b) 9 (c) 6 (d) -6

(49) If x-axis touches the circle $x^2+y^2+mx+4y+7-3m=0$, then m = ...

(a) 2 or 14 (b) -2 or -14 (c) 2 or -14 (d) -2 or 14

(50) If the straight line $3x-4y-12=0$ touches the circle $(x+3)^2+(y-1)^2=r^2$,

then the circumference of the circle = ... length unit (in terms of π)

(a) 5π (b) 10π (c) 15π (d) 20π

(51) If the straight line $y=mx$ touches the circle $(x-2)^2+(y-6)^2=4$, then m = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$

(52) The straight line $y=5-2x$ .... the circle whose equation :

$x^2+y^2-8x-4y+15=0$

(a) touch (b) intersect (c) outside (d) passes the centre

(53) The two circles $c_1:(x+2)^2+(y-1)^2=4$, $C_2:(x-5)^2+(y-3)^2=9$ ....

(a) distant. (b) touching externally.

(c) touching internally. (d) intersecting.

(54) The two circles $C_1:(x+2)^2=1-y^2$, $C_2:x^2+y^2-2x-8y-19=0$ are ....

(a) intersecting. (b) touching internally.

(c) distant. (d) touching externally.

(55) If the straight line $L:3x+4y+9=0$ touches the circle M :

$x^2+y^2-22x-4y-c=0$, then c = ...

(a) 15 (b) -20 (c) 25 (d) -25

(56) The length of the tangent segment to the circle : $x^2+y^2=9$ from the point (5 , 0)

equals .... length unit.

(a) 14 (b) 3 (c) 5 (d) 4

(57) Length of the tangent segment to the circle : $x^2 + y^2 = r^2$ from the point $(0 , 2r)$ equals ....

(a) r (b) $2r$ (c) $\sqrt{3}\,r$ (d) $\frac{\sqrt{3}}{2}\,r$

(58) If $\overleftrightarrow{AB}$ is a tangent to the circle $x^2 + y^2 + 6x - 8y + 15 = 0$ at the point $A(-2 , 1)$, then the equation of $\overleftrightarrow{AB}$ is ....

(a) $x - 3y + 5 = 0$ (b) $x - 3y = 5$ (c) $3x - y - 5 = 0$ (d) $3y - x + 5 = 0$

(59) If x-axis intersects the circle whose equation $x^2 + y^2 = 49$ at the two points A and B , then AB = ... length unit.

(a) 49 (b) 7 (c) 2 (d) 14

(60) The intersection point of the circle $(x - 2)^2 + y^2 = 16$ with the x-axis is ....

(a) $(6 , 0) , (-2 , 0)$ (b) $(-6 , 0) , (2 , 0)$

(c) $(4 , 0) , (-4 , 0)$ (d) $(2 , 0) , (-2 , 0)$

(61) If the line $y = 2$ intersects the circle whose equation $(x - 3)^2 + (y - 2)^2 = 25$ at the two points A and B , then AB = ... length unit.

(a) $\sqrt{13}$ (b) 7 (c) 8 (d) 10

(62) If the straight line : $y - 2x + 5 = 0$ cuts the circle $x^2 + y^2 - 4x - 8y = 0$ at the two points A and B , then the distance between the centre and the chord $\overline{AB}$ = ...

(a) 3 (b) 4 (c) 5 (d) $\sqrt{5}$

(63) A circle , its centre $M = (5 , 4)$ and its radius length = 5 length units and it intersects x-axis at the two points A and B , then the area of Δ MAB = ... square units.

(a) 6 (b) 9 (c) 12 (d) 18

(64) If the straight line $\overleftrightarrow{AB}$ is the axis of symmetry of the circle whose equation : $x^2 + y^2 = k^2$, and A , B $\in$ the circle where $A = (-2 , 5)$, then B = ...

(a) $(2 , -5)$ (b) $(2 , 5)$ (c) $(0 , 0)$ (d) $(5 , -2)$

(65) Area of the square whose vertices lie on the circle : $x^2 + y^2 - 4x + 6y + 4 = 0$ is .... square units.

(a) 6 (b) 9 (c) 12 (d) 18

(66) The circle passes through the three points $A = (-2 , 0)$, $B = (2 , -1)$, $C = (3 , 3)$ has a diameter of length .... length unit.

(a) $\sqrt{34}$ (b) $\sqrt{33}$ (c) $\sqrt{32}$ (d) $\sqrt{31}$

(67) If a circle with radius length 4 cm. and it passes through the vertices of a regular hexagon , then the area of the hexagon = ········· cm^2

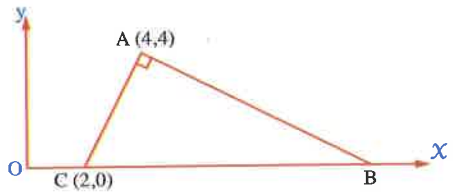
(a) $8\sqrt{3}$ (b) $16\sqrt{3}$ (c) 16 (d) $24\sqrt{3}$

(68) The area of a regular polygon with 12 sides. If the circle $x^2 + y^2 - 16 = 0$ passes through its vertices equal ········· square unit.

(a) 24 (b) 36 (c) 48 (d) 72

(69) In the opposite figure :

The equation of the circle which passes through the vertices of the triangle ABC is ·········



(a) $x^2 + (y-7)^2 = 25$

(b) $(x-7)^2 + y^2 = 25$

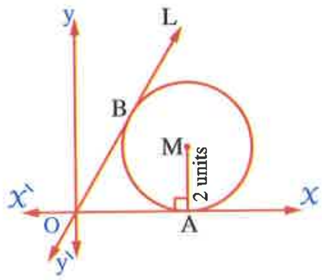
(c) $(x-4)^2 + (y-4)^2 = 16$

(d) $(x-4)^2 + y^2 = 16$

(70) In the opposite figure :

If OB = 5 length unit

, then the equation of the circle M is ·········



(a) $(x-2)^2 + (y-5)^2 = 25$

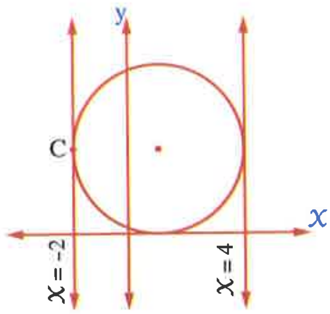
(b) $(x-2)^2 + (y-5)^2 = 4$

(c) $(x-5)^2 + (y-2)^2 = 25$

(d) $(x-5)^2 + (y-2)^2 = 4$

(71) In the opposite figure :

The equation of the circle is ·········



(a) $(x+2)^2 + (y-4)^2 = 36$

(b) $(x-1)^2 + (y-3)^2 = 36$

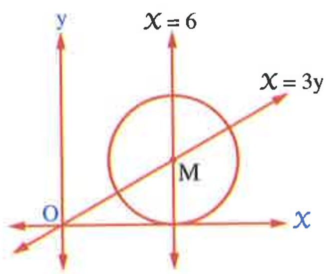
(c) $(x-1)^2 + (y-3)^2 = 9$

(d) $(x+1)^2 + (y+3)^2 = 9$

(72) In the opposite figure :

If the circle M touches the x-axis ,

then the equation of the circle M is ·········



(a) $(x-6)^2 + (y-2)^2 = 4$

(b) $(x-6)^2 + (y-3)^2 = 9$

(c) $(x-6)^2 + (y-4)^2 = 16$

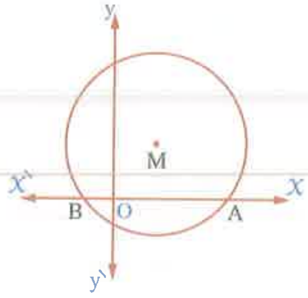
(d) $(x-8)^2 + (y-2)^2 = 4$

(73) In the opposite figure :

If the equation of the circle is

$(x-2)^2 + (y-3)^2 = 25$

, then AB = ... length unit.



(a) 8 (b) 4

(c) 6 (d) 5

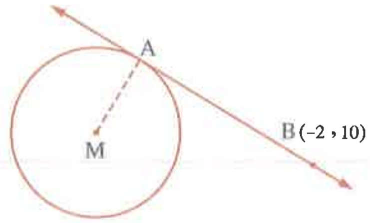
(74) In the opposite figure :

If the equation of the circle M is

$(x-3)^2 + (y+2)^2 = 25$,

$\overleftrightarrow{AB}$ is a tangent to the circle M at A where

B = (– 2 , 10) , then AB = ... length unit.



(a) 13 (b) $\sqrt{194}$ (c) 12 (d) 5

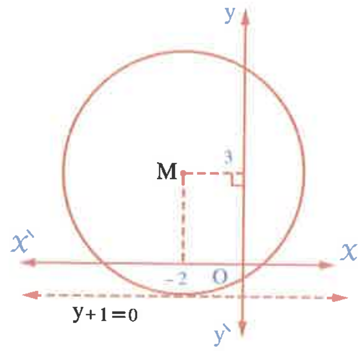
(75) Which of the following circle equations does represent the circle in the opposite figure ?

(a) $(x-3)^2 + (y+2)^2 = 16$

(b) $(x+2)^2 + (y-3)^2 = 16$

(c) $(x+2)^2 + (y-3)^2 = 4$

(d) $(x+2)^2 + (y-3)^2 = 9$



(76) In the opposite figure :

M and N are two congruent circles , the radius length of each is r

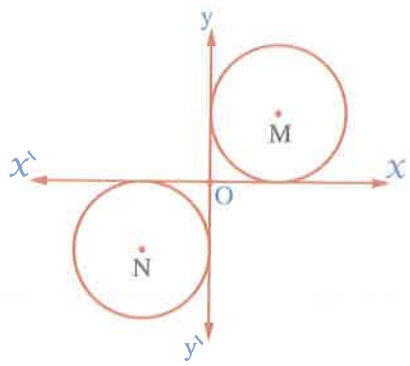
, then MN = ... length unit.

(a) 2 r

(b) $\sqrt{2}$ r

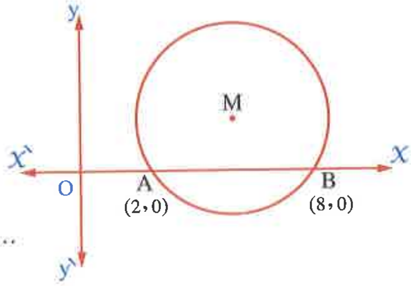
(c) $2\sqrt{2}$ r

(d) $\sqrt{5}$ r



(77) In the opposite figure :

If M is a circle where its circumference = 10π length unit , intersects x-axis at the two points A (2 , 0) , B (8 , 0) , then the equation of the circle M is ....



(a) $(x+5)^2+(y+4)^2=25$

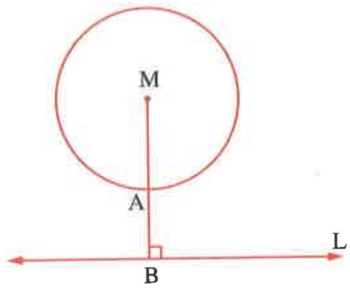
(b) $(x-5)^2+(y-4)^2=25$

(c) $(x-5)^2+(y-4)^2=9$

(d) $(x-5)^2+(y-4)^2=36$

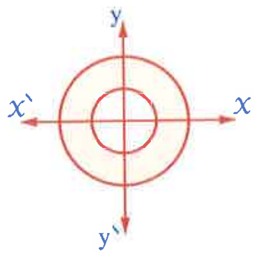
(78) In the opposite figure :

If the equation of the circle M is $x^2+y^2-6x+4y-12=0$, $\overline{MB} \perp$ the straight line L where the equation of L is $3x-4y+23=0$, $A \in \overline{MB}$, then the length of $\overline{AB}$ = ... length unit.



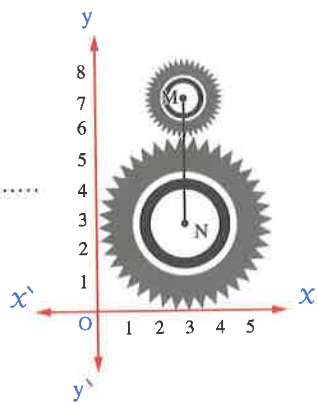
(a) 3 (b) 4 (c) 5 (d) 2.5

(79) The opposite figure represents a disc of a machine. It is required to make another one like it , if the cost of one square unit from the surface of the disc is 5 pounds and the equation of the smaller disc is $x^2+y^2=4$ the length of the diameter of the greater circle is 10 length units , then the cost of the disc $\simeq$ .... pounds.



(a) 440 (b) 660 (c) 220 (d) 330

(80) The opposite figure represents two gears in a machine , their centres are M and N , $\overline{MN}$ // the y-axis. If the radius of the smaller gear = $\frac{1}{3}$ the radius of the greater gear , then the equation of the smaller gear is ....



(a) $(x-3)^2+(y-1)^2=9$

(b) $(x-3)^2+(y-7)^2=1$

(c) $x^2+y^2-6x-14y+58=0$

(d) $(x-1)^2+(y-1)^2=1$

Second Essay questions

1 Find the equation of the circle whose centre is (M) and its radius length (r) unit in each of the following cases :

(1) M = (2 , 3) , r = 5

(2) M = (0 , 0) , r = 3

(3) M = (0 , – 1) , $r = 2\sqrt{3}$

(4) M = (– 4 , – 3) , $r = \frac{3}{2}$

2 Write the general form of the equation of the circle if :

(1) Its centre M (– 2 , 3) and its diameter length equals 8 length unit.

(2) Its centre M (5 , – 12) and it passes through the origin point.

(3) Its centre M (7 , – 5) and passes through the point A = (3 , 2)

(4) $\overline{AB}$ is a diameter in the circle where A = (6 , – 4) and B = (0 , 2)

(5) Its centre is the point (– 3 , – 2) and touches x-axis.

(6) Its centre is the point (3 , 0) and touches y-axis.

(7) Its centre is the point (5 , – 5) and touches the two coordinate axes.

(8) It passes through the two points A = (6 , 2) , B = (0 , – 1) and the two tangents to the circle at A and B are parallel.

(9) Its centre lies on x-axis and it passes through the two points (2 , 0) , (8 , 0)

(10) Its radius length = 6 length units and it touches the two axes given that the circle lies in the fourth quadrant.

3 Find the coordinates of the centre , also find the radius length for each of the following circles :

(1) $x^2 + y^2 - 8 = 0$

(2) $(x + 3)^2 + (y - 5)^2 = 49$

(3) $(x + 4)^2 + y^2 = 9$

(4) $x^2 + (y + 7)^2 = 24$

(5) $x^2 + y^2 - 4x + 6y - 12 = 0$

(6) $x^2 + y^2 + 4y = 8$

(7) $x^2 + y^2 - 4x - 2y = 0$

(8) $x^2 + y^2 - 8x = 12$

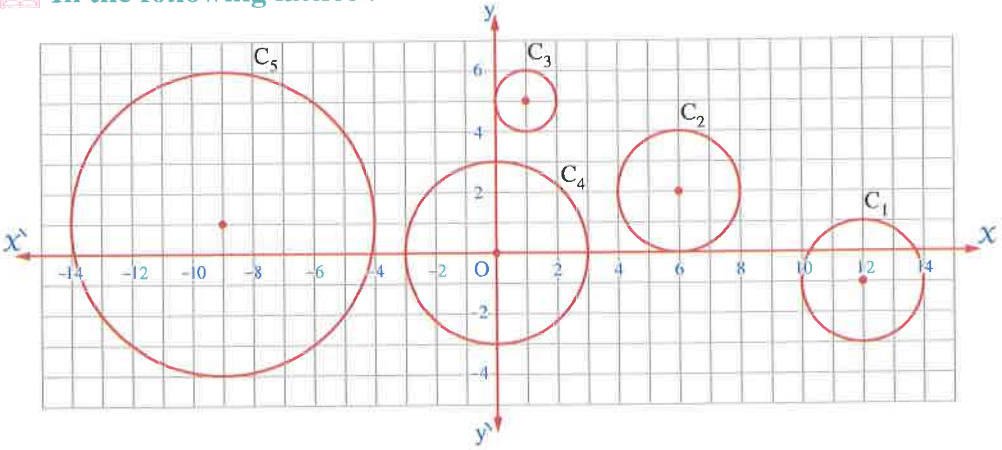
4 Show which two circles in the following are congruent and why ?

(1) $x^2 + y^2 - 4x + 8y = 0$, $x^2 + y^2 + 12y + 16 = 0$

(2) $x^2 + y^2 + 14y = 1$, $x^2 + y^2 + 10x - 25 = 0$

(3) $x^2 + y^2 - 2x + 4y - 3 = 0$, $x^2 + y^2 + 6x - 11 = 0$

5 **In the following lattice :**



(1) Write the equation of each circle.

(2) Which of the previous circles are congruent ? Explain your answer.

6 The opposite figure shows the two circles C_1 and C_2

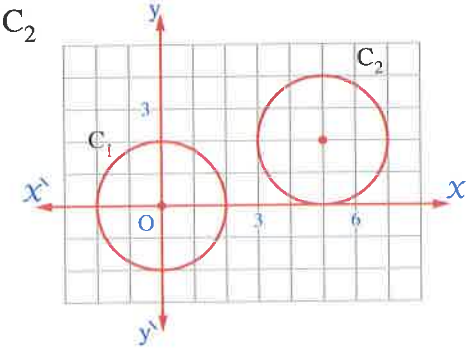
Prove that the two circles are congruent

, then find the equation of each of them.

If the circle C_3 is the image of the circle C_1

by translation $(-4 , 3)$

Find the equation of the circle C_3



7 **Show with reasons which of the following equations represents a circle and which of them does not represent a circle :**

(1) $x^2 + x\,y + y^2 = 25$

(2) $x^2 + y^2 + 8x - 16y - 1 = 0$

(3) $x^2 + 2y^2 + 6x - 5y = 0$

(4) $2x^2 + 2y^2 + 3y - 8 = 0$

(5) $(x + y)^2 - 3x + 6y - 4 = 0$

(6) $x^2 + y^2 + x + 2y + 7 = 0$

(7) $x^2 + y^2 + 2x - 4y + 5 = 0$

(8) $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$

(9) $x^2 + x = y^2 + y + 7$

8 M_1 and M_2 are the two centers of two circles where $M_1 = (2 , -1)$, $M_2 = (-1 , 3)$

Find the equation of each circle given that each of them passes through the centre of the other.

9 Prove that the two circles : $x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y + 15 = 0$ are concentric and find the radius length of each of them.

« 3 , 2.5 length unit »

10 Show which of the following points belongs to the circle C whose equation : $(x-6)^2+(y+1)^2=25$, then determine the position of each of the other points with respect to the circle C where : A (9 , 3) , B (7 , 5) , C (3 , 3) , D (2 , − 3)

11 A circle of centre (2 , − 1) passes through the point A = (− 1 , 3). Show the positions of the following points with respect to the circle M : B = (2 , 4) , C = (− 3 , 1) , D = (1 , 2)

12 Determine the position of the straight line with respect to the circle $(x+3)^2+(y-4)^2=9$

If the equation of the straight line is :

(1) $L_1 : 3x-4y+5=0$ (2) $L_2 : 6x-8y+23=0$ (3) $L_3 : 3x-4y+10=0$

13 Determine the position of the straight line $L : 5x-12y+13=0$ with respect to the circle $x^2+y^2-6x+4y-12=0$

14 Determine the position of the circle $C_1 : (x-5)^2+(y+2)^2=4$ with respect to the circle , $C_2 : (x+7)^2+(y-3)^2=1$

15 Are the two circles $C_1 : x^2+y^2-10x-8y+16=0$ and $C_2 : x^2+y^2+14x+10y-26=0$ touching externally ? Explain your answer.

16 Prove that the two circles : $(x+2)^2=1-y^2$, $x^2+y^2-2x-8y-19=0$ are touching internally.

17 If the two circles $C_1 : (x+2)^2+(y+11)^2=k$, $C_2 : (x-3)^2+(y-1)^2=16$ are touching. Find the value of k « 81 or 289 »

18 Prove that the two circles : $x^2+y^2-6x-4y+12=0$, $x^2+y^2+2x-4y-4=0$ touch each other and find the coordinates of the point of tangency, then find the circle equation whose centre is the point of tangency and passes through the center of the second circle.

19 Write the equation of the unit circle and if the point (2 a cos θ , 2 a sin θ) belongs to this circle. Find the real values of a (*i.e.* $a \in \mathbb{R}$)

20 **Find the value of $h \in \mathbb{R}$ which makes each of the following represents an equation of circle :**

(1) $x^2+y^2-2x-4y-h+2=0$

(2) $x^2+y^2+4x-6y-h^2+4=0$

(3) $x^2+y^2-4hx-2hy+10(h-1)=0$

(4) $x^2+y^2+6x+8y+h^2-3h+15=0$

(5) $x^2+y^2+2hx-6hy-2h^2+12h-3=0$

21 **Find the value of a in the equation :**

$x^2 + y^2 - 2x + 4y + 2a - 3 = 0$ in each of the following cases :

(1) The equation represents a circle.

(2) The equation represents a circle passing through the origin point.

(3) The equation represents a circle touching x-axis.

(4) The equation represents a circle touching y-axis.

(5) The equation represents a circle touching the straight line : $3x + 4y + 15 = 0$

(6) The equation represents a circle of diameter length 14 length unit.

22 **Write the general form of the equation of a circle if :**

(1) Its centre M (5 , 4) and touches the straight line $x = 2$

(2) Its centre M (5 , 3) and touches the straight line passing through the two points (3 , 7) , (– 1 , 3)

(3) Its centre M lies in the first quadrant and its radius length = 3 length unit and the two straight lines $x = 1$, $y = 2$ are tangents to it.

(4) Its radius length = 5 length unit and touches x-axis at the point (4 , 0)

(5) Its radius length = $3\frac{1}{2}$ unit and touches y-axis at the point (0 , – 4)

(6) Touches the two coordinate axes and passes through the point (– 2 , – 4)

(7) Touches x-axis at the point (– 3 , 0) and touches also y-axis

(8) Touches x-axis at the point (– 2 , 0) and intercepts from the positive part of y-axis a chord of length $4\sqrt{3}$ length unit.

(9) Touches y-axis at the point (0 , – 1) and intercepts from the negative part of x-axis a chord of length $4\sqrt{6}$ length unit.

(10) Touches the x-axis and passes through the two points (2 , 1) , (– 5 , 2)

(11) Touches y-axis and passes through the two points (– 4 , 2) , (– 1 , 2)

(12) Its centre lies on x-axis and passes through the two points A (1 , 3) , B (2 , – 4)

(13) Passes through the origin point and intercepts from the two positive parts of the x-axis and y-axis two parts of lengths 12 , 16 length units respectively.

(14) Its centre lies on the straight line : $y - x = 1$ and passes through the two points A = (– 2 , 4) , B = (6 , 8)

(15) Its radius length $= \sqrt{85}$ length unit and passes through the two points $A = (-1, 2)$, $B = (3, 4)$

(16) Its diameter $\overline{AB}$ where A and B are the points of intersection between the circle $x^2 + y^2 + 2x + 4y = 0$ and x-axis.

23 Find the area of the equilateral triangle which its circumcircle is :

$x^2 + y^2 + x - 4y - 2 = 0$ « $\frac{75\sqrt{3}}{16}$ square units »

24 Find to the nearest cm² the surface area of a regular pentagon. If the circle : $x^2 + y^2 + 6x - 12y + 5 = 0$ passes through its vertices knowing that each unit in the coordinate plane represents 5 cm. « 2378 cm² »

25 Find the surface area of the regular hexagon which its circumcircle is :

$x^2 + y^2 - 10x + 6y + 25 = 0$ « $\frac{27\sqrt{3}}{2}$ square units »

26 Find the surface area of a regular polygon of 12 sides and the circle : $x^2 + y^2 - 16 = 0$ passes through its vertices. « 48 square units »

27 Find the equation of the circle whose radius length = 5 length unit and the equations of two straight lines carrying two diameters in it are : $3x + y + 2 = 0$, $4x - y - 16 = 0$, then prove that the point $(5, -4)$ belongs to the circle.

28 Find the equation of the circle whose radius length equals the radius length of the circle :

$x^2 + y^2 - 2x\cos\theta - 2y\sin\theta - 8 = 0$

and the equations of two straight lines carrying two diameters in it are

$x + y = 0$, $\vec{r} = (1, 5) + k(1, 2)$

29 Find the equation of the circle which passes through the two points of intersection of the two circles $x^2 + y^2 - 10x = 0$ and $x^2 + y^2 + 2x - 12 = 0$ and whose center

(1) The origin (2) The point $(2, 0)$

30 Prove that the points : $A = (0, -1)$, $B = (-1, 0)$, $C = (-9, 0)$ lie on circle whose centre is $M = (-5, -5)$, then find the equation of this circle.

31 If the points : $A = (3 , -2)$, $B = (3 , 8)$, $C = (-1 , 0)$ belong to one circle, prove that $\overline{AB}$ is a diameter in it , then write the general form of its equation.

32 Prove that the triangle whose vertices are $A (8 , 0)$, $B (0 , 6)$, $C (0 , 0)$ is right-angled , then find the equation of the circle which passes through its vertices.

33 Prove that the points : $A = (-2 , 0)$, $B = (4 , 0)$, $C = \left(1 , 3\sqrt{3}\right)$ are the vertices of the equilateral triangle ABC , then find the equation of the circumcircle of ΔABC

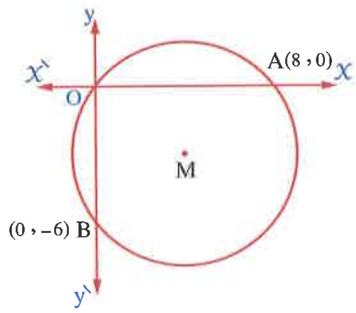
34 Find the equation of the circle which passes through the points : $A = (2 , -1)$, $B = (-2 , 0)$, $C = (0 , -9)$ and determine its centre and its radius length.

35 If $A = (3 , 0)$, $B = (0 , 9)$, $C = (0 , 1)$, $D = (-1 , 2)$

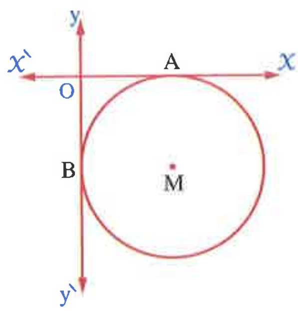
Prove that the quadrilateral ABCD is cyclic.

36 **Find the general form of the equation of the circle M in each of the following figures :**

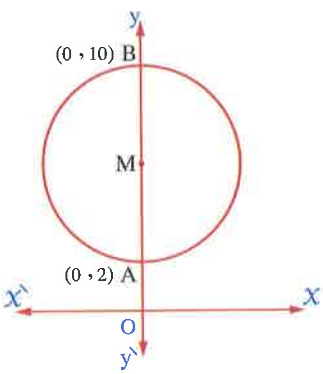
(1) The circle passes through the origin point and passes through the two points A and B



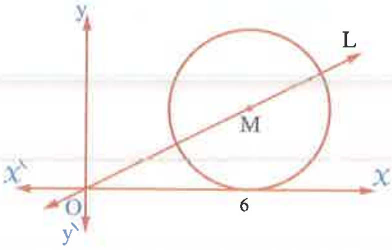
(2) The circle touches the two coordinate axes at A and B and the length of $\overline{MO} = 2\sqrt{2}$



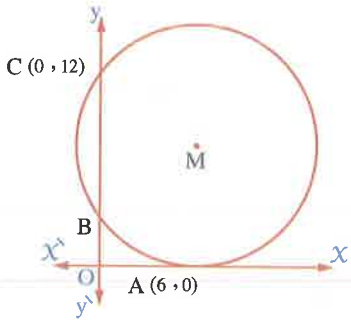
(3) The centre of the circle lies on y-axis and the circle intersects y-axis at A and B



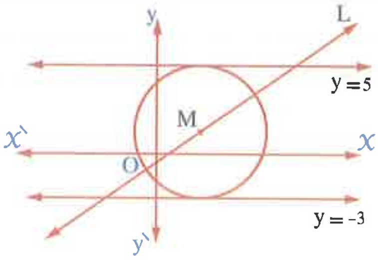
(4) The straight line whose equation is $x - 3y = 0$ passes through the centre of the circle and the origin point.



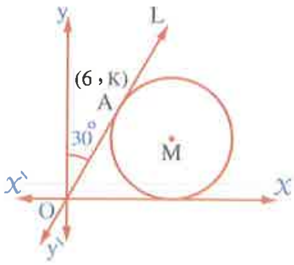
(5) The circle touches x-axis at A and intersects y-axis at B and C



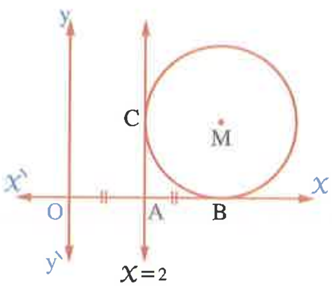
(6) The straight line L : $2x - 3y = 1$ passes through the centre of the circle and the two straight lines $y = 5$, $y = -3$ are tangents to the circle.



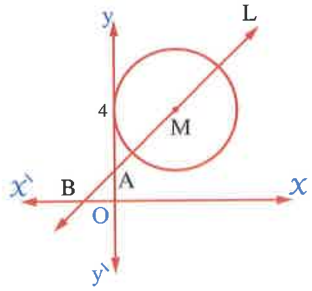
(7) The straight line L touches the circle at A (6 , k) and makes an angle of measure 30° with the positive direction of y-axis and the circle touches x-axis also.



(8) The circle touches x-axis at B and touches the straight line $x = 2$ at C



(9) The circle touches y-axis at the point (0 , 4) and the straight line L passes through the centre of the circle and the two points A (0 , 2) and B (– 1 , 0)

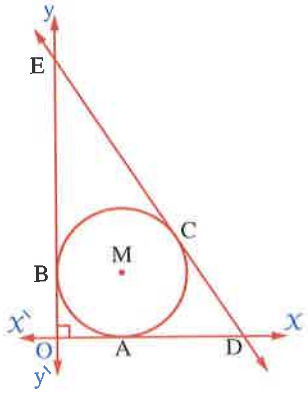


37 In the opposite figure :

The two coordinate axes touch the circle M at A and B.

If the straight line $4X + 3y - 12 = 0$ is a tangent to the circle M at C

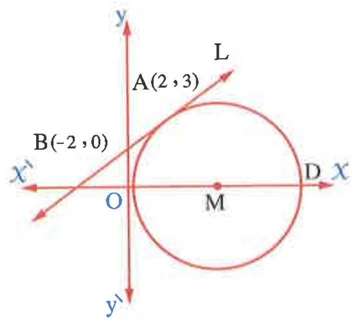
Find the equation of the circle.



38 In the opposite figure :

The straight line L touches the circle at A (2 , 3) and intersect X-axis at B (– 2 , 0)

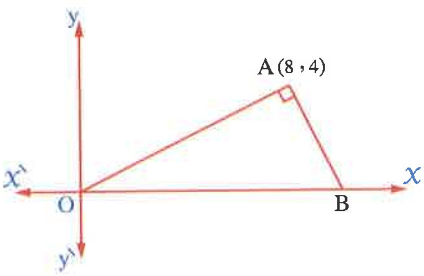
Find the equation of the circle M



39 In the opposite figure :

If $\overline{OA} \perp \overline{AB}$, A (8 , 4)

Find the equation of the circle which passes through the points A , B and O



Third Higher skills

1 Choose the correct answer from those givens :

(1) The equation : $(k-2)x^2+(2-k)y^2-kx+3ky-25=0$ ....

(a) represents a circle when $k=2$

(b) represents a circle when $k \neq 2$

(c) represents a circle when $k \in \mathbb{R}$

(d) does not represent a circle whatever the value of k.

(2) The height of a right circular cone is 6 length units and the equation of its circular base is $x^2+y^2=64$ in the xy – plane , then the volume of the cone = ... cubic units.

(a) 96π (b) $\frac{640}{3}\pi$ (c) 128π (d) $\frac{128}{3}\pi$

(3) The least distance between the y-axis and a point on the circle whose equation : $(x-7)^2+(y-5)^2=16$ is .... length units.

(a) 11 (b) 3 (c) 5 (d) 7

(4) Number of circles touch the coordinate axes and their centres lie on the circle : $x^2+y^2=25$ equals ....

(a) zero (b) 1 (c) 2 (d) 4

(5) In the opposite figure :

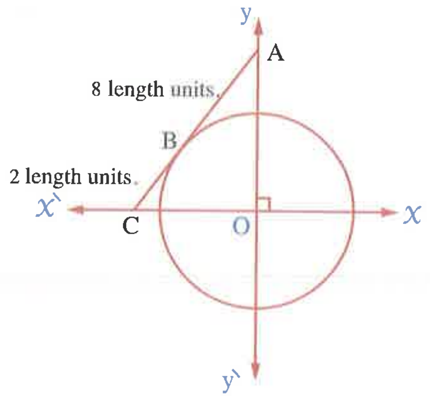
The equation of the circle is ....

(a) $x^2+y^2=4$

(b) $x^2+y^2=16$

(c) $x^2+y^2=64$

(d) $x^2+y^2=100$



(6) In the opposite figure :

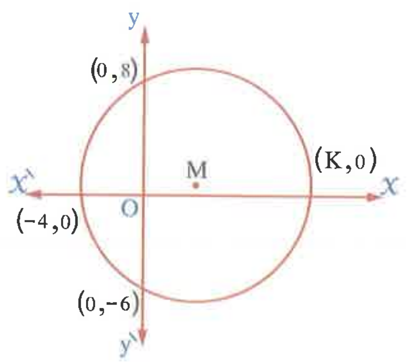
The equation of the circle is ....

(a) $(x+4)^2+(y+1)^2=65$

(b) $(x-6)^2+(y-2)^2=64$

(c) $(x-4)^2+(y-1)^2=65$

(d) $(x-4)^2+(y-2)^2=64$



(7) If O is the origin , $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two tangents to the circle $x^2 + y^2 - 10x + 4y + 6 = 0$, then the centre of the circumcircle of Δ AOB is ....

(a) $\left(\frac{7}{4}, 2\right)$ (b) $\left(\frac{5}{2}, -1\right)$ (c) $\left(\frac{7}{4}, -1\right)$ (d) $\left(\frac{5}{2}, 2\right)$

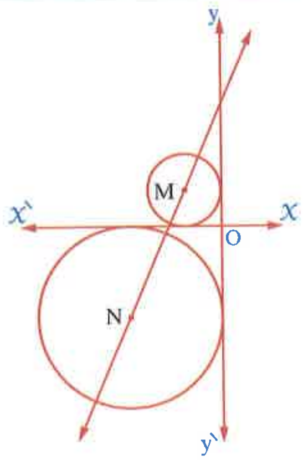
(8) The length of the common chord of the two circles $x^2 + y^2 - 10x - 10y = 0$ and $x^2 + y^2 + 6x + 2y - 40 = 0$ equals .... length unit.

(a) $5\sqrt{2}$ (b) 10 (c) 12 (d) $10\sqrt{2}$

2 **In the opposite figure :**

If each of the two circles M and N touches the two coordinate axes and the equation of the line of centres $\overleftrightarrow{MN}$ is : $y = 2x + 1$

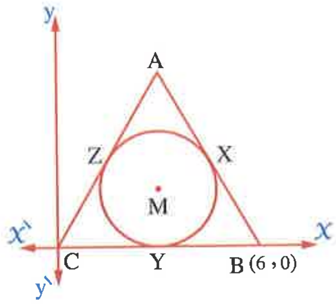
Find the equation of each of the two circles M and N



3 **In the opposite figure :**

ABC is an equilateral triangle its sides touch the circle M

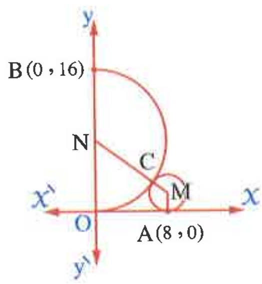
Find the equation of the circle M



4 **In the opposite figure :**

A semicircle , its centre N lies on y-axis and touches a circle M at C and the circle M touches x-axis at A where A = (8 , 0) If B = (0 , 16)

Find the general form of the equation of the circle M



● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Life applications

1 **Town designing :**

In the drawing for one of the cities in a perpendicular coordinate axes plane , where each unit in it represents 5 metres. It is found that the circle : $x^2 + y^2 - 6x + 8y + 11 = 0$ represents one of its squares. Find to the nearest squared metre the area of the square $\left(\pi = \frac{22}{7}\right)$ « 1100 m^2 »

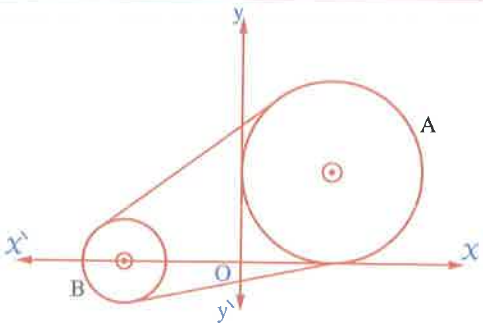
2 **Marine navigation :** A radar is located in the position A (7 , – 9) and cover a circular region. The length of its radius equals 30 length unit. Write the equation of the circle that determine the radar range in the coordinates plane. Can the radar observe a ship in the position B (25 , – 30) ? Explain your answer.

3 **Architectural design :**

An architect designs a building in the form of a regular octagon. Its vertices passes by a circle $x^2 + y^2 - 4x + 12y - 60 = 0$ Calculate the area of the building to the nearest squared unit. « $200\sqrt{2}$ square units »

4 **Industry :**

The opposite figure shows a pulley A in a machine touching the two coordinate axes , it rotates by a wire passing on a small pulley B which the equation of its circle is : $x^2 + y^2 + 14x + 45 = 0$



Find :

(1) The equation of the circle of the pulley A given that its radius length = 5 units.

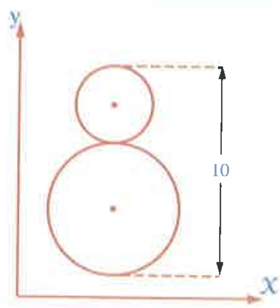
(2) The distance between the two centres of the two pullies if the plane unit represents 6 cm. « 78 cm. »

5 **Industry :**

The opposite figure shows two gears in a machine such that their centres lie on a straight line parallel to y-axis and the maximum distance between their edges is 10 units.

Find the equation of the circle of the small gear given that the equation of the great gear is :

$x^2 + y^2 - 10x - 8y + 32 = 0$

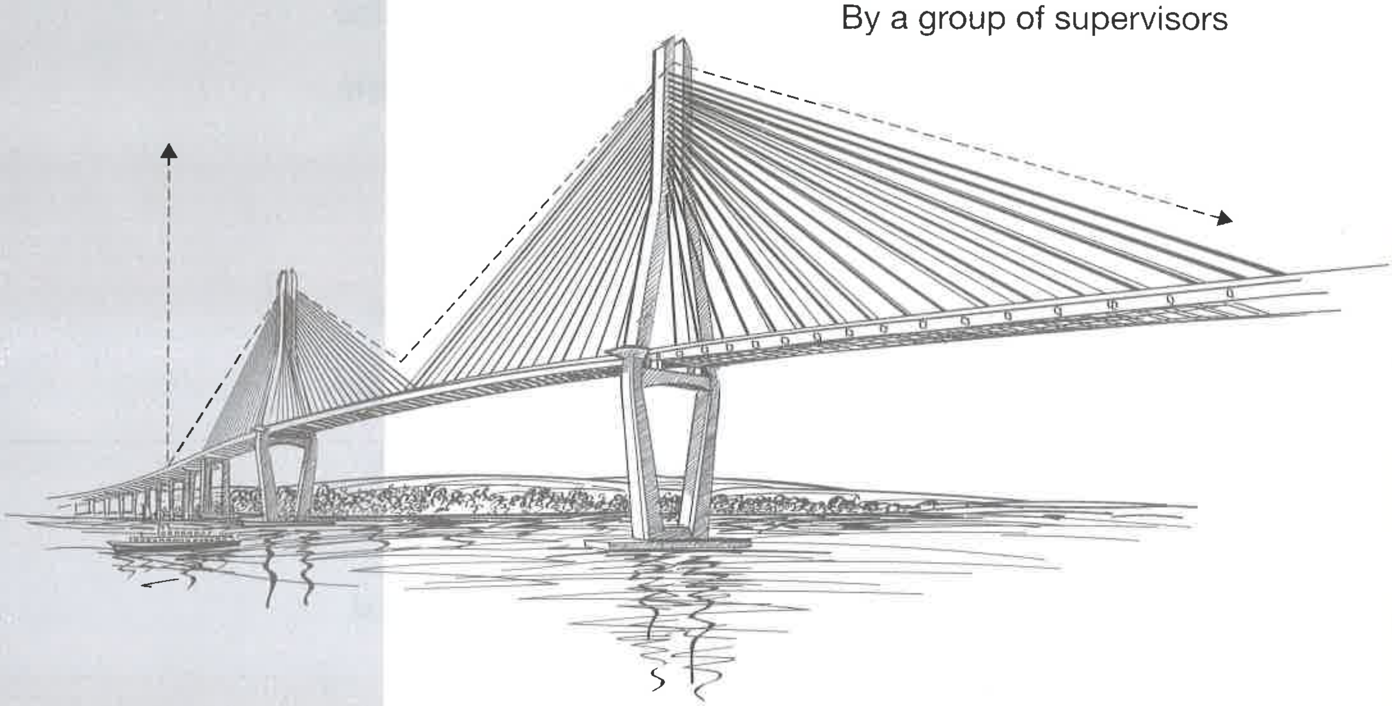


EL-MOASSER

Mathematics

Applications

By a group of supervisors



FIRST TERM

2 SEC.

EXAMINATIONS

SCIENTIFIC SECTION

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CONTENTS

Accumulative quizzes

- **First :** Accumulative quizzes on statics.
- **Second :** Accumulative quizzes on geometry and measurement.

Total mark

Quiz 1 on lesson 1 – unit 1

$\frac{\ldots}{10}$

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) $\overrightarrow{F_1} = 2\,\hat{i} + 3\,\hat{j}$, $\overrightarrow{F_2} = \hat{i} + \hat{j}$ where F_1 , F_2 are measured in newton then the magnitude of their resultant .... newton.

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{13}$ (d) 5

(2) Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton , then the magnitude of each is .... newton.

(a) $\frac{3}{2}$ (b) $\sqrt{3}$ (c) 3 (d) $3\sqrt{3}$

(3) The resultant of two forces acting at a point is maximum when the included angle between them is equal to ....

(a) zero (b) 60° (c) 120° (d) 180°

(4) The magnitude of the resultant of two forces 3 , 5 newton and the measure of their included angle is 60° equals .... newton.

(a) 2 (b) 6 (c) 7 (d) 8

Second question *3 marks*

The magnitude of two forces are F , 4 newton acting at a point , and the measure of the angle between them is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton , find the magnitude of $\overrightarrow{F}$ and the angle measure between their resultant and the force $\overrightarrow{F}$

Third question *3 marks*

The magnitude of two forces are 4 , F newton acting at a point , and the measure of the angle between them is 120° , their resultant is perpendicular on the first force. Find the value of F

Total mark

$\overline{}$ 10

Quiz 2 till lesson 2 – unit 1

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) Two forces of magnitude 3 F and 2 F intersecting at a point and their resultant is 5 F , then the measure of the angle between them is ....

(a) zero° (b) 60° (c) 20° (d) 180°

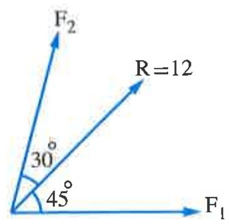
(2) As resolving the force $\overrightarrow{R}$ into two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ making with $\overrightarrow{R}$ two angles of measure θ_1 and θ_2 on both sides of $\overrightarrow{R}$ respectively , then the magnitude of $\overrightarrow{F_1}$ = ...

(a) $\frac{R \sin \theta_1}{\sin (\theta_1 + \theta_2)}$ (b) $\frac{R \sin \theta_2}{\sin (\theta_1 - \theta_2)}$ (c) $\frac{R \sin \theta_2}{\sin (\theta_1 + \theta_2)}$ (d) $\frac{R \sin (\theta_1 + \theta_2)}{\sin \theta_2}$

(3) Two forces of equal magnitudes , inclosing between them an angle of measure 90° If the magnitude of their resultant is 8 N , then the value of each force measured in newton is ....

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

(4) **In the given figure :**



F_1 = ...

(a) 12 cos 75° (b) 12 cos 45°

(c) 6 sec 45° (d) 6 csc 75°

Second question *3 marks*

Two forces of magnitudes 4 , F newton act at a point and the measure of their included angle is 135° Given that their resultant makes angle 45° with the force F , find F and the magnitude of their resultant.

Third question *3 marks*

Resolve a force 100 newton in two directions the first inclines by 60° to the force and the other by 30° in the other side of the given force.

Total mark

10

Quiz 3 till lesson 3 – unit 1

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) A body of weight (W) is placed on an inclined plane makes angle of measure θ to the horizontal then the component of its weight in direction of line of greatest slope equals ....

(a) $W \cos \theta$ (b) $W \sin \theta$ (c) $W \tan \theta$ (d) W

(2) Two perpendicular forces of magnitude 12 newton , 5 newton act at a point , then the magnitude of their resultant = ... newton.

(a) 17 (b) 7 (c) 13 (d) 14

(3) Given : $\overrightarrow{F_1} = 3\hat{i} - 2\hat{j}$, $\overrightarrow{F_2} = a\hat{i} - \hat{j}$, $\overrightarrow{F_3} = 4\hat{i} - b\hat{j}$ and their resultant $\overrightarrow{R} = 6\hat{i} - 4\hat{j}$, then a + b = ...

(a) 2 (b) – 2 (c) zero (d) – 1

(4) Given : $\overrightarrow{F_1} = 5\hat{i}$, $\overrightarrow{F_2} = 7\hat{i} - 5\hat{j}$, $\overrightarrow{R}$ is their resultant then $\|\overrightarrow{R}\|$ = ...

(a) $\sqrt{5} + \sqrt{74}$ (b) 49 (c) 13 (d) $\sqrt{12} - \sqrt{5}$

Second question *3 marks*

Three coplanar forces of magnitudes 85 , 75 , $50\sqrt{2}$ kg.wt. act at a point , the first acts towards East , the second towards 30° West of the North and the third towards West South. Find the magnitude of their resultant.

Third question *3 marks*

Two forces act at a point , the maximum value of their resultant is 32 kg.wt. and the minimum value of their resultant is 12 kg.wt. Find the magnitude of each force , then find the magnitude of their resultant when the angle between the two forces = 60°

Total mark

......

10

Quiz 4 till lesson 4 – unit 1

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) Three equal forces in magnitudes act at a point and the forces are in equilibrium , then the measure of the angle between any two forces = ...

(a) 60° (b) 90° (c) 120° (d) 150°

(2) The maximum and minimum value respectively of the resultant of the two forces of magnitudes 8 , 13 newton are .... newton.

(a) 13 , 8 (b) 13 , 5 (c) 21 , 8 (d) 21 , 5

(3) Two forces act at a point of magnitudes 5 , 3 newton and the measure of the angle between them is 60° then the magnitude of their resultant (R) equals .... newton.

(a) 2 (b) 7 (c) 8 (d) 5

(4) Two forces of equal magnitudes , the magnitude of their resultant is 3 newton and the measure of the angle between them is $\frac{\pi}{3}$, then the magnitude of each .... newton.

(a) $\sqrt{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $3\sqrt{3}$

Second question *3 marks*

A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ The body is prevented from sliding by a force makes with the line of the greatest slope an angle of measure 30° upwards.

Find the magnitude of the force and the reaction of the plane.

Third question *3 marks*

If $\overrightarrow{F_1} = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$, $\overrightarrow{F_3} = -14\,\overrightarrow{i} + b\,\overrightarrow{j}$ are three coplanar forces meeting at a point and their resultant is $\overrightarrow{R} = \left(10\sqrt{2}\ ,\ \frac{3\pi}{4}\right)$, then find the values of a and b

Total mark

Quiz 5 till lesson 5 – unit 1

10

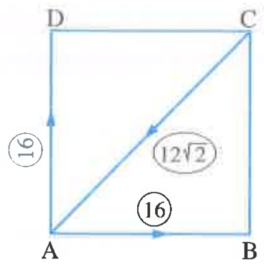
Answer the following questions :

First question *2 marks*

The magnitudes of two forces F , $\sqrt{2}$ F newton act at a point and their resultant is perpendicular to the first force. Find the angle between the two forces and prove that the magnitude of their resultant equals F

Second question *2 marks*

The opposite figure represents the forces 16 , 16 , $12\sqrt{2}$ newton which act in the square ABCD in the directions $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CA}$ respectively. Find the magnitude and direction of their resultant.



Third question *4 marks*

A smooth sphere of radius length 30 cm. and of weight 10 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 30 cm. , one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the tangency point of the sphere and the wall.

Find the magnitudes of the tension in the string and the reaction of the wall

Fourth question *2 marks*

Three coplanar forces of magnitudes 5 , 10 , $4\sqrt{7}$ newton act at a point , the measure of the angle between the first two forces equals 60° , find the greatest and the smallest magnitude of their resultant.

Total mark

10

Quiz 1 on lesson 1 – unit 2

Answer the following questions :

First question *5 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) All the following cases determine a plane except ....

(a) a straight line and a point not on it. (b) two different parallel straight lines.

(c) two intersecting straight lines. (d) two skew straight lines.

(2) The number of planes which passes through 3 non-collinear points equals ....

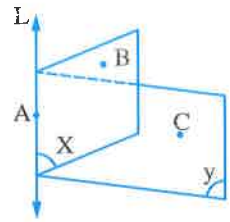
(a) 1 (b) 3 (c) 6 (d) infinite numbers.

(3) The skew lines ....

(a) never intersect. (b) are not perpendicular.

(c) are not parallel. (d) are neither parallel nor intersecting.

(4) **In the opposite figure :**



The plane X $\cap$ the plane Y $\cap$ the plane ABC = ...

(a) $\{A\}$ (b) the straight line L

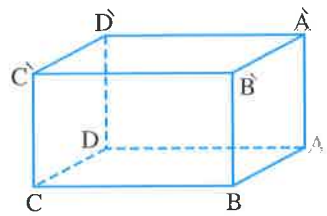
(c) $\overleftrightarrow{AC}$ (d) $\overleftrightarrow{AB}$

(5) If $\overleftrightarrow{AB}$ // plane X , then $\overleftrightarrow{AB} \cap X =$ ....

(a) $\overline{AB}$ (b) $\overrightarrow{AB}$ (c) $\overleftrightarrow{AB}$ (d) $\varnothing$

Second question *5 marks* **1 mark for each item**

By using the opposite figure state :



(1) Two parallel planes.

(2) Two intersecting planes.

(3) Two skew straight lines.

(4) A straight line and a plane which are parallel.

(5) The intersection line of the plane ABB̀À with the plane ACD

Total mark

Quiz 2 till lesson 2 – unit 2

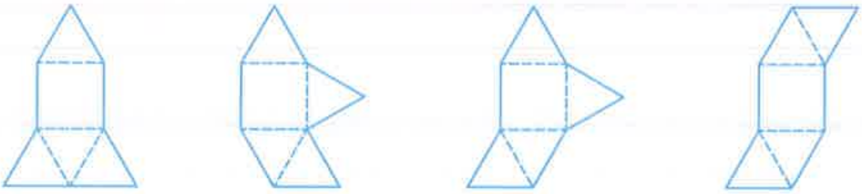
10

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



(a) (b) (c) (d)

(2) The volume of a regular quadrilateral pyramid 12 cm^3. and its height 4 cm. then the length of its base side = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

(3) A regular quadrilateral pyramid , the length of its base side is 10 cm. , and its lateral height is 13 cm. , then its volume in cm^3. = ...

(a) $\frac{1}{2} \times (10)^2 \times 13$ (b) $\frac{1}{3} \times (10)^2 \times 12$

(c) $\frac{1}{2} \times (12)^2 \times 13$ (d) $\frac{1}{3} \times (13)^2 \times 10$

(4) If the sum of edge lengths of a triangular regular faces pyramid equals 18 cm. , then its total area = ... cm^2.

(a) $\frac{27\sqrt{2}}{4}$ (b) $\frac{27\sqrt{3}}{4}$ (c) $\frac{27\sqrt{3}}{2}$ (d) $9\sqrt{3}$

Second question *3 marks* **$1\frac{1}{2}$ marks for each item**

The side length of the base of regular quadrilateral pyramid is 20 cm. and its height is $10\sqrt{3}$ cm.

Find : (1) The lateral area. (2) The volume of the pyramid.

Third question *3 marks*

A regular hexagonal pyramid , the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm. Find its total area.

Total mark

....

10

Quiz 3 till lesson 3 – unit 2

Answer the following questions :

First question *4 marks* **1 mark for each item**

Choose the correct answer from those given :

(1) The lateral area of a right cone whose base radius length is 6 cm.

and its height = 8 cm. is .... cm^2.

(a) 60π (b) 28π (c) 10π (d) 48π

(2) A regular quadrilateral pyramid of base side length 10 cm. and its lateral height 13 cm. , its lateral area = ...

(a) 260 cm^2. (b) 360 cm^2. (c) 130 cm^2. (d) 520 cm^2.

(3) The number of planes passes through 3 collinear points is ....

(a) zero (b) 1 (c) 3 (d) infinite.

(4) The volume of a regular quadrilateral pyramid whose base perimeter 36 cm. and its height 10 cm. equals .... cm^3.

(a) 810 (b) 180 (c) 360 (d) 270

Second question *3 marks*

The base length of a regular quadrilateral pyramid is 18 cm. , its volume is 1296 cm^3. , Find its lateral height and its lateral area.

Third question *3 marks*

Find the radius length of the base of right circular cone whose total area 616π cm^2. and the length of its drawer is 30 cm.

Total mark

10

Quiz 4 till lesson 4 – unit 2

Answer the following questions :

First question *4 marks* 1 mark for each item

Choose the correct answer from those given :

(1) The centre of the circle : $x^2 + y^2 - 6x + 8y = \text{zero}$ is the point ....

(a) $(3, -4)$ (b) $(4, -3)$ (c) $(-3, 4)$ (d) $(-4, 3)$

(2) The circumference of a circle whose equation : $(x-3)^2 + (y+2)^2 = 25$ equals ....

(a) 2π (b) 3π (c) 10π (d) 25π

(3) The lateral area of a right cone whose base radius length 6 cm. and its height 8 cm. equals .... cm^2.

(a) 60π (b) 28π (c) 40π (d) 48π

(4) The point which lies on the circle : $(x-2)^2 + y^2 = 13$

(a) $(2, 3)$ (b) $(3, -2)$ (c) $(2, 5)$ (d) $(4, 3)$

Second question *3 marks*

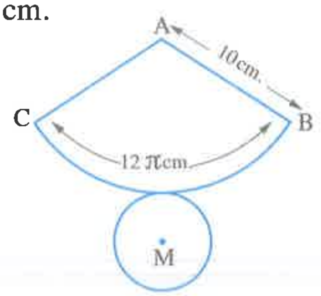
Find the general form of the circle whose centre $(-2, 5)$ and passes through $(3, 2)$

Third question *3 marks*

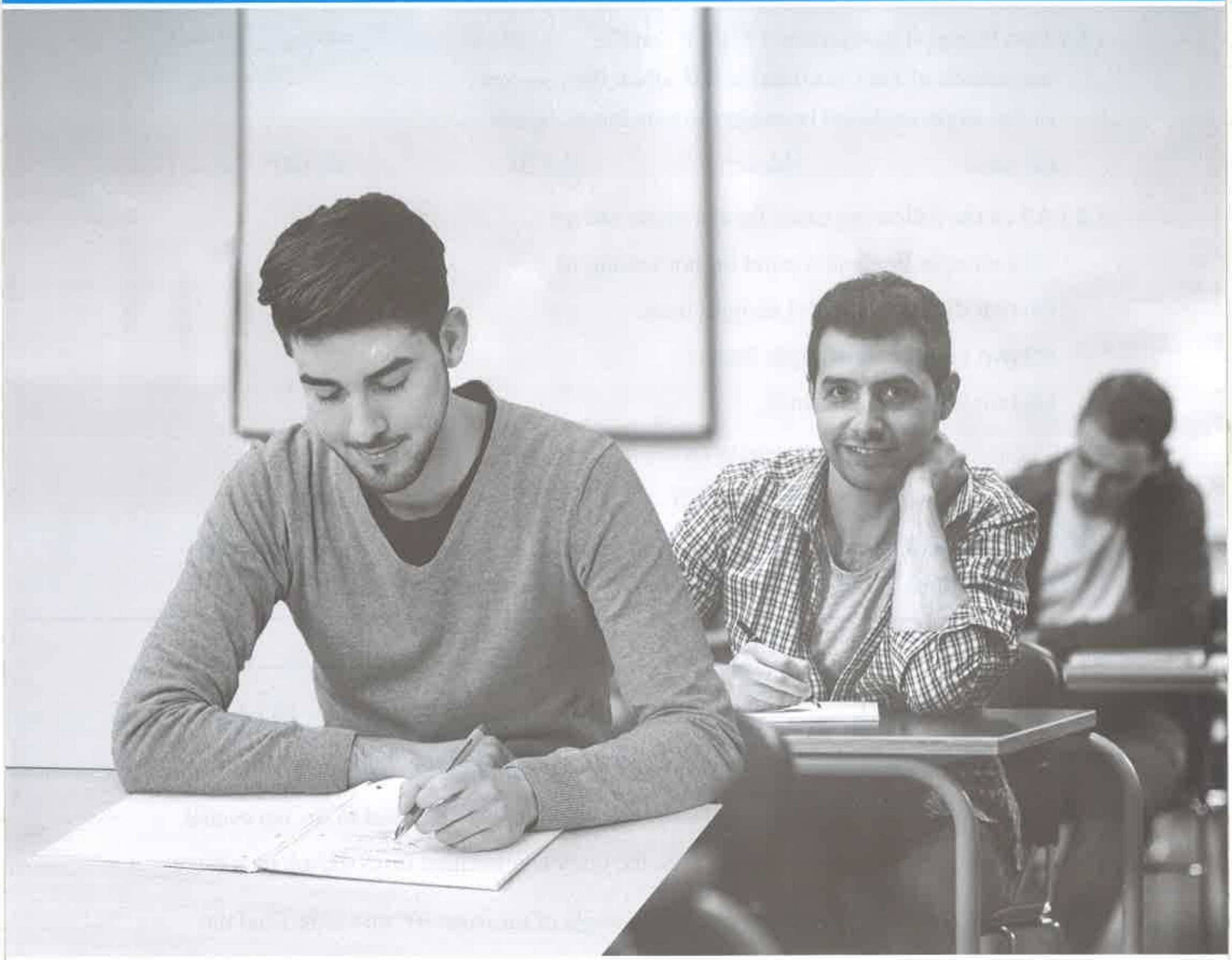
The opposite figure represents the net of a solid where $\overset{\frown}{BC} = 12\pi$ cm. , AB = 10 cm. , **calculate :**

(1) The total area of this solid.

(2) The volume of the solid.



School book examination



School book examination

Answer the following questions :

The questions of the first term in the school book examinations are collected to form one test.

1 **Choose the correct answer from the given ones :**

(1) Two forces of magnitude 3 F , 2 F and the magnitude of their resultant is 5 F , then the measure of the angle enclosed between the two forces equals ....

(a) zero° (b) 60° (c) 20° (d) 180°

(2) All of the following cases form a plane except ....

(a) a straight line and a point do not belong to it.

(b) two different parallel straight lines.

(c) two intersected straight lines.

(d) two skew straight lines.

(3) The point that lies on the circle $(x-2)^2 + y^2 = 13$ ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 5) (d) (4 , 3)

(4) Two forces of magnitudes 5 , 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant R equals ....

(a) 2 (b) 7 (c) 8 (d) 5

2 (a) If the three coplanar forces $\overrightarrow{F}_1 = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F}_2 = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$, $\overrightarrow{F}_3 = -14\,\overrightarrow{i} + b\,\overrightarrow{j}$ act at a point and their resultant $\overrightarrow{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$ Find the values of a and b

(b) A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ the body is prevented from sliding by a force form with the line of the greatest slope an angle of measure 30° upwards. Find the magnitude of the force and the reaction of the plane.

3 (a) Find the general form of the equation of a circle whose centre (2 , – 1) and the length of its radius is 3 cm.

(b) A uniform smooth sphere of weight 10 gm.wt. and radius length 30 cm. is hanged from a point on its surface by a light string of length 30 cm. and the other end of the string is fixed in a point on a vertical smooth wall. **Find in the case of equilibrium each of :**

(1) The tension in the string. (2) The reaction of the wall on the sphere.

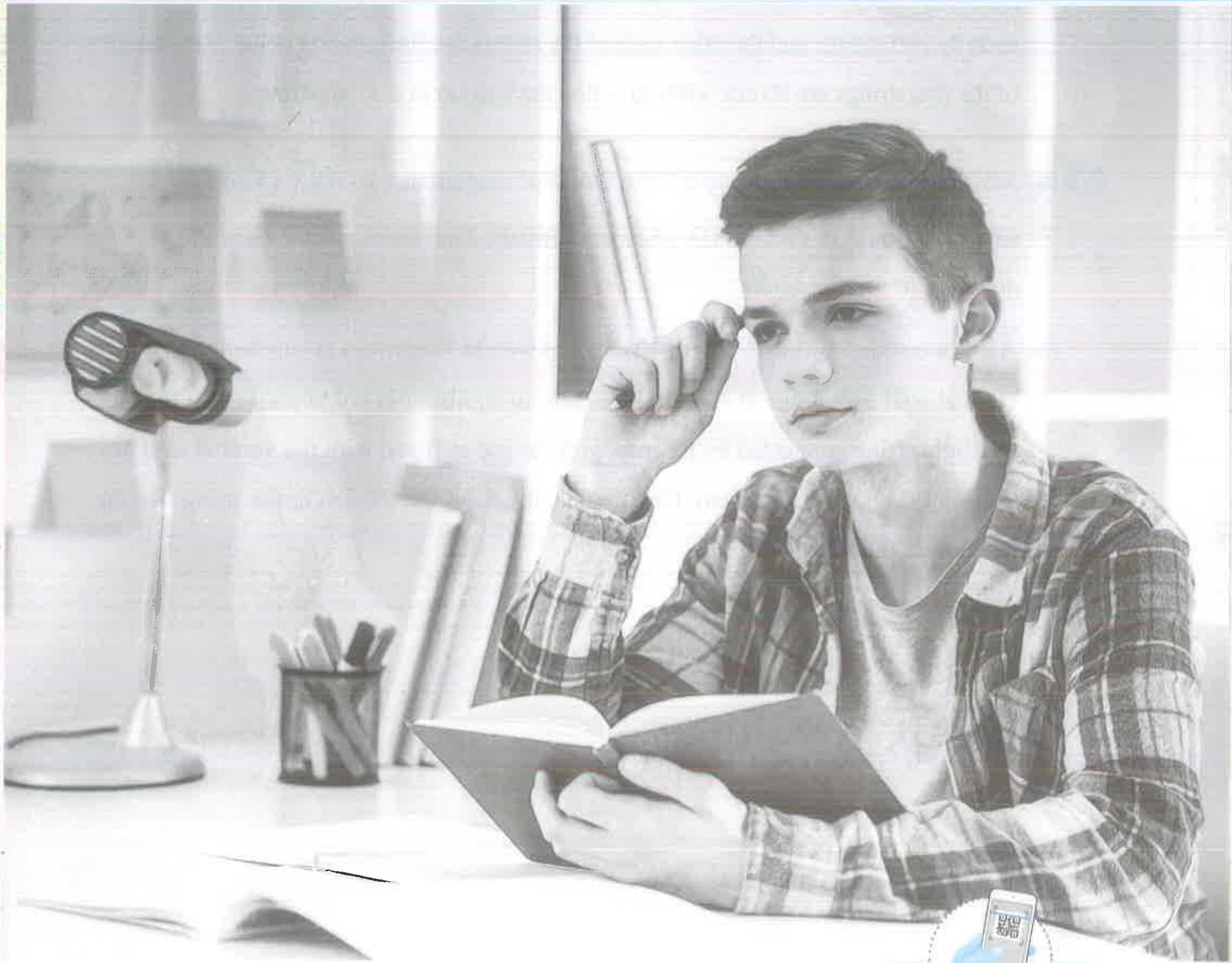
4 (a) A cube of wax with edge length 30 cm. transfer into a right circular cone of height 45 cm. Find the length of the radius of the base of the cone, if 8 % of the wax loss during milting and transferring processes.

(b) A uniform rod of length 100 cm. and weight 150 gm.wt. is suspended freely from its ends by two strings and the other ends of the strings are fixed in one point. If the lengths of the two strings are 80 cm. , 60 cm. , find the tension in the two strings.

5 (a) ABCDEF is a uniform hexagon , the forces of magnitudes 8 , $6\sqrt{3}$, 5 and $4\sqrt{3}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

(b) $\overline{AB}$ is a uniform rod with length 40 cm. and weight 30 newton is attached with a vertical wall by a hinge at A , the rod is kept in equilibrium horizontally by a mean of a light string connected by its ends with the rod at B and with the vertical wall at the point C above A by 40 cm. Find the magnitude of the tension in the string and the reaction of the hinge at A

Final examinations



- **First : Examination models**
- **Second : Multiple choice examinations**

Scan the QR codes to solve interactive tests

First Examination Models

Model 1

Interactive test 1

Answer the following questions :

1 The total surface area of a right circular cone which its drawer length equal the diameter length of its base is ....

(a) $4 \pi r^2$ (b) $3 \pi r^2$ (c) $3 \pi r^3$ (d) $4 \pi r^3$

2 If A , B and C are three points identify a plane , then ....

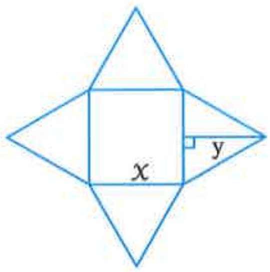
(a) $AB = BC = CA$ (b) $AB + BC = AC$

(c) $AB + BC > AC$ (d) $AB + BC < AC$

3 Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton , then the magnitude of each is .... newton.

(a) $\sqrt{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $3\sqrt{3}$

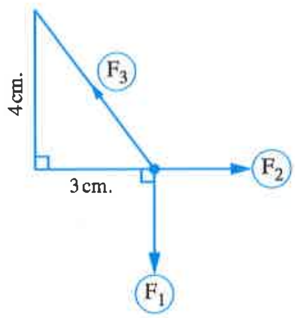
4 The opposite figure represents a regular quadrilateral pyramid its height (h) , then the relation between x , y and h is ....



(a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$

(c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$

5 **In the opposite figure :**



A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton , and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 =$ ....

(a) 3 : 4 : 5 (b) 3 : 5 : 4

(c) 4 : 5 : 3 (d) 4 : 3 : 5

6 ABCDHE is a regular hexagon. Forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AH}$, $\overrightarrow{AE}$ respectively.
Find the magnitude and the direction of their resultant.

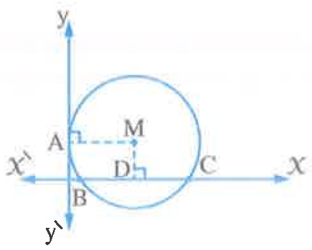
7 Volume of a regular quadrilateral pyramid is 400 cm^3 and its height is 12 cm. , then its lateral surface area = ... cm^2.

(a) 240 (b) 260 (c) 300 (d) 360

8 The area of base of a right circular cone is 36π cm^2 and the length of its drawer is 10 cm. **, find its :**

(1) Lateral surface area. (2) Total surface area. (3) Volume.

9 **In the opposite figure :**



If B (2 , 0) , C (8 , 0) , then the equation of the circle is ....

(a) $(x-5)^2+(y-4)^2=25$
(b) $(x+5)^2+(y-4)^2=36$
(c) $(x-5)^2+(y-4)^2=36$
(d) $(x+5)^2+(y-4)^2=25$

10 Two forces of magnitude 6 , F kg.wt. act at a point and measure of the angle between them 135° , if its line of action inclined by an angle 45° , with the line of action of the force F , the magnitude of the resultant = ... kg.wt.

(a) 6 (b) $6\sqrt{2}$ (c) $6\sqrt{3}$ (d) 10

11 A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. then the magnitude of the weight ....

(a) 36 (b) $72\sqrt{3}$ (c) 72 (d) $36\sqrt{3}$

12 If $\overrightarrow{R}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{R'}$ is the resultant of the two forces $\overrightarrow{F_1}$, $-\overrightarrow{F_2}$, then ....

(a) $\overrightarrow{R}+\overrightarrow{R'}=2\overrightarrow{F_1}$
(b) $\overrightarrow{R}=\overrightarrow{R'}+2\overrightarrow{F_2}$
(c) $R^2+R'^2=2(F_1^2+F_2^2)$
(d) all of previous.

13 **The equation of the circle which it is the image of the circle :**

$x^2 + y^2 - 12x + 6y + 20 = 0$ by translation $(x + 2, y - 2)$

(a) $x^2 + y^2 - 10x + 4y + 20 = 0$

(b) $x^2 - 16x + 10y + 20 = 0$

(c) $(x - 6)^2 + (y + 3)^2 = 20$

(d) $(x - 8)^2 + (y + 5)^2 = 25$

14 A force of magnitude $5\sqrt{3}$ newton act in direction 30° east of north , is resolved into two perpendicular components , then the magnitude of its component in direction the east = ... newton.

(a) 5

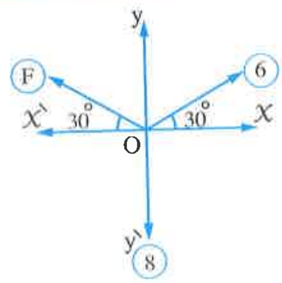
(b) 7.5

(c) $\frac{5\sqrt{3}}{2}$

(d) 15

15 $\overline{AB}$ is a uniform rod of weight 20 kg.wt. the end A attached to a hinge fixed on a vertical wall a horizontal force F acts at B , the body is an equilibrium when it inclined by angle 30° with vertical , find the magnitude of each of the force and reaction of the hinge.

16 **In the opposite figure :**



If the resultant of the forces (in newton) acts along y-axis

, then F = ... newton.

(a) 8

(b) 6

(c) 14

(d) 2

17 A cube made of wax , its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. then the radius length of the base of the cone = ... cm. given that 12% from wax had been lost during melting and reforming. $\left(\pi \simeq \frac{22}{7}\right)$

(a) $\frac{20\sqrt{110}}{11}$

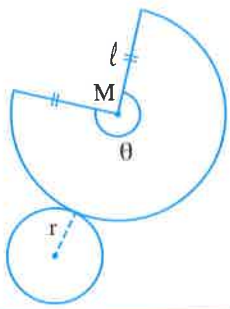
(b) $10\sqrt{2}$

(c) 160

(d) $8\sqrt{5}$

18 The opposite figure represents net of a cone where the central angle of its circular sector = θ , 180° < θ < 360°

, then ....



(a) $\ell < 2r$

(b) $\ell = r$

(c) $\ell = 2r$

(d) $\ell > 2r$

19 Which of the following system of forces can not be equilibrium ?

(a) 10 newton , 10 newton , 5 newton.

(b) 4 newton , 6 newton , 8 newton.

(c) 11 newton , 7 newton , 8 newton.

(d) 8 newton , 4 newton , 14 newton.

20 If the equation $\begin{pmatrix} x & y & 25 \end{pmatrix} \begin{pmatrix} x \\ y \\ -4 \end{pmatrix} = O$ represents a circle , then the length of its diameter is .... length unit.

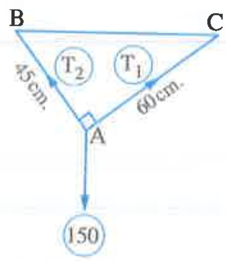
(a) 10 (b) 20 (c) 100 (d) 200

21 The magnitudes of two forces, meeting at a point, are 5F , 3F , then the magnitude of their resultant can not be equal to ....

(a) 2 F (b) $3\sqrt{2}$ F (c) 8 F (d) $5\sqrt{3}$ F

22 **In the opposite figure :**

The weight of a body is 150 gm. wt.
It is tied by two perpendicular strings their lengths are 60 cm. , 45 cm. and the other ends are fixed at C and B on the same horizontal line, then $T_2 - T_1 =$ .... gm.wt.



(a) 120 (b) 90 (c) 60 (d) 30

23 Two lines are skew if ....

(a) they are not parallel. (b) they are not intersecting.
(c) they are not coincident. (d) they are not on the same plame.

24 The point which lies on the circle $(x-2)^2 + y^2 = 13$ is ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 0) (d) (4 , 3)

Model 2

Interactive test 2

Answer the following questions :

1 Any three points are non-collinear identify ....

(a) 1 plane. (b) 2 planes. (c) 3 planes. (d) 4 planes.

2 When the two forces 6 and 8 newton are perpindicular , then the sine of inclinition angle of the resultant with the first force equals ....

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

3 The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) $(3, -4)$ (b) $(-4, 3)$ (c) $(-3, 4)$ (d) $(-3, -4)$

---

4 Three forces are equal in magnitude and meeting at a point are in equilibrium , then the measure of the angle between any two of them is ....

(a) $60°$ (b) $120°$ (c) $90°$ (d) $150°$

---

5 The volume of the right cone , if the circumference of its base is 44 cm. and its height is 15 cm. equals .... cm^3. $\left(\pi \simeq \frac{22}{7}\right)$

(a) 77 (b) 105 (c) 110 (d) 770

---

6 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

---

7 The forces of magnitudes 2 F , 3 F and 4 F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order.
then the magnitude of the resultant of these forces = ... newton.

(a) 5F (b) $2\sqrt{3}$ F (c) $\sqrt{3}$ F (d) F

---

8 ABCDEF is a regular hexagon , the force of magnitude 20 newton acts along $\overrightarrow{AD}$, is resolved into two components in directions $\overrightarrow{AC}$, $\overrightarrow{AF}$, then the component in direction $\overrightarrow{AF}$ equals .... newton.

(a) 10 (b) $10\sqrt{3}$ (c) 20 (d) $20\sqrt{3}$

---

9 The equation of the circle which its centre is $(2, -3)$ and touches the straight line which its equation is : $3x - 4y + 2 = 0$ is ....

(a) $(x-2)^2 + (y+3)^2 = 2$ (b) $(x+2)^2 + (y-3)^2 = 4$

(c) $x^2 + y^2 - 4x + 6y = 12$ (d) $(x-2)^2 + (y+3)^2 = 16$

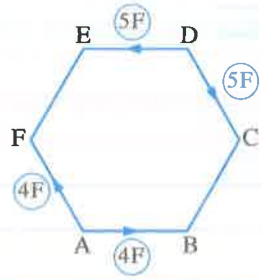
---

10 If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume ....

(a) will doubled. (b) will be three times.

(c) will be four times. (d) will not change.

11 **In the opposite figure :**



ABCDEF is a regular hexagon , then the resultant of these forces should be in direction ....

(a) $\overrightarrow{AD}$ (b) $\overrightarrow{DA}$

(c) $\overrightarrow{AC}$ (d) $\overline{EA}$

12 A regular quadrilateral pyramid , the side length of its base = 40 cm. , and its lateral height is 25 cm. , **find :**

(1) Height of the pyramid. (2) The lateral surface area.

(3) The total surface area. (4) Its volume.

13 If $\overrightarrow{F_1} = 5\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = -7\hat{i} + 2\hat{j}$, $\overrightarrow{F_3} = 2\hat{i} + \hat{j}$, then $\overrightarrow{R}$ = ...

(a) $7\hat{i} - 2\hat{j}$ (b) $14\hat{i} - 4\hat{j}$ (c) $-14\hat{i} + 4\hat{j}$ (d) $\vec{0}$

14 The ball of a pendulum of weight 600 gm.wt. is displaced unitl the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string. Then the magnitude of the force = ... dynes.

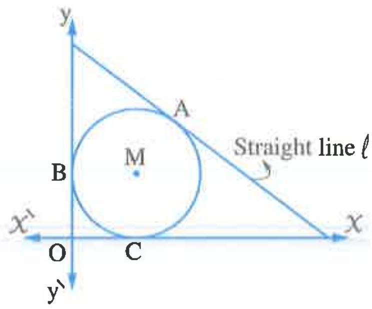
(a) $300\sqrt{3}$ (b) 1200 (c) 300 (d) $300\sqrt{2}$

15 Two forces F , F act at a particle and the magnitude of their resultant is F , then the measure of the included angle of the two forces = ...

(a) 60° (b) 45° (c) 120° (d) 135°

16 A circular sector made of paper its radius length is 36 cm. and its central angle is 210° , folded to a right circular cone , find its height.

17 **In the opposite figure :**



If the equation of the straight line ℓ is $\frac{x}{8} + \frac{y}{6} = 1$, then the equation of the circle is ....

(a) $(x-2)^2 + (y-2)^2 = 4$

(b) $(x-2)^2 + (y-2)^2 = 16$

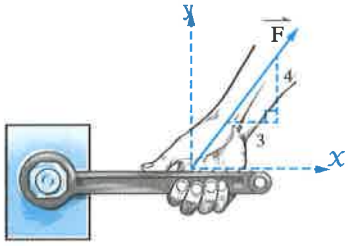
(c) $(x+2)^2 + (y+2)^2 = 4$

(d) $(x+2)^2 + (y+2)^2 = 16$

18 The ratio between volume of a regular triangular pyramid and volume of greatest cone can put it inside the pyramid equals ....

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

19 **In the opposite figure :**



If vertical component of the force $(\overrightarrow{F})$ of a person uses a spanner is 60 newton , then the horizontal component of $\overrightarrow{F}$ equals .... newton.

(a) 30 (b) 45
(c) 60 (d) 75

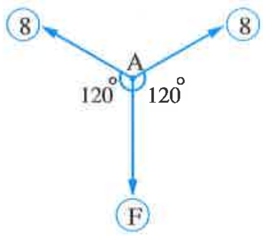
20 The magnitude of two forces 4 , 6 N. and the magnitude of the resultant is 10 N. , then the measure of the angle between the two forces equals ....

(a) 0° (b) 90° (c) 180° (d) 45°

21 The lateral area of a right cone, its base radius length is r , and its drawer ℓ equals ....

(a) $2\pi r \ell$ (b) $2\pi r^2 \ell$ (c) $\pi r \ell$ (d) $\pi r^2 \ell$

22 **In the opposite figure :**



Particle A is kept in equilibrium under action of the three forces, as shown in the figure, where $\overrightarrow{F}$ is in equilibrium with two forces each of magnitude 8 N. and it makes with each an angle of measure 120°, then F = ... N.

(a) zero (b) 8 (c) 16 (d) 8 sin 120°

23 The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) (3 , – 4) (b) (4 , – 3) (c) (– 3 , 4) (d) (– 4 , 3)

24 Which of the following statements is not true ?

(a) Any two points in the space have only one plane passing through them.
(b) Any three non-collinear points in the space determine a plane.
(c) The vertices of a triangle determine a plane.
(d) Every two intersecting straight lines are contained in one plane.

Model 3

Interactive test 3

Answer the following questions :

1 Two forces of magnitudes 5 , 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant $\mathbb{R}$ equals ....

(a) 2 (b) 7 (c) 8 (d) 5

2 The height of a right circular cone is 12 cm. and the length of its drawer is 15 cm. , then its volume = ... cm^3.

(a) 324π (b) 715π (c) 32π (d) 180π

3 The minimum value of resultant of two forces of magnitudes 5 , 9 newton and meeting at a point equals .... newton.

(a) zero (b) 9 (c) 4 (d) 5

4 The least number of planes can determine a solid is .... planes.

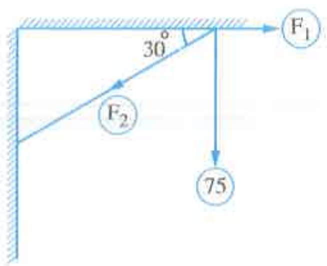
(a) three. (b) four. (c) two. (d) five.

5 A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.

6 The volume of a regular quadrilateral pyramid if the side length of its base = 18 cm. and the lateral height is 15 cm. is equal to .... cm^3.

(a) 1156 (b) 1254 (c) 1308 (d) 1296

7 **In the opposite figure :**



A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal (F_1) and the other F_2 , then F_2 = ... newton.

(a) 75 (b) $75\sqrt{3}$

(c) 150 (d) $150\sqrt{3}$

8 Two forces of magnitudes 6 , 12 newton act at a particle , enclosed between them an angle of measure 120° , then the measure of the angle between the resultant and the first forces = ...

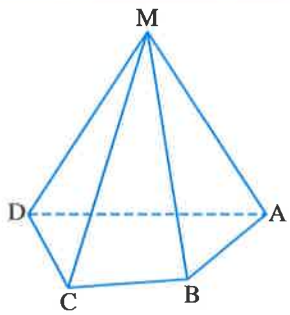
(a) 120° (b) 60° (c) 90° (d) 30°

9 **In the opposite figure :**

The plane ABD $\cap$ the plane MCD = ...

(a) $\overleftrightarrow{AM}$ (b) $\overleftrightarrow{CD}$

(c) $\{D\}$ (d) $\overleftrightarrow{MC}$



10 The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point , the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.

11 If the geometric centre of a regular hexagon is the origin and its area = $3\sqrt{3}$ cm^2 , then the equation of its circumcircle is ....

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$

(c) $x^2 + y^2 = 6$ (d) $x^2 + y^2 = 8$

12 **In the opposite figure :**

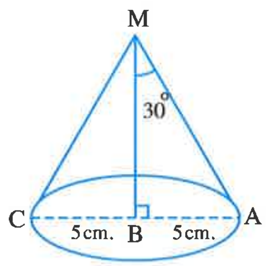
A right circular cone in which m ($\angle$ AMB) = 30°

, the radius length of the base = 5 cm.

, then its total area = ... cm^2

(a) 50π (b) 75π

(c) 100π (d) 125π



13 Two forces of magnitudes 6 , 2.5 newton , the magnitude of their resultant = 6.5 newton , then the included angle between the two forces is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

14 A body of weight 100 newton is placed on a smooth plane inclines to the horizontal by an angle 30° , the body kept in equilibrium by a horizontal force. F N. and the reaction of the plane on the body. is R N. then F + R = ... N.

(a) $100\sqrt{3}$ (b) $\frac{100\sqrt{3}}{3}$ (c) $200\sqrt{3}$ (d) $\frac{200\sqrt{3}}{3}$

15 In a trianglular pyramid of regular faces , if the sum of lengths of its edges = 36 cm. , then the height of the pyramid = ... cm.

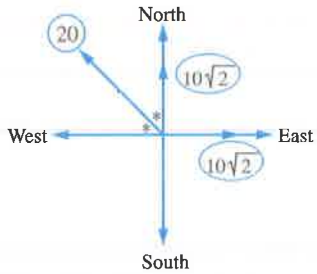
(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) 6 (d) 4

16 **Prove that the two circles :**

$x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y + 15 = 0$

are concentric circles , and find length of radius of each of them.

17 **In the opposite figure :**



The resultant of the forces $10\sqrt{2}$, $10\sqrt{2}$, 20 newton acts in direction ....

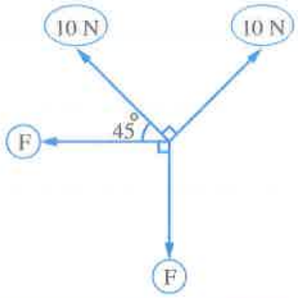
(a) the eastern north. (b) the north.

(c) the western north. (d) the western south.

18 If the volume of hemisphere of radius length "r" equals the volume of a cone the length of radius of its base = r and its height = h , then ....

(a) $h = \frac{2}{3} r$ (b) $h = 2r$ (c) $h = 2r^2$ (d) $h = 4r$

19 **In the opposite figure :**



The condition of equilibrium of the given forces is ....

(a) F = 10 newton. (b) $F = 10\sqrt{2}$ newton.

(c) $F = 5\sqrt{2}$ newton. (d) the system will not be equilibrium.

20 The circumference of the circle whose equation : $x^2 + y^2 = 8$ is ....

(a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

21 Two planes coincide if they have .... in common.

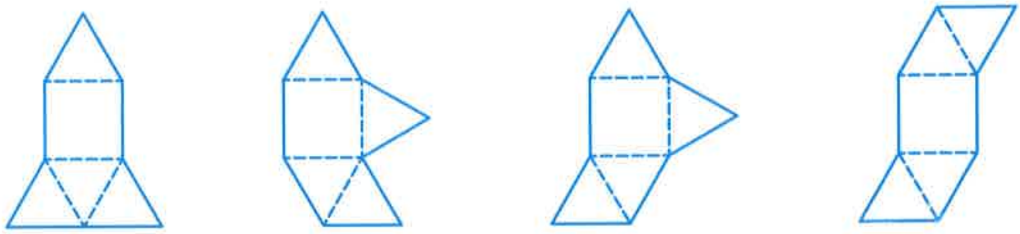
(a) one point (b) two points

(c) three collinear points (d) three non-collinear points

22 Force of magnitude $4\sqrt{2}$ N. acts in east direction. It is resolved into two perpendicular directions, then its component in the direction of north of the east equals .... N.

(a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

23 Which of the following nets cannot form a pyramid ?



(a) (b) (c) (d)

24 If a body is kept in equilibrium under action of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then ....

(a) $\overrightarrow{F_1} = \overrightarrow{F_2}$ (b) $F_1 = F_2$

(c) $\overrightarrow{F_1} + \overrightarrow{F_2} \neq \overrightarrow{O}$ (d) $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are not on the same straight line.

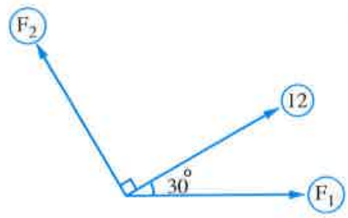
Model 4

Interactive test 4

Answer the following questions :

1 **In the opposite figure :**

The force of magnitude 12 newton is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ make angles of measures 30° , 90° , then F_2 = ... newton.



(a) 10 (b) $10\sqrt{3}$

(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

2 The height of a regular quadrilateral pyramid is 9 cm. and its volume = 300 cm^3 , then the side length of its base equals .... cm.

(a) 5 (b) 10 (c) 15 (d) 20

3 Two perpendicular forces of magnitudes 12 newton , 5 newton , act at a point , then the magnitude of their resultant .... newton.

(a) 5 (b) 12 (c) 13 (d) 17

4 ABCD is a rectangle which AB = 6 cm. , BC = 8 cm. , a point E $\in \overline{AD}$ where AE = 6 cm. , the forces of magnitudes F , 5 , K , $6\sqrt{10}$ newton act along $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$ and $\overrightarrow{EC}$ respectively. If the system of forces are in equilibrium , then find value of each of F and K

5 All of the following cases form a plane except ....

(a) a straight line and a point do not belong to it.

(b) two different parallel straight lines.

(c) two intersected straight lines.

(d) two skew straight lines.

6 ABC is right-angle triangle at B where AB = 3 cm. , BC = 4 cm. , then the volume of the solid which generated by rotation of the triangle complete turn about $\overleftrightarrow{BC}$ is .... cm^3.

(a) 16π (b) 18π (c) 15π (d) 12π

7 A right circle cone , its base on the coordinate plane with equation $x^2 + y^2 = 36$ if the height of the cone = 8 length unit , **find :**

(1) Volume of the cone. (2) Total surface area.

8 The equation $(x \quad y \quad 8)\begin{pmatrix} x \\ y \\ -2 \end{pmatrix} = O$

represents a circle its diameter length = ... length unit.

(a) 2 (b) 4 (c) 6 (d) 8

9 Two forces of magnitudes 4 , F newton act at a particle , the measure of included angle is 120° , if line of action of the resultant is perpendicular to the first force , then magnitude of the resultant = ... newton.

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$

10 A body of weight (W) newton is suspended by two light strings inclined to the vertical by angles θ° and 30° the body becomes in equilibrium when the tension of the first string equal 12 newton. and the other is $12\sqrt{3}$ newton , then the weight of the body W = ... N.

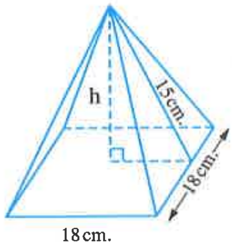
(a) 60 (b) 25 (c) 36 (d) 24

11 If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces , then the measure of the angle enclosed between $\overrightarrow{F_1}$ and the resultant of the two forces $(\overrightarrow{F_1} + \overrightarrow{F_2})$, $(\overrightarrow{F_1} - \overrightarrow{F_2})$ equals ....

(a) zero. (b) $\tan^{-1}\left(\frac{F_1}{F_2}\right)$

(c) $\tan^{-1}\left(\frac{F_2}{F_1}\right)$ (d) $\tan^{-1}\left(\frac{F_1 - F_2}{F_1 + F_2}\right)$

12 **In the opposite figure :**



The volume of the regular quadrilateral pyramid in which the side length of its base = 18 cm. and the lateral height = 15 cm. is .... cm^3.

(a) 1296 (b) 1620

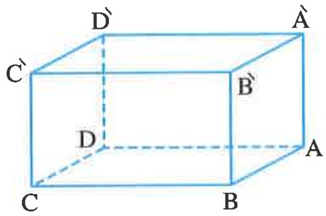
(c) 540 (d) 1944

---

13 Find the equation of the circle which passes through the two points (1 , 3) , (2 , – 4) and its centre lies on x-axis.

---

14 **In the opposite figure :**



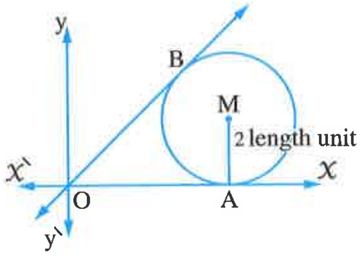
The plane $AA`B`$ $\cap$ the plane $A`CC`$ = ...

(a) $\overleftrightarrow{AA`}$ (b) $\overleftrightarrow{BB`}$

(c) $\overleftrightarrow{CC`}$ (d) $\overleftrightarrow{A`C}$

---

15 **In the opposite figure :**



If OB = 5 length unit ,

then the equation of the circle M is ....

(a) $(x-2)^2+(y-5)^2=25$

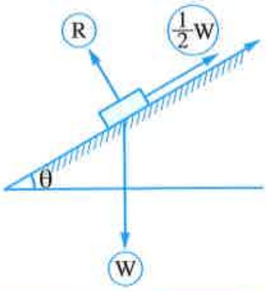
(b) $(x-2)^2+(y-5)^2=4$

(c) $(x-5)^2+(y-2)^2=25$

(d) $(x-5)^2+(y-2)^2=4$

---

16 **In the opposite figure :**



If the body is in equilibrium under acting of the shown forces , then m (∠ θ) = ...

(a) 30° (b) 60°

(c) 45° (d) 15°

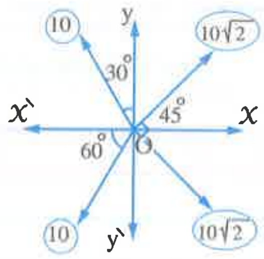
---

17 The radius length of the base of a right circular cone = 5 cm. and its total surface area = 90 π cm^2 , then its volume = ... cm^3.

(a) 105 π (b) 95 π (c) 100 π (d) 120 π

18 **In the opposite figure :**

The resultant of the system of forces "R" = ... newton.



(a) 20 (b) $10\sqrt{2}$

(c) 10 (d) zero.

---

19 The equation $\begin{vmatrix} x & -y \\ y & x \end{vmatrix} = 36$ represents the equation of a circle with radius length equals .... length unit.

(a) 3 (b) 6 (c) 9 (d) 18

---

20 Three equal forces, intersecting at one point, are in equilibrium, then the measure of the angle between any two forces = ...

(a) 60° (b) 90° (c) 120° (d) 150°

---

21 The ratio between the edge length of a uniform triangular pyramid : its height = ...

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$

(c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

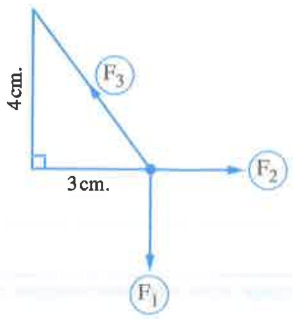
---

22 Two forces of magnitude 8 , F newton , the measure of the angle between them $\in]0 , \pi[$ If their resultant bisects the angle between them , then F = ... newton.

(a) $2\sqrt{2}$ (b) 4 (c) 8 (d) 16

---

23 The opposite figure represnts a body remains in equilibrium under action of three forces F_1 , F_2 , F_3 N. and the sides of the right angled triangle are parallel to the lines of action of these forces and in one cyclic order then $F_1 : F_2 : F_3$ = ...



(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

24 **In the opposite figure :**

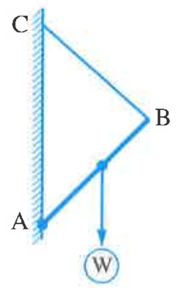
$\overline{AB}$ is a uniform rod, its weight W,

its end A is connected to a hinge fixed at a vertical wall,

$\overline{BC}$ is a light string , its end B fixed to the rod

and the other end C fixed at the vertical wall above A ,

then the reaction of the hinge ....



(a) is in $\overrightarrow{AB}$ direction. (b) is perpendicular to $\overline{BC}$

(c) is perpendicular to the wall. (d) bisects $\overline{BC}$

Model 5

Interactive test 5

Answer the following questions :

1 The two straight lines are skew if they are ....

(a) not parallel. (b) not intersecting.

(c) not coincident. (d) not contained in the same plane.

---

2 The lateral surface area of a right circular cone , radius length of its base = 6 cm. and its height = 8 cm. equals .... cm^2.

(a) $60\,\pi$ (b) $28\,\pi$ (c) $10\,\pi$ (d) $48\,\pi$

---

3 Two forces of magnitudes 5 F , 2 F and their resultant is 7 F newton , then the measure of the angle between them = ...

(a) 180° (b) 60° (c) 20° (d) zero.

---

4 If $\vec{F}$ be equilibrium with two perpendicular forces of magnitudes 8 newton , 15 newton , then F = ... newton.

(a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$

---

5 If the three coplanar forces $\vec{F_1} = 5\,\hat{i} + 3\,\hat{j}$, $\vec{F_2} = a\,\hat{i} + 6\,\hat{j}$, $\vec{F_3} = -14\,\hat{i} + b\,\hat{j}$ act at a point and their resultant $\vec{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$, then a + b = ...

(a) – 1 (b) 1 (c) zero. (d) 14

6 The total surface area of a right circular cone is 96π cm², the length of its drawer is 10 cm. Find the radius length of its base and its volume.

7 A homogeneous smooth sphere its radius length is 10 cm., its weight = 30 gm.wt. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at the point in vertical smooth wall, find the tension of the string and the reaction of the wall on the sphere.

8 The total surface area of a triangular regular faces pyramid which its edge length = ℓ cm. is equal to .... cm².

(a) $2\sqrt{3}\,\ell^2$ (b) $\sqrt{3}\,\ell^2$ (c) $\frac{\sqrt{3}}{3}\ell^2$ (d) $3\sqrt{2}\,\ell^2$

9 The area of any of the lateral faces of a regular quadrilateral pyramid equals to the area of its base, if side length of its base = 6 cm., then its volume = ... cm³.

(a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$

10 ABCD is a square of side length = 10 cm., E is the midpoint of $\overline{AB}$, forces of magnitudes 2, $7\sqrt{5}$, $4\sqrt{2}$ and 4 newton in directions $\overrightarrow{CB}$, $\overrightarrow{CE}$, $\overrightarrow{CA}$ and $\overrightarrow{CD}$ respectively, find magnitude and direction of resultant of this forces.

11 Two forces of magnitude F, $F\sqrt{3}$ newton, meeting at a point and magnitude of their resultant = R_1 when the measure of included angle = 90° and the resultant became R_2 when the measure of the included angle = 150°, then ....

(a) $R_1 = R_2$ (b) $R_1 = 2R_2$ (c) $R_1 = \frac{3}{5}R_2$ (d) $R_1 = \frac{1}{2}R_2$

12 The general form of the circle which its diameter $\overline{AB}$, where A (2, 3), B (–4, 9) is ....

(a) $x^2 + y^2 - 4x - 6y + 18 = 0$ (b) $(x+4)^2 + (y-9)^2 = 72$

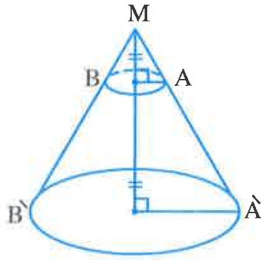
(c) $x^2 + y^2 - 2x + 12y + 19 = 0$ (d) $x^2 + y^2 + 2x - 12y + 19 = 0$

13 The maximum value of the resultant is 25 newton and minimum value of their resultant is 13 newton of two forces, then their magnitudes are ....

(a) 25, 13 (b) 19, 6 (c) 13, 12 (d) 7, 20

14 **In the opposite figure :**

The ratio between the lateral surface area of the cone MAB to the lateral surface area of the cone M$\grave{A}\grave{B}$ equals ....

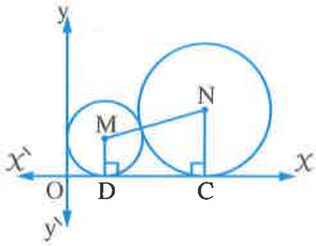


(a) 1 : 2 (b) 1 : 4

(c) 1 : 6 (d) 1 : 8

15 **In the opposite figure :**

M , N are two circles touching externally their equations are $(x-2)^2 + (y-2)^2 = 4$ and $(x-a)^2 + (y-b)^2 = 64$, then a + b = ...



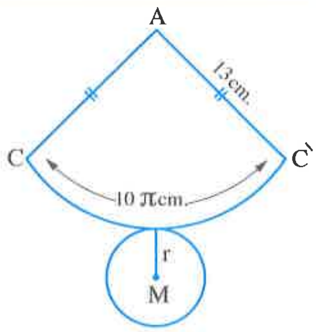
(a) 8 (b) 10

(c) 18 (d) 28

16 **In the opposite figure :**

Net of a solid

, its volume = ... cm^3.



(a) 25 π (b) 50 π

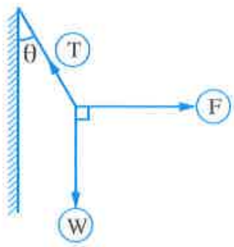
(c) 75 π (d) 100 π

17 A force of magnitude 40 newton acts vertically upwords was resolved into two components one of them is horizontal and its magnitude 20 newton , then the magnitude of the other = ... newton.

(a) 20 (b) $20\sqrt{2}$ (c) 10 (d) $20\sqrt{5}$

18 **In the opposite figure :**

A weight of magnitude (W) newton is suspended in one end of a string and the other end of the string fixed in a point on a vertical wall , the weight is pulled by a horizontal force of magnitude F newton till the string makes an angle θ with vertical.



Which of the following statements is not correct in equilibrium state ?

(a) $F = W \tan\theta$ (b) $\overrightarrow{W} + \overrightarrow{F} + \overrightarrow{T} = \overrightarrow{O}$

(c) $T^2 = F^2 + W^2$ (d) $T = F + W$

19 A force $\overrightarrow{F}$ is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ to make with $\overrightarrow{F}$ two angles of measures θ_1 and θ_2 from its both sides respectively , then magnitude of $\overrightarrow{F_1}$ is ....

(a) $\frac{F \sin \theta_1}{\sin(\theta_1 + \theta_2)}$ (b) $\frac{F \sin \theta_2}{\sin(\theta_1 - \theta_2)}$ (c) $\frac{F \sin \theta_2}{\sin(\theta_1 + \theta_2)}$ (d) $\frac{F \sin(\theta_1 + \theta_2)}{\sin \theta_2}$

20 Three coplanar forces intersecting at one point and in equilibrium. If 3 and 7 N. are two forces of them , then the magnitude of the third force could be equals .... N.

(a) 11 (b) 2 (c) 5 (d) 3

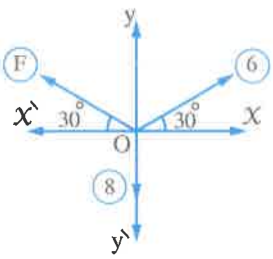
21 If a plane intersects a regular quadrilateral pyramid , parallel to its base , then the cross section shape is ....

(a) a triangle. (b) a square. (c) a rectangle. (d) a circle.

22 The point on the circle $x^2 + (y - 3)^2 = 16$ is ....

(a) (0 , 3) (b) (3 , – 2) (c) (2 , 0) (d) (4 , 3)

23 If the resultant of the forces shown in the figure acts along the y-axis , then : F = ... N.



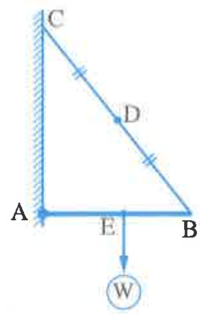
(a) 2 (b) 6

(c) 8 (d) 14

24 **In the opposite figure :**

$\overline{AB}$ is a rod fixed to a hinge at A to a vertical wall.

It's kept horizontally by a string fixed to point B and the other end of the string is fixed to point C on the wall above A.



Which of the following is the triangle of force ?

(a) Δ DBE (b) Δ DEA (c) Δ ADE (d) Δ ACD

Model 6

Interactive test 6

Answer the following questions :

1 The lateral surface area of right cone , which the radius length of its base is r , and length of its drawer ℓ equals ....

(a) $2\pi\ell r$ (b) $2\pi\ell r^2$ (c) $\pi\ell r$ (d) $\pi\ell r^2$

2 Which two forces from the following pairs , could not have resultant with magnitude = 4 newton ?

(a) 2 newton , 4 newton (b) 3 newton , 3 newton

(c) 2 newton , 6 newton (d) 3 newton , 8 newton

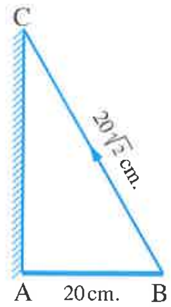
3 The point which lies on the circle $(x-2)^2 + y^2 = 13$ is ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 5) (d) (4 , 3)

4 Number of planes which carry faces of pentagon pyramid is ....

(a) 5 (b) 6 (c) 10 (d) infinite.

5 **In the opposite figure :**



$\overline{AB}$ is a uniform rod with length 20 cm. and its weight = 30 newton attached by a smooth hinge fixed on a vertical wall in the end A and at the end B suspended by a light string with length $20\sqrt{2}$ cm. its other end fixed at point C on the wall above point A , if the rod is in equilibrium in the horizontal position , then the reaction of the hinge ....

(a) act in direction $\overrightarrow{AB}$

(b) its line of action distant 10 cm. from the wall.

(c) bisects $\overline{BC}$

(d) is of magnitude = 15 newton.

6 A body of weight 340 gm.wt. is suspended by two strings with lengths 16 cm. , 30 cm. from two points on same horizontal line , the distance between them 34 cm. , then the magnitude of the tension of each of the two strings respectively = ... N.

(a) $100\sqrt{3}$, $60\sqrt{3}$ (b) $150\sqrt{2}$, $80\sqrt{2}$ (c) 300 , 160 (d) 300 , 100

7 The general form of the equation of circle its centre is $(5 , -4)$ and touches x-axis is ....

(a) $x^2 + y^2 - 10x + 8y + 25 = 0$
(b) $x^2 + y^2 - 5x + 4y = 0$
(c) $x^2 + y^2 - 10x + 8y = 25$
(d) $x^2 + y^2 + 10x - 8y + 25 = 0$

---

8 A uniform rod of length 100 cm. , and its weight 150 gm.wt. is suspended from its ends by two strings , the other end of each string fixed on the same point , if the lengths of the two strings are 80 cm. , 60 cm. , then find the magnitude of the tension of each of them.

---

9 If $\vec{R}$ is the resultant of $\vec{F_1}$, $\vec{F_2}$, $\vec{R} \perp \vec{F_1}$ and $R = \frac{1}{2} F_2$, then the measure of the angle between the two forces $\vec{F_1}$, $\vec{F_2}$ is ....

(a) 40°
(b) 120°
(c) 135°
(d) 150°

---

10 A regular quadrilateral pyramid , the side length of its base 18 cm. If its volume is 1296 cm^3. , then find the lateral height and lateral surface area.

---

11 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. If the angle between the 1st and the 2nd forces measures 120° and between the 2nd and the 3rd measures 90° , then the value of K = ... newton.

(a) $30\sqrt{3}$
(b) $30\sqrt{2}$
(c) 30
(d) 60

---

12 A right cone of volume 27π cm^3. , circumference of its base 6π cm. , then its height. = ... cm.

(a) 27
(b) 3
(c) $3\sqrt{3}$
(d) 9

---

13 The ratio between the lateral surface area of the triangular pyramid of regular faces to its total surface area = ...

(a) 1 : 3
(b) 1 : 4
(c) 3 : 4
(d) 1 : 2

---

14 ABCDEO is regular hexagon the forces of magnitudes 2 , $4\sqrt{3}$, $4\sqrt{3}$, 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AE}$, $\overrightarrow{AO}$ respectively. , then the resultant of this forces acts in direction of ....

(a) $\overrightarrow{AC}$
(b) $\overrightarrow{AE}$
(c) $\overrightarrow{AD}$
(d) $\overrightarrow{AO}$

15 The length of the tangent segment which drawn from the point $(0, 2r)$ to the circle $x^2 + y^2 = r^2$ is .... length unit.

(a) r (b) 2 r (c) $\sqrt{3}$ r (d) $\frac{\sqrt{3}}{2}$ r

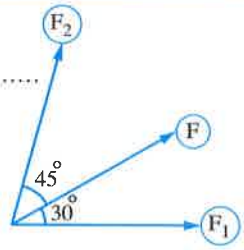
16 ABC is an isosceles triangle where AB = AC = 10 cm. , BC = 12 cm.
, rotates a complete revolution about $\overline{BC}$
, calculate the volume of the solid which generated by rotation.

17 ABCD $\grave{A}\grave{B}\grave{C}\grave{D}$ is a cube of edge length = 20 cm. a right circular cone is put inside the cube such that the vertex of the cone is the centre of cube base $\grave{A}\grave{B}\grave{C}\grave{D}$, and base of the cone touches the sides of the base ABCD , then the ratio between volume of each the cone and cube is ....

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{1}{3}$ (d) $\frac{12}{\pi}$

18 **In the opposite figure :**

The force $\overrightarrow{F}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\frac{F_1 + F_2}{F}$ = ...



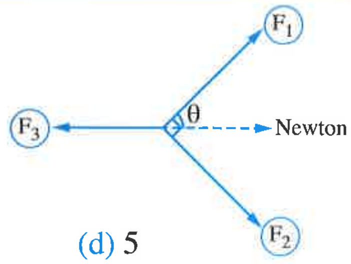
(a) sin 30° + sin 45° (b) $\frac{\sin 75° + \sin 30°}{\sin 75°}$

(c) $\frac{\sin 45° + \sin 30°}{\sin 75°}$ (d) $\frac{\sin 75°}{\sin 30°} + \frac{\sin 75°}{\sin 45°}$

19 Two forces meeting at a point of magnitudes F_1 , F_2 where $0 \le F_1 \le 13$, $8 \le F_2 \le 17$, the measure of the included angle is 180° and magnitude of their resultant R , then ....

(a) $3 \le R \le 4$ (b) $0 \le R \le 4$ (c) $0 \le R \le 17$ (d) $5 \le R \le 17$

20 The opposite figure represents three forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ of magnitudes 4 , 3 and 2 newton respectively , if $\sin \theta = \frac{3}{5}$, then magnitude of their resultant equals .... newton.



(a) 1 (b) 2 (c) 3 (d) 5

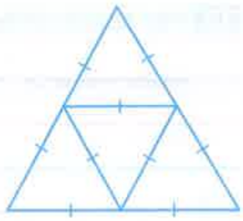
21 The vertical straight lines in the space are ....

(a) parallel. (b) skew. (c) lie in one plane. (d) intersecting.

22 If $\overrightarrow{F}$ is in equilibrium with two perpendicular forces of magnitude 8 N. , 15 N. , then F = ... N.

(a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$

23 Which of the following can be unfolded to form the opposite net ?



(a) quadrilateral pyramid.

(b) regular quadrilateral pyramid.

(c) regular triangular pyramid.

(d) otherwise.

---

24 The magnitude of two forces 3 and 5 N. and thier resultant is 2 N. , then the measure of the angle between the resultant and the second force = ...

(a) 180° (b) 90° (c) 0° (d) 30°

Model 7

Interactive test 7

Answer the following questions :

1 If the resultant of two forces acting at a point reaches the maximum value , then the measure of the angle between their line of actions equals ....

(a) 180° (b) 120° (c) 0° (d) 60°

---

2 A regular quadrilateral pyramid , the side length of its base 10 cm. , and its lateral height 13 cm. , then its lateral area = ... cm^2

(a) 260 (b) 360 (c) 130 (d) 520

---

3 The centre of the circle $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) (3 , – 4) (b) (4 , – 3) (c) (– 3 , 4) (d) (– 4 , 3)

---

4 If the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ are three forces measured by newton are in equilibrium and meeting at a point and $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 3\hat{i} + 5\hat{j}$, then $\overrightarrow{F_3}$ = ... newton.

(a) $5\hat{i} + 2\hat{j}$ (b) $-5\hat{i} - 2\hat{j}$ (c) $\sqrt{29}$ (d) $\sqrt{34}$

---

5 A body of weight W newton is placed on smooth inclined plane , where the angle of the inclination of the plane with the horizontal is 30° , the body kept in equilibrium by a force of magnitude 36 newton and acts in the direction of the line of greatest slope upward , then the magnitude of the weight = ... N.

(a) $36\sqrt{3}$ (b) $36\sqrt{2}$ (c) 72 (d) $72\sqrt{3}$

6 The general form of the equation of the circle which its centre is (– 2 , 5) and passes through (3 , 2) is ……………

(a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$

(c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

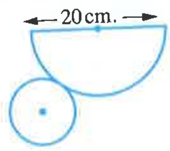
7 If the straight line L // the plane X , A ∈ X , then $L \cap X$ = ……………

(a) $\varnothing$ (b) L (c) {A} (d) X

8 The lateral surface area of a regular quadrilateral pyramid 240 cm^2, and its slant height is 12 cm. , **find :**

(1) Height of the pyramid. (2) Volume of the pyramid.

9 If we folded the opposite net to become a cone , then the radius length of its base = ……………



(a) 10 cm. (b) 8 cm.

(c) 5 cm. (d) 2.5 cm.

10 A metal sphere of weight 400 kg.wt. acts in its centre , placed between two smooth planes , one of them is vertical and the other inclined 60° with vertical , then find the reaction of each plane.

11 The volume of right cone , where the length of its drawer 15 cm. and the total surface area = 216 π cm^2 equals …………… cm^3.

(a) 205 π (b) 320 π (c) 380 π (d) 324 π

12 If R is the resultant of the two forces F_1 , F_2 where $F_2 > F_1$, then which of the following conditions is enough to make $R \perp F_1$?

(a) $R^2 = F_1^2 + F_2^2$ (b) $R^2 = F_2^2 - F_1^2$ (c) $\overrightarrow{F_1} \perp \overrightarrow{F_2}$ (d) all of previous.

13 ABCD is a square of side length 12 cm. H ∈ $\overline{BC}$ where BH = 5 cm. forces of magnitudes 2 , 13 , $4\sqrt{2}$, 9 gm.wt. act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ respectively.

Find the resultant of these forces.

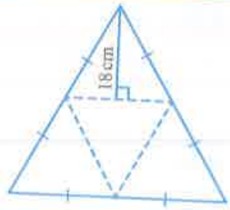
14 If $x^2 + y^2 + 2(\cos\theta)x - 2(\sin\theta)y - 8 = 0$ represents the equation of a circle , then r = …………… length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) 8

15 Four coplanar forces of magnitudes F_1 , $6\sqrt{2}$, $6\sqrt{2}$, F_2 gm.wt. acting at a point in direction of east , the eastern north , western north and south respectively. If the resultant of this forces equal 7 gm.wt. and acts in direction of east , then $(F_1 , F_2) =$ ....

(a) $(7 , 0)$ (b) $(7 , 12)$ (c) $\left(7 , 12\sqrt{2}\right)$ (d) $\left(6\sqrt{2} , 6\sqrt{2}\right)$

16 When we fold the opposite net , then the total surface area of the produced solid is .... cm^2.



(a) $108\sqrt{3}$ (b) $324\sqrt{3}$

(c) 758 (d) $432\sqrt{3}$

17 ABCDE is a regular pentagon , a force of magnitude 20 newton acts along $\overrightarrow{AC}$, then was resolved in two directions $\overrightarrow{AB}$ and $\overrightarrow{AE}$, then the magnitude of the component in direction $\overrightarrow{AB}$ equals .... newton.

(a) 10 (b) 20 (c) $20\sqrt{3}$ (d) 12.4

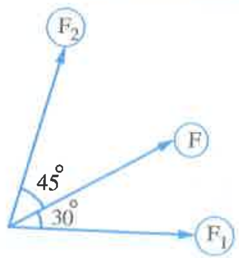
18 The radius length of the base of a right circular cone is 15 cm. , and its height = 20 cm. , then its lateral area = ... cm^2.

(a) $600\,\pi$ (b) $375\,\pi$ (c) $1875\,\pi$ (d) $5625\,\pi$

19 If the force of magnitude F is in equilibrium with the two forces 5 , 3 N. and their included angle is 60° , then F = ... N.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

20 **In the opposite figure :**



The force $\overrightarrow{F}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\dfrac{F_1 + F_2}{F} =$ ....

(a) $\sin 30° + \sin 45°$ (b) $\dfrac{\sin 75° + \sin 30°}{\sin 75°}$

(c) $\dfrac{\sin 45° + \sin 30°}{\sin 75°}$ (d) $\dfrac{\sin 75°}{\sin 30°} + \dfrac{\sin 75°}{\sin 45°}$

21 The centre of the circle $2x^2 + 2y^2 - 6x + 8y = 0$ is the point ....

(a) $(3 , -4)$ (b) $(-4 , 3)$ (c) $\left(\frac{3}{2} , -2\right)$ (d) $(-3 , -4)$

22 Two perpendicular forces of magnitude 6 , 8 N. , then the sine of angle between the resultant and first force = ...

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

23 The least number of planes could form a solid is ....

(a) 1 (b) 2 (c) 3 (d) 4

24 If $\overrightarrow{F_1} = 2\hat{i} - 2\hat{j}$, $\overrightarrow{F_2} = 4\hat{i} - 8\hat{j}$ and their resultant $\overrightarrow{R} = 2a\hat{i} - 3b\hat{j}$, then a + b = ...

(a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12

Model 8

Interactive test 8

Answer the following questions :

1 Two forces of magnitudes 8 , F gm.wt. and the measure of the included angle $\in]0 , \pi[$, the line of action of their resultant bisects the included angle , then F = ... gm.wt.

(a) $2\sqrt{2}$ (b) 4 (c) 8 (d) 16

2 The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. is .... cm^3.

(a) 810 (b) 180 (c) 360 (d) 270

3 The circumference of the circle which its equation is $x^2 + y^2 = 8$ is ....

(a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

4 If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the .... of the included angle between the two other forces.

(a) cosine (b) sine (c) tangent (d) cotangent

5 Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton , then the measure of the included angle = ...

(a) 0° (b) 30° (c) 60° (d) 120°

6 The volume of regular hexagon pyramid is $8\sqrt{3}$ cm^3 and its height is 4 cm. , find the perimeter of its base.

---

7 Force of magnitude $10\sqrt{2}$ gm.wt. acts in direction the eastern south , it was resolved into two perpendicular components , then the component in the south direction = ... gm.wt.

(a) $10\sqrt{3}$ (b) $10\sqrt{2}$ (c) 10 (d) 5

---

8 The general form of the equation of the circle where its centre is (2 , – 1) and radius length is 3 cm. is ....

(a) $x^2 + y^2 - 4x + 2y - 4 = 0$ (b) $x^2 + y^2 - 2x + y - 4 = 0$

(c) $x^2 + y^2 + 4x - 2y - 4 = 0$ (d) $x^2 + y^2 - 4x + 2y - 16 = 0$

---

9 A body of weight 24 newton is suspended at one end of a string of length 130 cm. , the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. , then the magnitude of horizontal force = ... newton when the body is at a distance = 50 cm. from the wall.

(a) 26 (b) 22 (c) 13 (d) 10

---

10 **In the opposite figure :**

The central angle of the sector which if it is folded becomes this cone is ....

(a) acute. (b) obtuse. (c) straight. (d) reflex.

---

11 The forces of magnitudes F , 80 , K , 50 , $80\sqrt{3}$ newton act at a point in the directions of east , 30° east of north , north , west and south respectively.
Find the values of F and K if the resultant is 40 newton in magnitude in the direction of 60° north of east.

---

12 Number of the planes which passes through two given points is ....

(a) zero. (b) 1 (c) 2 (d) infinite.

---

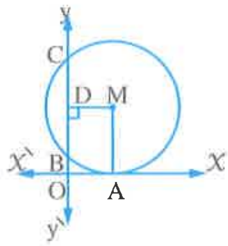
13 A right circular cone , length of its drawer 17 cm. , and its height 15 cm. , then the radius length of its base = ... cm.

(a) 8 (b) 13 (c) 7 (d) 12

14 **In the opposite figure :**

Circle M touches X-axis at A , OB = 2 length units

, BC = 6 length units , then equation of the circle M is ....



(a) $(X+4)^2+(y+5)^2=16$

(b) $(X-4)^2+(y-5)^2=25$

(c) $(X-4)^2+(y-5)^2=16$

(d) $(X+4)^2+(y+5)^2=25$

15 A body of weight 6 kg.wt. is placed on a smooth plane inclines to the horizontal by an angle of measure 30° and kept in equilibrium by a horizontal force. , then the magnitude of the reaction of the plane on the body = ... kg.wt.

(a) $2\sqrt{3}$ (b) $4\sqrt{3}$ (c) $12\sqrt{3}$ (d) $8\sqrt{3}$

16 Find K which makes the two circle $C_1 : (X+2)^2+(y+11)^2=K$, $C_2 : (X-3)^2+(y-1)^2=16$ are touching each other.

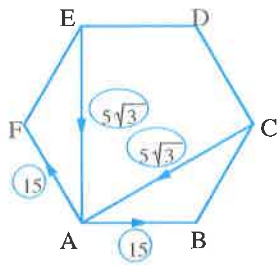
17 If the resultant of two perpendicular forces , inclined to the greatest one by angle of measure θ , then which of the following values is suitable value of θ ?

(a) 90° (b) 70° (c) 45° (d) 10°

18 **In the opposite figure :**

ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 act along $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$ and $\overrightarrow{AF}$

, then the magnitude of the resultant R = ... newton.



(a) 5 (b) 10

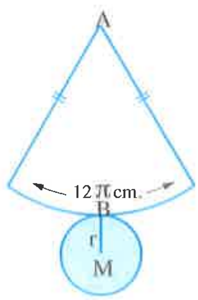
(c) 25 (d) zero.

19 ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of $\overrightarrow{AD}$, then the components of the force in direction of $\overrightarrow{AC}$, $\overrightarrow{AF}$ respectively are ....

(a) $10\sqrt{3}$, 10 (b) $5\sqrt{3}$, 10 (c) 10 , $10\sqrt{3}$ (d) $20\sqrt{3}$, 20

20 The opposite net describes a solid its volume = 96 π cm^3.

, then its total area = ... cm^2.



(a) 16 π (b) 32 π

(c) 48 π (d) 96 π

21 Which of the following statements is true ?

(a) The lateral faces of the right pyramid are congruent.

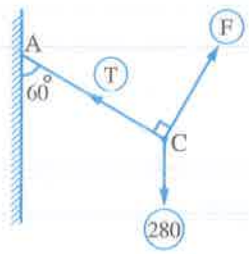
(b) The regular pyramid is a right pyramid.

(c) The heights of the lateral faces of the right pyramid are equal.

(d) The least number of planes that can determine a solid = 3 planes.

22 **In the opposite figure :**

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T}$ = ...



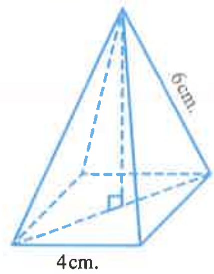
(a) 2 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

23 The resultant of two forces F , 2 F is perpendicular to one of them , then R = ...

(a) $\sqrt{5}$ F (b) $\sqrt{3}$ F (c) 3 F (d) F

24 The opposite figure represents a regular quadrilateral pyramid of height = ... cm.



(a) $7\sqrt{2}$ (b) $2\sqrt{7}$

(c) $4\sqrt{2}$ (d) $2\sqrt{5}$

Model 9

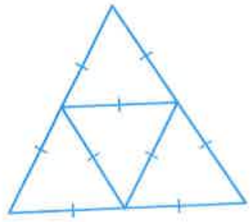
Interactive test 9

Answer the following questions :

1 Two perpendicular forces of magnitudes 2 F – 5 , F + 2 newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton , then F = ... newton

(a) 2 (b) 3 (c) 4 (d) 5

2 Which solid , its net is the opposite figure ?



(a) Quadrilateral pyramid.

(b) Regular quadrilateral pyramid.

(c) Triangular pyramid with regular faces.

(d) Otherwise.

3 Volume of right circular cone is 100 cm^3 , then its volume when its height is doubled becomes .... cm^3.

(a) 100 (b) 200 (c) 400 (d) 800

4 A body of weight 18 kg.wt. is placed on a smooth plane inclines to the horizontal at angle of measure 30° , the body kept in equilibrium by a force F inclines to line of greatest slope upward by an angle of measure 30° , then the magnitude of this force = ... kg.wt.

(a) 12 (b) 9 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$

5 Force of magnitude $4\sqrt{2}$ acts in east direction it was resolved into two perpendicular component , then the magintude of the component in direction of eastern north equals .... newton.

(a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$

6 A regular quadrilateral pyramid. The perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = ... cm^2.

(a) 200 (b) 240 (c) 260 (d) 320

7 The equation of the circle which the straight line : $x + y = 2$ touches it , and its centre is (3 , 5) is ....

(a) $(x-3)^2 + (y-5)^2 = 3\sqrt{2}$

(b) $(x+3)^2 + (y+5)^2 = 18$

(c) $(x-3)^2 + (y-5)^2 = 12$

(d) $(x-3)^2 + (y-5)^2 = 18$

8 A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude F newton acts on the weight in a perpendicular direction of the string till it becomes in equilibrium when the string is inclined to the wall with an angle of measure 30° , then the magnitude of the tension in the string. = ... newton.

(a) 8 (b) $8\sqrt{2}$ (c) $8\sqrt{3}$ (d) 12

9 A uniform rod $\overline{AB}$ of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A The rod is kept horizontally by attaching it at a point C on the rod (where AC = 4 metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge.

10 The equation of the circle which touch the X-axis at the point $(-2, 0)$ and intersept from the positive part of y-axis a chord of length $4\sqrt{3}$ length unit is ....

(a) $(X+2)^2 = 48$
(b) $(X+2)^2 + (y-4)^2 = 48$
(c) $(X-2)^2 + (y+4)^2 = 24$
(d) $(X+2)^2 + (y-4)^2 = 16$

11 Two forces are equal in magnitude and the magnitude of their resultant is 24 newton and the measure of the angle between the resultant and one of the two forces is 30° , then the magnitude of each of the two forces = ... newton.

(a) 8
(b) $8\sqrt{3}$
(c) $8\sqrt{2}$
(d) 12

12 A circular sector , the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone.

13 The ratio between length of the edge of triangular pyramid of regular faces to its height = ...

(a) $\sqrt{2} : \sqrt{3}$
(b) $\sqrt{3} : 2$
(c) $\sqrt{6} : 2$
(d) $\sqrt{3} : 3$

14 Three forces of magnitudes 10 , 20 , 30 newton act at a particle , the first in direction of east and the second in direction of 30° west of north and third in direction of 60° south of west. Find the magnitude and direction of the resultant of these forces.

15 Right circular cone , area of its base = 25π cm^2, length of its drawer = 13 cm. , then its lateral area = ... cm^2

(a) 50π
(b) 65π
(c) 90π
(d) 100π

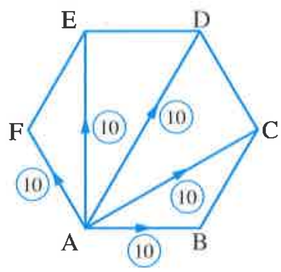
16 Two forces of magnitudes F , 2 F newton act at a particle , and the line of action of its resultant is perpendicular to one of the two forces , then the measure of the included angle between the two forces = ...

(a) 60°
(b) 90°
(c) 120°
(d) 135°

17 The point which lies on the circle : $(x-2)^2+y^2=13$ is ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 5) (d) (4 , 3)

18 Five forces equal in magnitude each equals 10 newton act on one of vertices of a regular hexagon in direction of the other vertices as shown in the opposite figure , then the resultant of this forces is .... newton.



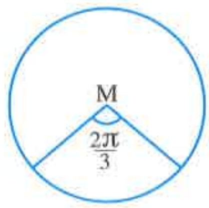
(a) 50 (b) 20

(c) $30\sqrt{3}$ (d) $\left(20+10\sqrt{3}\right)$

19 **In the opposite figure :**

A circle is divided into two circular sectors such that they form two right cone nets

, then : $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}}=$ ....



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

20 Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

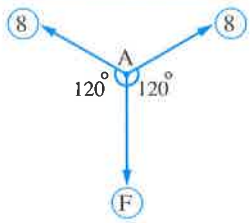
21 Two forces equal in magnitude, the measure of the angle between them is 90° and the magnitude of their resultant is 8 N. , then the magnitude of each one = ... N.

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

22 The centre of the circle : $x^2+y^2-6x+8y=0$ is the point ....

(a) (3 , – 4) (b) (4 , – 3) (c) (– 3 , 4) (d) (– 4 , 3)

23 A is a particle balanced under the effect of the three forces as shown in the opposite figure where $\vec{F}$ is balanced with two forces the magnitude of each is 8 newton and the angle between them is of measure 120° , then F = ... newton.



(a) zero (b) 8 (c) 16 (d) 8 sin 120°

24 Two non parallel planes intersect at ....

(a) a point. (b) a straight line. (c) a plane. (d) a ray.

Model 10

Interactive test 10

Answer the following questions :

1 The point which lies on the circle : $x^2 + (y - 5)^2 = 20$ is ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 5) (d) (4 , 3)

2 Two forces of magnitudes 3 , 4 newton their resultant is 7 newton , then the measure of the angle between them is ....

(a) 0° (b) 60° (c) 180° (d) 90°

3 If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ = ...

(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

4 Two forces of magnitudes 8 , F newton act at a particle , if the measure of the included angle is 120° , and their resultant $F\sqrt{3}$ newton , then F = ... newton.

(a) 4 (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 8

5 The length of the drawer of a right circular cone is 17 cm. and its height = 15 cm. , then its total surface area = ... cm^2.

(a) 200 π (b) 136 π (c) 320 π (d) 400 π

6 The length of the base side of a regular quadrilateral pyramid is 20 cm. and its height is $10\sqrt{3}$ cm. , **then find :**

(1) The lateral surface area. (2) The volume of the pyramid.

7 If O is the origin of perpendicular Cartisian coordinate plane and $\overrightarrow{F}$ = (8 kg.wt. , 135°) is a force acts at the point O , then the component of $\overrightarrow{F}$ in direction of y-axis equals ....

(a) $-4\sqrt{2}$ (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 4

8 ABCDEF is a regular hexagon , forces of magnitudes $6\sqrt{3}$, 5 , $6\sqrt{3}$ newton in directions $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ respectively. then the magnitude and direction of the resultant of these forces is ....

(a) 18 N. in $\overrightarrow{AD}$ direction. (b) 23 N. in $\overrightarrow{AD}$ direction.

(c) 20 N. in $\overrightarrow{AE}$ direction. (d) 23 N. in $\overrightarrow{AC}$ direction.

9 A body of weight 32 newton is suspended at the end of a string with length 10 cm. and the other end of the string is fixed at a point on a vertical wall and the body is pulled by horizontal force to make the body in equilibrium when it distant 6 cm. from the wall.
, then the magnitude of this force = ... newton.

(a) 24 (b) 40 (c) 36 (d) 28

---

10 A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by angle of measure 30° and kept in equilibrium by a horizontal force of magnitude F newton.
, then the magnitude of the reaction of the plane on the body = ... newton.

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $10\sqrt{3}$

---

11 The equation of the circle which its centre $(-4, 3)$ and passes through the origin point is ....

(a) $(x+4)^2 + (y-3)^2 = 5$ (b) $(x-4)^2 + (y+3)^2 = 25$

(c) $(x+4)^2 + (y-3)^2 = 625$ (d) $(x+4)^2 + (y-3)^2 = 25$

---

12 A cylindrical shaped vessel contains water , a metallic body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. is completely immersed in it raising the surface of the water in the vessel with 1 cm.
Find the length of base diameter of the vessel.

---

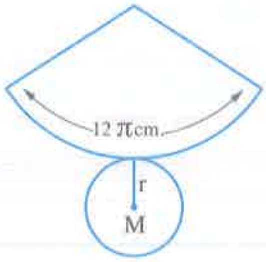
13 Which of the following nets does not make a regular quadrilateral pyramid when it is folded ?

(a) (b) (c) (d)

---

14 Five coplanar forces meeting at a point their magnitudes are 12 , 9 , $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt. act due east , north , western north , western south and south respectively , prove that the system is in equilibrium.

15 The opposite figure describes a solid its volume = 96π cm^3, its total surface area = ... cm^2

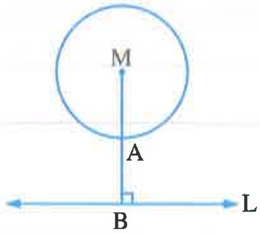


(a) 96π (b) 48π

(c) 32π (d) 16π

---

16 **In the opposite figure :**



If the equation of the circle is : $x^2 + y^2 - 6x + 4y - 12 = 0$

, $\overline{MB} \perp L$ where $L : 3x - 4y + 23 = 0$,

$\overline{MB}$ intersects the circle at A ,

then length of $\overline{AB}$ = ... length units.

(a) 3 (b) 5 (c) 8 (d) 12

---

17 Two forces of magnitudes F , $F\sqrt{3}$ newton act at a particle , the magnitude of their resultant R = F newton , and θ_1 is the measure of the angle between 1^{st} force and the resultant and θ_2 is the angle between the 2^{nd} force and the resultant , then ....

(a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2}\theta_2$ (c) $\theta_1 = 3\theta_2$ (d) $\theta_1 = 4\theta_2$

---

18 Which of the following statements is not true ?

(a) Any two different parallel straight line identify a plane ?

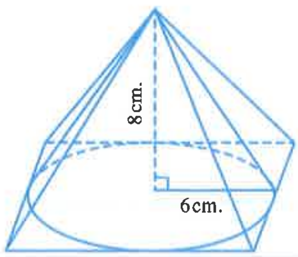
(b) Any two intersecting different straight lines have a common point.

(c) The two skew lines aren't contained in one plane.

(d) Any three non collinear points , there is at least one plane passes through them.

---

19 **In the opposite figure :**



A right regular pyramid and right circular cone are common in the vertex and base such that base of the cone touches the sides of the base of the pyramid internally ,

then the ratio between the lateral area of the right circular cone and the lateral area of the pyramid = ...

(a) $\frac{4}{\pi}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{\pi}{4}$

20 Force of magnitude $5\sqrt{3}$ N. acts in direction 30° east of the north. It is resolved into two perpendicular components, then the component in direction of the east equals .... N.

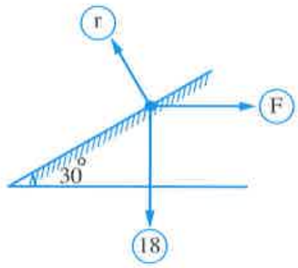
(a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

21 If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ are three forces intersect at one point and in equilibrium, then the magnitude of the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ = ...

(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

22 **In the opposite figure :**

A body of weight 18 N. placed on a smooth inclined plane, inclined to the horizontal at an angle of measure 30° and kept in equilibrium by a horizontal force F N., then F + r = ... N.



(a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$

23 The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals .... length units.

(a) 3 (b) 6 (c) 12 (d) 24

24 The ratio between the edge length of regular triangular pyramid and its hight = ...

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

Second Multiple choice examinations

Model 1

Answer the following questions : *(Calculators are allowed)*

Choose the correct answer from the given answers :

1 Two forces of magnitudes 3 F and 2 F newton act at a point and their resultant is 5 F newton , then the measure of the angle between them = ...

(a) $0°$ (b) $60°$ (c) $20°$ (d) $180°$

2 A force of magnitude $5\sqrt{3}$ newton acts in the direction of 30° East of North , is resolved into two perpendicular components , then the magnitude of its component in the East direction = ... newton.

(a) 5 (b) $7\frac{1}{2}$ (c) $\frac{5\sqrt{3}}{2}$ (d) 15

3 A uniform smooth sphere of weight 1.5 gm.wt. and radius length 25 cm. is suspended at a point on its surface by a light string of length 25 cm. and the other end of the string is fixed at the point in vertical smooth wall. If the sphere is in equilibrium , then the tension in the string = ... gm.wt.

(a) $\sqrt{3}$ (b) 6 (c) $2\sqrt{3}$ (d) 3

4 If the three coplanar forces $\overrightarrow{F_1} = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$, $\overrightarrow{F_3} = -14\,\overrightarrow{i} + b\,\overrightarrow{j}$ act at a point and their resultant $\overrightarrow{R} = \left(10\sqrt{2}\,,\,\frac{3}{4}\pi\right)$, then a + b = ...

(a) – 1 (b) 1 (c) zero (d) 14

5 Two forces of magnitudes 5 , 3 newton act at a point and the measure of the angle between them is 60° , then the magnitude of their resultant R equals ....

(a) 2 (b) 7 (c) 8 (d) 5

6 Three forces are equal in magnitude and meeting at a point are in equilibrium , then the measure of the angle between any two forces of them is ....

(a) $60°$ (b) $120°$ (c) $90°$ (d) $150°$

7 Two forces of magnitudes 4 , F newton act at a particle , the measure of the angle between them is 120°. If line action of the resultant is perpendicular to the first force , then magnitude of the resultant = ... newton.

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$

8 ABCDEF is regular hexagon , then forces of magnitudes 4 , $8\sqrt{3}$, $4\sqrt{3}$, 8 newton act at a point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ respectively , then the magnitude of their resultant = ... newton.

(a) 12 (b) $12\sqrt{3}$ (c) 576 (d) 24

9 The minimum value of resultant of two forces of magnitude 5 , 9 newton and meeting at a point equals .... newton.

(a) zero (b) 9 (c) 4 (d) 5

10 A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by an angle of measure 30° , then body kept in equilibrium by a horizontal force of magnitude F newton , then F = ... newton.

(a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) 6 (d) 18

11 If $\overrightarrow{F_1} = 5\vec{i} - 3\vec{j}$, $\overrightarrow{F_2} = -7\vec{i} + 2\vec{j}$, $\overrightarrow{F_3} = 2\vec{i} + \vec{j}$, then $\overrightarrow{R} =$ ....

(a) $7\vec{i} - 2\vec{j}$ (b) $4\vec{i} - 4\vec{j}$ (c) $-14\vec{i} + 4\vec{j}$ (d) $\vec{O}$

12 All of the following cases determine a plane except ....

(a) a straight line and a point not belong to it.

(b) two different parallel straight lines.

(c) two intersected straight lines. (d) two skew straight lines.

13 The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) (3 , – 4) (b) (– 4 , 3) (c) (– 3 , 4) (d) (– 3 , – 4)

14 If the equation $(x \quad y \quad 25)\begin{pmatrix} x \\ y \\ -4 \end{pmatrix} = O$ represents a circle , then the length of its diameter = ... length unit..

(a) 10 (b) 20 (c) 100 (d) 200

15 The volume of the right cone , the circumference of its base is 44 cm. and its height is 15 cm. = ... cm^3. $\left(\pi = \frac{22}{7}\right)$

(a) 77 (b) 105 (c) 110 (d) 770

16 The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. equals .... cm^3.

(a) 810 (b) 180 (c) 360 (d) 270

17 The two straight lines are skew if they are ....

(a) not parallel. (b) not intersecting.

(c) not coincident. (d) not contained in the same plane.

18 If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume = ...

(a) will doubled. (b) will be three times. (c) will be four times. (d) will not change.

19 The length of the tangent segment which drawn of the circle : $x^2 + y^2 = r^2$ from the point $(0 , 2 r)$ is ....

(a) r (b) 2 r (c) $\sqrt{3}$ r (d) $\frac{\sqrt{3}}{2}$ r

20 The lateral surface area of a right circular cone , radius length of its base = 6 cm. , and its height = 8 cm. equals .... cm^2.

(a) 60π (b) 28π (c) 10π (d) 48π

21 The two circles : $x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y - 60 = 0$ are ....

(a) touching externally. (b) touching internally.

(c) concentric. (d) distant.

Model 2

Answer the following questions :

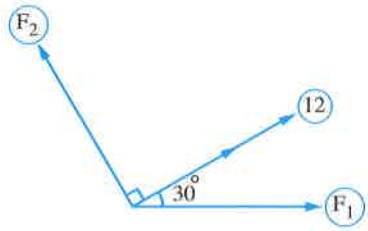
Choose the correct answer :

1 The centre of the circle $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) $(3 , -4)$ (b) $(4 , -3)$ (c) $(-3 , 4)$ (d) $(-4 , 3)$

2 **In the opposite figure :**

The force of magnitude 12 newton is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ make angles of measures 30° , 90° , then F_2 = ... newton.



(a) 10 (b) $10\sqrt{3}$

(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

---

3 If the straight line L // the plane X , A ∈ X , then L ∩ X = ...

(a) ∅ (b) L (c) {A} (d) X

---

4 If the three coplanar forces $\overrightarrow{F_1} = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$, $\overrightarrow{F_3} = -14\,\overrightarrow{i} + b\,\overrightarrow{j}$ act at a point and their resultant $\overrightarrow{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$, then a + b = ...

(a) – 1 (b) 1 (c) zero (d) 14

---

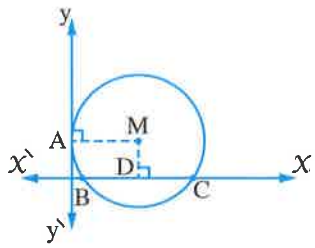
5 Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton , then the measure of the included angle = ...

(a) 0 (b) 30° (c) 60° (d) 120°

---

6 **In the opposite figure :**

If B (2 , 0) , C (8 , 0) , then the equation of the circle is ....



(a) $(x-5)^2 + (y-4)^2 = 25$

(b) $(x+5)^2 + (y-4)^2 = 36$

(c) $(x-5)^2 + (y-4)^2 = 36$

(d) $(x+5)^2 + (y-4)^2 = 25$

---

7 Two forces of magnitudes 3 , 4 newton their resultant is 7 newton , then the measure of the angle between them is ....

(a) zero (b) 60° (c) 180° (d) 90°

---

8 Number of the planes which passes through two given points is ....

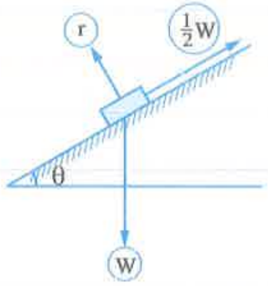
(a) zero (b) 1 (c) 2 (d) infinite.

9 Two forces of magnitudes 4 , F newton act at a particle , the measure of included angle is 120° , if line action of the resultant is perpendicular to the first force , then magnitude of the resultant = ... newton.

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$

---

10 **In the opposite figure :**



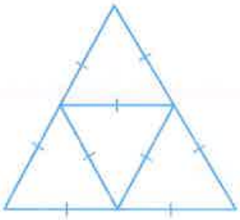
If the body is in equilibrium under acting of the shown forces , then m (∠ θ) = ...

(a) 30° (b) 60°

(c) 45° (d) 15°

---

11 Which solid , its net is the opposite figure ?



(a) Quadrilateral pyramid.

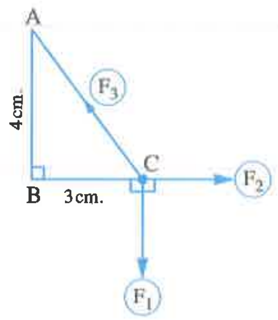
(b) Regular quadrilateral pyramid.

(c) Triangular pyramid with regular faces.

(d) Otherwise.

---

12 **In the opposite figure :**



A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton , and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3$ = ...

(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

---

13 If the resultant of two forces acting at a point reaches the maximum value , then the measure of the angle between their line of actions equals ....

(a) 180° (b) 120° (c) 0° (d) 60°

---

14 The radius length of the base of a right circular cone = 5 cm. and its total surface area = 90 π cm^2. , then its volume = ... cm^3.

(a) 105 π (b) 95 π (c) 100 π (d) 120 π

15 Which of the following system of forces can not be equilibrium ?

(a) 10 newton , 10 newton , 5 newton. (b) 4 newton , 6 newton , 8 newton.

(c) 11 newton , 7 newton , 8 newton. (d) 8 newton , 4 newton , 14 newton.

16 A regular quadrilateral pyramid. The perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = ... cm^2.

(a) 200 (b) 240 (c) 260 (d) 320

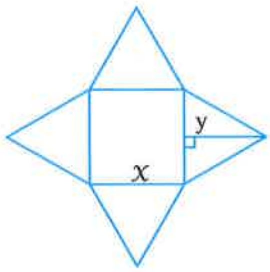
17 The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. is .... cm^3.

(a) 810 (b) 180 (c) 360 (d) 270

18 Two perpendicular forces of magnitudes $2F - 5$, $F + 2$ newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton , then F = ...

(a) 2 (b) 3 (c) 4 (d) 5

19 **In the opposite figure :**

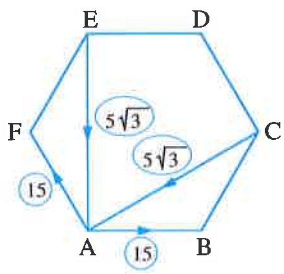


Represents a regular quadrilateral pyramid its height (h) , then the relation between x , y and h is = ...

(a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$

(c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$

20 **In the opposite figure :**



ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 act along $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$ and $\overrightarrow{AF}$, then the magnitude of the resultant R = ... newton.

(a) 5 (b) 10

(c) 25 (d) zero

21 The general form of the equation of the circle which its centre is (– 2 , 5) and passes through (3 , 2) is ....

(a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$

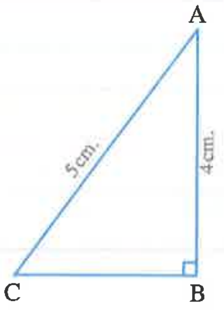
(c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

Model 3

Answer the following questions : *(Calculators are allowed)*

Choose the correct answer :

1 If the triangle ABC is rotated about $\overleftrightarrow{AB}$ with a full turn , then the volume of the formed solid = ... cm^2.



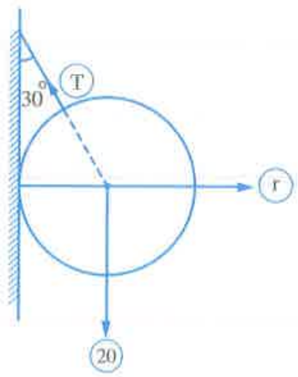
(a) 6π (b) 8π
(c) 10π (d) 12π

2 The volume of regular quadrilateral pyramid is 200 cm^3 and the length its height is 6 cm. , then its lateral surface area = ... cm^2

(a) $30\sqrt{61}$ (b) $20\sqrt{61}$ (c) $10\sqrt{61}$ (d) $40\sqrt{61}$

3 **In the opposite figure :**

A smooth sphere of weight 20 newton rests against a smooth vertical wall. It suspended at a point on its surface by means of a string and the other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall , if the string makes with the vertical an angle of measure 30° , then in case of equilibrium T : r = ...



(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{3} : 1$ (d) $2 : \sqrt{3}$

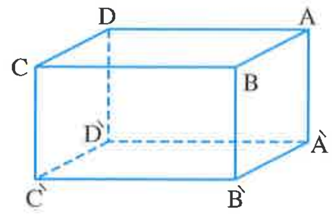
4 A force of magnitude $10\sqrt{2}$ newton acts in the direction of East. it is resolved into two perpendicular components , one in the direction of eastern north , then the components of the force in the perpendicular direction is .... newton.

(a) 10 (b) 20 (c) $10\sqrt{3}$ (d) $10\sqrt{2}$

5 Two forces $\overrightarrow{F_1} = 3\,\overrightarrow{i} + b\,\overrightarrow{j}$ and $\overrightarrow{F_2} = a\,\overrightarrow{i} + 2\,\overrightarrow{j}$ act at a particle and they are in equilibrium , then a + b = ...

(a) 6 (b) – 5 (c) 5 (d) zero

6 **In the opposite figure :**



All the following expressions are true except ....

(a) the plane ABB̀ À ⋂ then plane ABCD = $\overleftrightarrow{AB}$

(b) The two straight lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{DD`}$ are two skew straight lines.

(c) $\overleftrightarrow{BB`} \cap \overleftrightarrow{AD} = \varnothing$

(d) m (∠ ADC) > 90°

7 If the equation of the circle is : $x^2 + y^2 - 4x - 6y + 4 = 0$, then each of the following is correct except ....

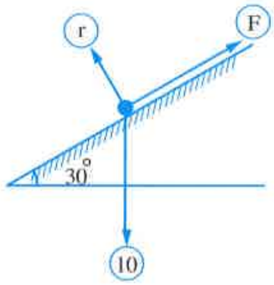
(a) the centre of the circle is (2 , 3)

(b) the circumeference of the circle = 6π length unit.

(c) the equation of the circle by translation of magnitude two units in the positive direction of x-axis is : $(x - 4)^2 + (y - 3)^2 = 9$

(d) the equation of the diameter of the circle : $x + y = 7$

8 **In the opposite figure :**



The body is in equilibrium on the inclined plane , then all the following expressions are true except ....

(a) the measure of the angle between the reaction of the plane (r) and the weight of the body = 150°

(b) $F = 5\sqrt{3}$ newton.

(c) $r = \sqrt{3}\,F$

(d) The component of the weight in the direction of the plane downwards = 5 newton.

9 If the magnitude of the resultant of two forces acting at a point reached to its maximum value , then the measure of the included angle between the two forces = ...

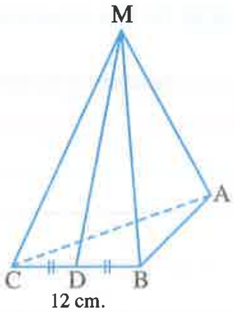
(a) 90° (b) 180° (c) 0° (d) 30°

10 Two forces of magnitudes 3 and 5 newton and enclose between them an angle of measure 60° , then the magnitude of their resultant = ... newton.

(a) 7 (b) 14 (c) 8 (d) 2

11 **In the opposite figure :**

Atriangular pyramid with regular faces , the length of its edge is 12 cm. , D is the midpoint of $\overline{BC}$, then all the following statements are true except ....



(a) $MD = 8\sqrt{3}$ cm.

(b) the height of the pyramid $= 4\sqrt{6}$ cm.

(c) the total area of the pyramid $= 144\sqrt{3}$ cm^2.

(d) the plane AMD $\cap$ the plane MBC $= \varnothing$

---

12 If R is the resultant of two forces , $R \in [4 , 12]$, then one of the following statements is not true ....

(a) the resultant of two forces $= 4\sqrt{3}$ force unit when the angle between the two forces $= 120°$

(b) the resultant of two forces $= 4\sqrt{5}$ force unit when the two forces are perpendicular.

(c) the resultant of two forces = 12 force unit when the angle between the two forces = 180°

(d) the resultant of two forces = 4 force unit when the angle between the two forces = 180°

---

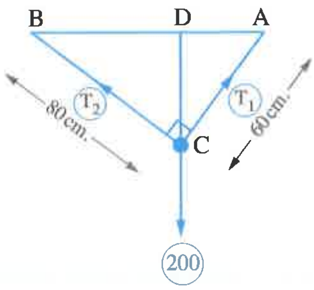
13 The least number of planes that can determine a solid is ....

(a) 2 planes. (b) 3 planes. (c) 4 planes. (d) 5 planes.

---

14 **In the opposite figure :**

A body of weight 200 newton is in equilibrium by suspending it by two perpendicular strings , AC = 60 cm. , CB = 80 cm. , then all following statements is false except :



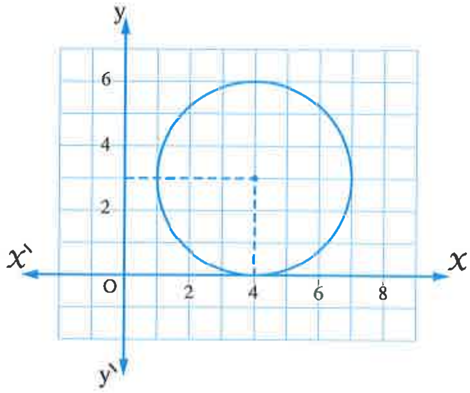
(a) AB = 200 cm.

(b) $T_1 = 120$ newton , $T_2 = 160$ newton

(c) $T_1 = 160$ newton , $T_2 = 120$ newton.

(d) The body can not be in equilibrium under the effect of these forces.

15 **In the opposite figure :**



The equation of the circle ....

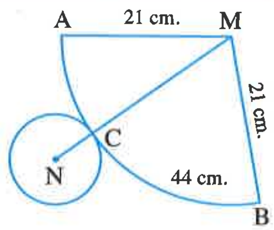
(a) $(X-3)^2+(y-4)^2=9$

(b) $(X-4)^2+(y-3)^2=9$

(c) $(X+3)^2+(y+4)^2=9$

(d) $(X+4)^2+(y+3)^2=9$

16 The opposite figure shows a net of a right cone



, then its height = ... cm. $\left(\pi=\frac{22}{7}\right)$

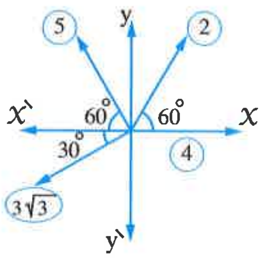
(a) 7

(b) $14\sqrt{2}$

(c) 14

(d) $7\sqrt{3}$

17 **In the opposite figure :**



The true one from the following is ....

(a) the sum of the algebraic components of the forces in direction $\overrightarrow{OX}=2\,\overrightarrow{i}$

(b) the sum of the algebraic components of the forces in direction $\overrightarrow{OY}=-2\sqrt{3}\,\overrightarrow{j}$

(c) $\overrightarrow{R}=-2\,\overrightarrow{i}+2\sqrt{3}\,\overrightarrow{j}$

(d) the set of forces in equilibrium with the force (4 , 120°)

18 The two points $\left(\frac{3}{5},-\frac{4}{5}\right)$ and $\left(-\frac{3}{5},\frac{4}{5}\right)$ are the ends of a diameter in a circle , then the false statement of the following statements is ....

(a) the equation of the circle is $X^2+y^2=1$

(b) the point (1 , 1) lies on the circle.

(c) the straight line $X=1$ touch the circle.

(d) the point $\left(\frac{5}{13},-\frac{12}{13}\right)$ lies on the circle.

19 The equation of the circle M is $x^2 + y^2 = 4$ and the equation of the circle N is $(x-3)^2 + y^2 = 9$ and , then ....

(a) the two circles M and N are touching externally.

(b) the two circles M and N are touching internally.

(c) the two circles M and N are two distant circles.

(d) the two circles M and N are two intersecting circles.

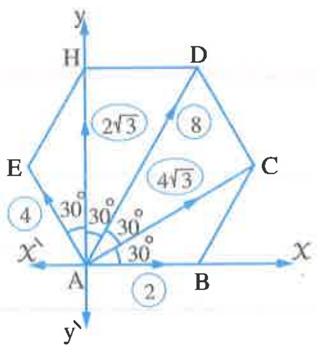
---

20 Two forces F and F , such that R = F , then the measure of the angle between the two forces = ...

(a) 30° (b) 60° (c) 120° (d) 150°

---

21 **In the opposite figure :**



The false statement is ....

(a) the sum of algebraic components of the forces in direction $\overrightarrow{OX} = 10$

(b) the sum of algebraic components of the forces in direction $\overrightarrow{OY} = 10\sqrt{3}$

(c) the resultant of the set of forces = $20\sqrt{3}$

(d) the resultant acts in the direction of $\overrightarrow{AD}$

Model 4

Answer the following questions :

Choose the correct answer :

1 The magnitudes of two forces acting on a particle are 5 , 8 newton , then the smallest value of their resultant = ... newton.

(a) 2 (b) 3 (c) 7 (d) 13

---

2 Two forces of magnitudes $3\sqrt{2}$ and 6 newton and the measure of the angle between them is 135° , then the measure of the angle between their resultant and the second forces is ....

(a) 30° (b) 45° (c) 60° (d) 90°

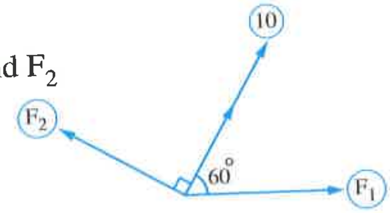
3 Two equal forces , the magnitude of each is 6 newton , the magnitude of their resultant is 6 newton , then the angle between them is ....

(a) 30° (b) 60° (c) 120° (d) 150°

4 Two forces of equal of magnitudes of , enclosing between them an angle of measure $\frac{\pi}{2}$ if their resultant is 8 newton , then the value of each force is .... newton.

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

5 **In the opposite figure :**



If the force 10 newton is resolved into two components F_1 and F_2 inclined to forces by 60° and 90° respectively

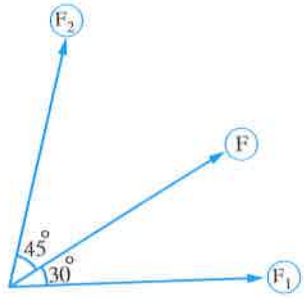
, then F_2 = ... newton.

(a) $5\sqrt{3}$ (b) 10 (c) $10\sqrt{3}$ (d) 20

6 A force of magnitude 6 newton acts in direction of North. It resolved into two perpendicular components , so its component in the direction of East = ... newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

7 **In the opposite figure :**



The force $\vec{F}$ is the resultant of the two forces $\vec{F_1}$ and $\vec{F_2}$

, then $\frac{F_1 + F_2}{F}$ = ...

(a) sin 30° + sin 45° (b) $\frac{\sin 75° + \sin 30°}{\sin 75°}$

(c) $\frac{\sin 45° + \sin 30°}{\sin 75°}$ (d) $\frac{\sin 75°}{\sin 30°} + \frac{\sin 75°}{\sin 45°}$

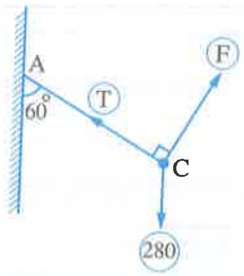
8 If $\vec{F_1} = 5\,\vec{i}$, $\vec{F_2} = 7\,\vec{i} - 5\,\vec{j}$, then $\|\vec{R}\|$ = ... force unit.

(a) 5 (b) $\sqrt{73}$ (c) 12 (d) 13

9 If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the .... of the included angle between the other two forces.

(a) tangent (b) sin (c) cos (d) cotangent.

10 A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T}$ = ...



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 2

11 If a body of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then F = ... newton.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

12 The radius length of the base of a right circular cone = 5 cm. and its total surface area = 90 π cm^2. , then its volume is .... cm^3.

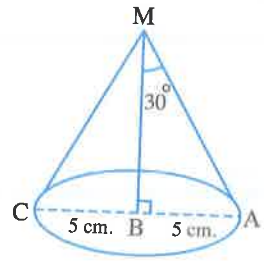
(a) 95 π (b) 100 π (c) 105 π (d) 120 π

13 **In the opposite figure :**

A right circular cone in which m (∠ AMB) = 30°

the radius length of the base = 5 cm.

, then its total area = ... cm^2.



(a) 50 π (b) 75 π

(c) 100 π (d) 125 π

14 The volume of a regular quadrilateral pyramid , where the perimeter of its base is 36 cm. and its height is 10 cm. is .... cm^3.

(a) 180 (b) 270 (c) 360 (d) 810

15 A regular quadrilateral pyramid , the perimeter of its base is 40 cm. and its height is 12 cm. , then its total surface area = ... cm^2.

(a) 200 (b) 240 (c) 260 (d) 360

16 The circumference of the circle which its equation is $x^2 + y^2 = 8$ is ....

(a) 8 π (b) 64 π (c) $2\sqrt{2}\,\pi$ (d) $4\sqrt{2}\,\pi$

17 If the straight line L // the plane X , A $\in$ X , then L $\cap$ X = ...

(a) $\varnothing$ (b) L (c) $\{A\}$ (d) X

18 The general form of the equation of a circle which its center is (– 2 , 5) and passes through the point (3 , 2) is ....

(a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$

(c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

19 The equation of a circle which its center is (– 4 , 3) and passes through the origin point ....

(a) $(x + 4)^2 + (y - 3)^2 = 5$ (b) $(x + 4)^2 + (y - 3)^2 = 625$

(c) $(x + 4)^2 + (y - 3)^2 = 25$ (d) $(x - 4)^2 + (y + 3)^2 = 25$

20 If we cut a regular quadrilateral pyramid by a plane parallel to its base , then the resulting section is ....

(a) triangle. (b) square. (c) rectangle. (d) circle.

21 The two lines be skew if they are ....

(a) not parallel. (b) not intersecting.

(c) not coincident. (d) not contained in the same plane.

Model 5

Answer the following questions :

Choose the correct answer from those given :

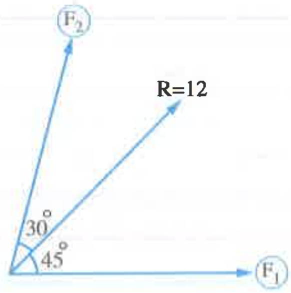
1 If the resultant of two forces acting at a point reaches the maximum value , then the measure of the angle between their line of actions equals ....

(a) 180° (b) 120° (c) zero° (d) 60°

2 The height of a regular quadrilateral pyramid is 4 cm. and its volume 12 cm^3 , then the side length of its base = ...

(a) 1 (b) 2 (c) 3 (d) 4

3 **In the opposite figure :**

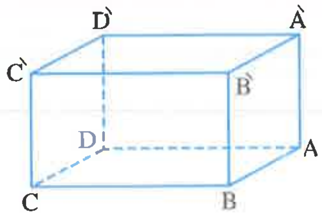


$F_1 = \dots\dots\dots$

(a) $12 \cos 75°$ (b) $12 \cos 45°$

(c) $6 \sec 45°$ (d) $6 \csc 75°$

4 **In the opposite figure :**



plane $A\grave{A}\grave{B} \cap$ plane $\grave{A}C\grave{C} = \dots\dots\dots$

(a) $\overleftrightarrow{A\grave{A}}$ (b) $\overleftrightarrow{B\grave{B}}$

(c) $\overleftrightarrow{C\grave{C}}$ (d) $\overleftrightarrow{\grave{A}C}$

5 Two forces of magnitude 4 , F newton act at a point and measure of the angle between them 120° and their resultant is perpendicular on 1st , then value of F = ... newton.

(a) 4 (b) 8 (c) 6 (d) 2

6 Right circular cone , length of its base radius is 6 cm. and its height 8 cm. , then its lateral area = ... cm^2.

(a) 60π (b) 28π (c) 10π (d) 48π

7 The circumference of the circle which its equation is : $(x-3)^2 + (y+2)^2 = 25$ equal .... length unit.

(a) 2π (b) 3π (c) 10π (d) 25π

8 If the three coplanar forces $\vec{F_1} = 5\,\vec{i} + 3\,\vec{i}$, $\vec{F_2} = a\,\vec{i} + 6\,\vec{j}$, $\vec{F_3} = -14\,\vec{i} + b\,\vec{j}$ act point and their resultant $\vec{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$, then $a + b = \dots\dots\dots$

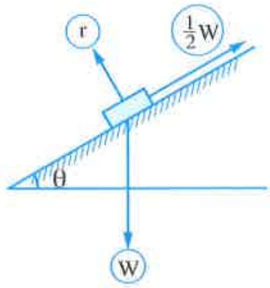
(a) -1 (b) 1 (c) zero (d) 14

9 In the triangular pyramid of regular faces , if the sum of lengths of its edges is 18 cm. , the its total area = ... cm^2.

(a) $\frac{27\sqrt{2}}{4}$ (b) $\frac{27\sqrt{3}}{4}$ (c) $\frac{27\sqrt{3}}{2}$ (d) $9\sqrt{3}$

10 **In the opposite figure :**

If the body is in equilibrium under acting of the shown forces , then m (∠ θ) = ...°



(a) 30 (b) 60

(c) 45 (d) 15

11 The volume of the right cone is 27 π cm^3. and the circumference of its base is 6 π cm. , then its height is .... cm.

(a) 27 (b) 18 (c) 9 (d) 6

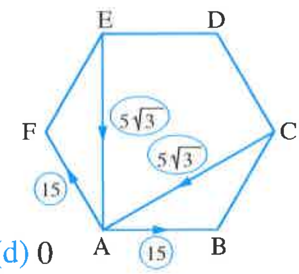
12 If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume ....

(a) will doubled. (b) will not change.

(c) will be three times. (d) will be four times.

13 **In the opposite figure :**

ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 newton act anlog $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$ and $\overrightarrow{AF}$, then the magnitude of the resultant R = ... newton.



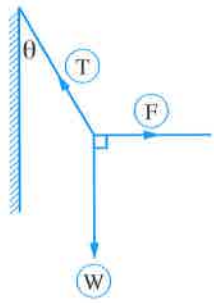
(a) 5 (b) 10 (c) 25 (d) 0

14 If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force proportional to the .... of the included angle between the two others forces.

(a) cosine (b) sine (c) tangent (d) contangent

15 **In the opposite figure :**

A weight of magnitude W newton is suspended in one end of string and the other end of the string fixed in a point on a vertical wall , the weight is pulled by a horizontal force of magnitude F newton till the string become makes an angle θ with vertical which of the following statements is not correct in equilibrium state ?



(a) F = W tan θ (b) $\overrightarrow{W} + \overrightarrow{F} + \overrightarrow{T} = \overrightarrow{O}$ (c) $T^2 = F^2 + W^2$ (d) T = F + W

16 The volume of regular hexagon pyramid is $8\sqrt{3}$ cm^3. and its height is 4 cm.
, then perimeter of its base = ...

(a) 2 (b) 12 (c) 6 (d) $6\sqrt{3}$

17 A forcc of magnitude $5\sqrt{3}$ newton act in direction 30° east of north is resolved into two perpendicular components in direction , then value of component in the east direction = ... newton.

(a) 5 (b) 7.5 (c) $\frac{5\sqrt{3}}{2}$ (d) 15

18 The equation of the circle which its center $(-4, 4)$ and touches X-axis and y-axis is ....

(a) $X^2 + y^2 + 8X - 8y + 16 = 0$ (b) $X^2 + y^2 = 16$

(c) $X^2 + y^2 - 8X + 8y + 16 = 0$ (d) $X^2 + y^2 = 8$

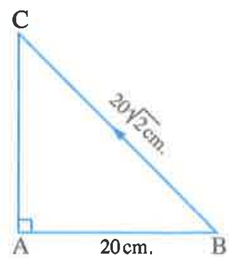
19 Two forces are equal act at a point and the measure of the angle between them is 90° and their resultant is 8 newton , then the magnitude of each is .... newton.

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

20 Volume of regular triangular pyramid is 12 cm^3. and area of its base is 4 cm^2.
, then its height = ...

(a) 3 (b) 6 (c) 9 (d) 2

21 **In the opposite figure :**



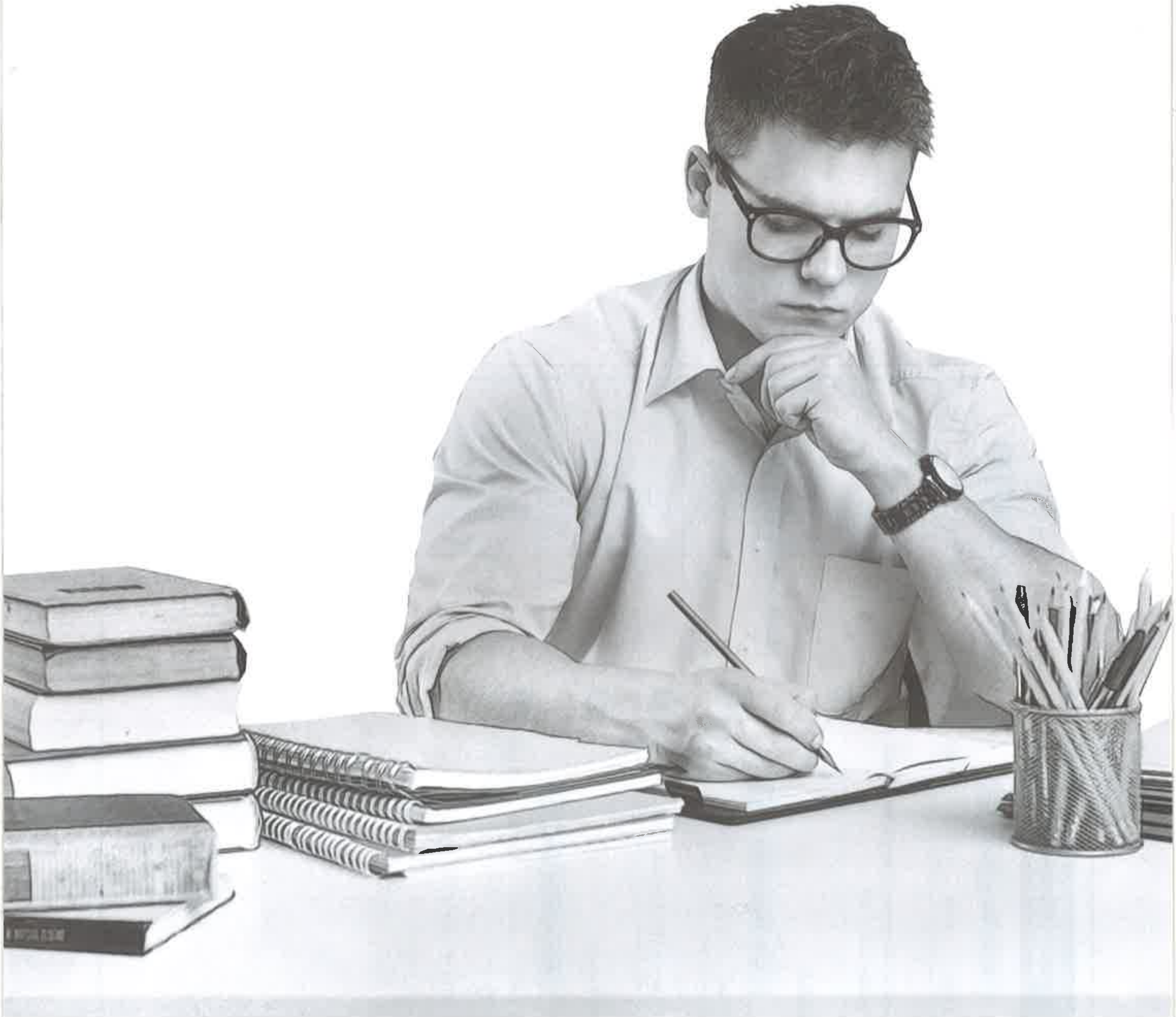
$\overline{AB}$ is a uniform rod with length 20 cm. and its weight = 30 newton attached by a smooth hinge fixed on a vertical wall in the end A and at the end B suspended by a light string with length $20\sqrt{2}$ cm. its other end fixed at point C on the wall above point A if the rod in equilibrium in the horizontal position , then the reaction of the hinge ....

(a) action direction $\overrightarrow{AB}$

(b) its line of action distant 10 cm. from the wall.

(c) bisects $\overline{BC}$ (d) is magnitude = 15 newton.

Answers



Answers of accumulative quizzes on Statics

Accumulative quiz 1

1 (1) (d) (2) (b) (3) (a) (4) (c)

2 F = 8 newton , the measure of the inclination angle of the resultant on $\overrightarrow{F}$ = 30°

3 F = 8 newton

Accumulative quiz 2

1 (1) (a) (2) (c) (3) (c) (4) (d)

2 $F = 4\sqrt{2}$ newton , R = 4 newton.

3 50 , $50\sqrt{3}$ newton.

Accumulative quiz 3

1 (1) (b) (2) (c) (3) (c) (4) (c)

2 $R \simeq 15.16$ kg.wt. , $\theta \simeq 99° \, 30'$

3 22 , 10 kg.wt. , $R = 2\sqrt{201}$ kg.wt.

Accumulative quiz 4

1 (1) (c) (2) (d) (3) (b) (4) (a)

2 $100\sqrt{3}$ gm.wt. , $100\sqrt{3}$ gm.wt.

3 a = – 1 , b = 1

Accumulative quiz 5

1 135° , prove by yourself.

2 $R = 4\sqrt{2}$ newton , in direction of $\overrightarrow{AC}$

3 $T = \frac{20\sqrt{3}}{3}$ gm.wt , $r = \frac{10\sqrt{3}}{3}$ gm.wt.

4 $9\sqrt{7}$ newton , $\sqrt{7}$ newton.

Answers of accumulative quizzes on Geometry and Measurement

Accumulative quiz 1

1 (1) (d) (2) (a) (3) (d) (4) (a) (5) (d)

2
(1) The two planes ABCD , $\grave{A}\grave{B}\grave{C}\grave{D}$ (there are other solutions)

(2) The two planes ABCD , $AB\grave{B}\grave{A}$ (there are other solutions)

(3) The two straight lines $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ (there are other solutions)

(4) $\overleftrightarrow{AB}$, the plane $\grave{A}\grave{B}\grave{C}\grave{D}$ (there are other solutions)

(5) $\overleftrightarrow{AB}$

Accumulative quiz 2

1 (1) (b) (2) (c) (3) (b) (4) (d)

2
(1) Lateral area = 800 cm².

(2) The volume = $\frac{4000\sqrt{3}}{3}$ cm³.

3 Total area = $576\sqrt{3}$ cm².

Accumulative quiz 3

1 (1) (c) (2) (a) (3) (d) (4) (d)

2 Lateral height = 15 cm. , lateral area = 540 cm².

3 14 cm.

Accumulative quiz 4

1 (1) (a) (2) (c) (3) (a) (4) (d)

2 $x^2 + y^2 + 4x - 10y - 5 = 0$

3
(1) Total area = 96 π cm².

(2) Volume = 96 π cm³.

Guide answers of school book examination

1

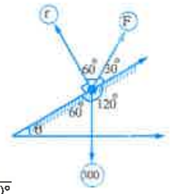
(1) (a) (2) (d) (3) (d) (4) (b)

2

(a) $5 + a - 14 = 10\sqrt{2}\cos 135° = -10$ $\therefore a = -1$

$, 3 + 6 + b = 10\sqrt{2}\sin 135° = 10$ $\therefore b = 1$

(b) Let the angle between the inclined plane and the horizontal be θ



$\therefore \tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30°$

$\therefore \frac{F}{\sin 150°} = \frac{r}{\sin 150°} = \frac{300}{\sin 60°}$

$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{300}{\frac{\sqrt{3}}{2}}$

$\therefore F = r = \frac{300 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 100\sqrt{3}$ gm.wt.

3

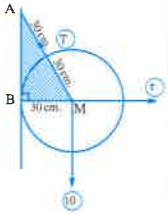
(a) $(X - 2)^2 + (y + 1)^2 = 3^2$

$\therefore X^2 + y^2 - 4X + 2y - 4 = 0$

(b) Δ MAB is the triangle of forces

where AM = 60 cm.

, MB = 30 cm.



$\therefore AB = \sqrt{(60)^2 - (30)^2}$

$= 30\sqrt{3}$

Applying the triangle of forces rule :

$\frac{r}{30} = \frac{T}{60} = \frac{10}{30\sqrt{3}}$

$\therefore r = \frac{10\sqrt{3}}{3}$ gm.wt. , $T = \frac{20\sqrt{3}}{3}$ gm.wt.

4

(a) Volume of the wax = volume of cube

$= (30)^3 = 27000$ cm^3.

, $\because$ 8% of wax had been lost during the melting and transferring

$\therefore$ The volume of the cone $= 92\% \times 27000$

$= 24840$ cm^3.

$\because$ volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore \frac{1}{3} \times \pi \times r^2 \times 45 = 24840$

$\therefore r \approx 22.959$ cm.

(b) $\because (AB)^2 = (BC)^2 + (AC)^2$

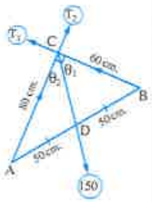
$\therefore m(\angle ACB) = 90°$, $\therefore CD = \frac{1}{2}AB = 50$ cm.

$\therefore CD = DB$, $\therefore m(\angle B) = \theta_1$

, $\because CD = AD$, $\therefore m(\angle A) = \theta_2$

$\frac{T_1}{\sin(90° + \theta_1)} = \frac{T_2}{\sin(90° + \theta_2)}$

$= \frac{150}{\sin 90°}$



$\therefore \frac{T_1}{\cos\theta_1} = \frac{T_2}{\cos\theta_2} = \frac{150}{1}$

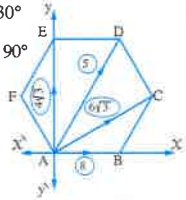
$\therefore \frac{T_1}{\frac{6}{10}} = \frac{T_2}{\frac{8}{10}} = 150$

$\therefore T_1 = 90$ gm.wt. , $T_2 = 120$ gm.wt.

5

(a) $X = 8\cos 0° + 6\sqrt{3}\cos 30°$

$+ 5\cos 60° + 4\sqrt{3}\cos 90°$



$= 8 \times 1 + 6\sqrt{3} \times \frac{\sqrt{3}}{2}$

$+ 5 \times \frac{1}{2} + 4\sqrt{3} \times 0$

$= \frac{39}{2}$

$, Y = 8\sin 0° + 6\sqrt{3}\sin 30° + 5\sin 60° + 4\sqrt{3}\sin 90°$

$= 8 \times 0 + 6\sqrt{3} \times \frac{1}{2} + 5 \times \frac{\sqrt{3}}{2} + 4\sqrt{3} \times 1 = \frac{19\sqrt{3}}{2}$

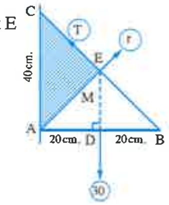
$\therefore \vec{R} = \frac{39}{2}\vec{i} + \frac{19\sqrt{3}}{2}\vec{j}$

$\therefore R = \sqrt{\left(\frac{39}{2}\right)^2 + \left(\frac{19\sqrt{3}}{2}\right)^2} = \sqrt{651}$ newton

$, \tan\theta = \frac{19}{39}\sqrt{3}$ $\therefore \theta \simeq 40° 9'$

(b) $\because$ The set of forces are in equilibrium

$\therefore \vec{r}$ passes through the point E



, $\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{AC}$

$\therefore$ E is the midpoint of $\overline{CB}$

, $\because BC = \sqrt{(40)^2 + (40)^2}$

$= 40\sqrt{2}$

$\therefore CE = 20\sqrt{2}$ and $AE = 20\sqrt{2}$

$\therefore \Delta$ AEC is triangle of forces

$\therefore \frac{r}{20\sqrt{2}} = \frac{T}{20\sqrt{2}} = \frac{30}{40}$

$\therefore r = T = 15\sqrt{2}$ newton.

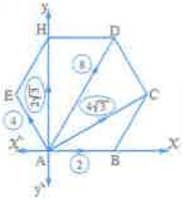
Guide answers of final models

Model 1

1 (b) **2** (c) **3** (a) **4** (c) **5** (d)

6

Let $\overrightarrow{AB}$ in the direction of $\overrightarrow{OX}$



$\therefore X = 2\cos 0° + 4\sqrt{3}\cos 30°$
$+ 8\cos 60°$
$+ 2\sqrt{3}\cos 90°$
$+ 4\cos 120°$
$= 2\times 1 + 4\sqrt{3}\times\frac{\sqrt{3}}{2}$
$+ 8\times\frac{1}{2} + 2\sqrt{3}\times 0 + 4\times\frac{-1}{2} = 10$

$, Y = 2\sin 0° + 4\sqrt{3}\sin 30° + 8\sin 60°$
$+ 2\sqrt{3}\sin 90° + 4\sin 120°$
$= 2\times 0 + 4\sqrt{3}\times\frac{1}{2} + 8\times\frac{\sqrt{3}}{2} + 2\sqrt{3}\times 1 + 4\times\frac{\sqrt{3}}{2}$
$= 10\sqrt{3}$

$\therefore \vec{R} = 10\,\hat{i} + 10\sqrt{3}\,\hat{j}$

$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20$ kg.wt.

$, \tan\theta = \frac{10\sqrt{3}}{10} = \sqrt{3}$ $\qquad \therefore \theta = 60°$

$\therefore$ The magnitude of $\vec{R} = 20$ kg.wt. and makes an angle of measure 60° with $\overrightarrow{OX}$

7 (b)

8

(1) The area of the base $= \pi r^2$

$\therefore 36\pi = \pi r^2$ $\qquad \therefore r = 6$ cm.

, the lateral area $= \pi r L = \pi\times 6\times 10 = 60\pi$ cm².

(2) The total area $= \pi r(L + r) = \pi\times 6(10 + 6)$
$= 96\pi$ cm².

$, h = \sqrt{(10)^2 - (6)^2} = 8$ cm.

(3) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\times 6^2\times 8 = 96\pi$ cm³.

9 (a) **10** (a)

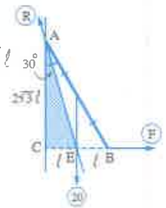
11 (c)

12 (d)

13 (d)

14 (c)

15



Suppose that :

$AB = 4\ell$ $\qquad \therefore CB = 2\ell$

$\therefore CE = \ell$ $\qquad \therefore AC = 2\sqrt{3}\,\ell$

In Δ ACE : $AE = \sqrt{13}\,\ell$

$\because \Delta$ ACE is the Δ of forces

$\frac{20}{2\sqrt{3}\,\ell} = \frac{F}{\ell} = \frac{R}{\sqrt{13}\,\ell}$

$\therefore F = \frac{10\sqrt{3}}{3}$ kg.wt.

$R = \frac{10\sqrt{39}}{3}$ kg.wt.

16 (b)

17 (d)

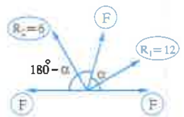
18 (a) **19** (d) **20** (b) **21** (d) **22** (d)

23 (d) **24** (d)

Model 2

1 (a) **2** (b) **3** (a) **4** (b) **5** (d)

6



$R_1 = 2F\cos\frac{\alpha}{2} = 12$

$\therefore F\cos\frac{\alpha}{2} = 6$ $\qquad (1)$

$, R_2 = 2F\cos\left(\frac{180° - \alpha}{2}\right) = 6$

$\therefore F\sin\frac{\alpha}{2} = 3$ $\qquad (2)$

squaring the two equations and adding them

$\therefore F^2\cos^2\frac{\alpha}{2} + F^2\sin^2\frac{\alpha}{2} = 6^2 + 3^2$

$\therefore F^2\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right) = 45$

$\therefore F = \sqrt{45} = 3\sqrt{5}$ kg.wt.

7 (c)

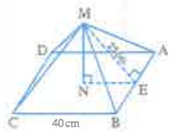
8 (a)

9 (d)

10 (c) **11** (b)

12

(1) In Δ MNE :



$MN = \sqrt{(25)^2 - (20)^2} = 15$ cm.

i.e. height = 15 cm.

(2) The lateral area

$= \frac{1}{2}\times$ base perimeter $\times$ slant height

$= \frac{1}{2}\times(4\times 40)\times 25$

$= 2000$ cm².

(3) The total area $= 2000 + (40)^2 = 3600$ cm².

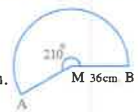
(4) The volume $= \frac{1}{3}\times(40)^2\times 15 = 8000$ cm³.

13 (d)

14 (c)

15 (c)

16



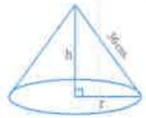
Length of the arc $\overset{\frown}{AB}$

$= MB\,.\,\theta^r = 36\times\frac{210}{180}\pi = 42\pi$ cm.

Circumference of the base of the cone $= 42\pi$

$\therefore 2\pi r = 42\pi$

$\therefore r = 21$



$\therefore h = \sqrt{36^2 - 21^2} = 3\sqrt{95}$ cm.
$\simeq 29.2$ cm.

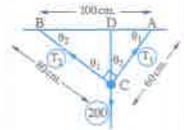
17 (a) **18** (a) **19** (b) **20** (a)

21 (c) **22** (b) **23** (a) **24** (a)

Model 3

1 (c) **2** (a) **3** (c) **4** (b)

5



$\because (60)^2 + (80)^2 = (100)^2$

$\therefore \Delta$ ACB is right-angled at C

From lami's rule

$\therefore \frac{200}{\sin 90°} = \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2}$

$\because \sin\theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}$

$, \sin\theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$

$\therefore \frac{200}{1} = \frac{T_1}{\frac{4}{5}} = \frac{T_2}{\frac{3}{5}}$

$\therefore T_1 = 200\times\frac{4}{5} = 160$ gm.wt.

$, T_2 = 200\times\frac{3}{5} = 120$ gm.wt.

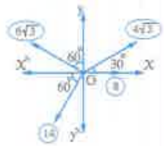
6 (d)

7 (c)

8 (c)

9 (b)

10



Suppose $\overrightarrow{OX}$ is the direction of the first force

$X = 8\cos 0° + 4\sqrt{3}\cos 30°$
$+ 6\sqrt{3}\cos 150° + 14\cos 240°$
$= 8\times 1 + 4\sqrt{3}\times\frac{\sqrt{3}}{2} + 6\sqrt{3}\times\left(-\frac{\sqrt{3}}{2}\right) + 14\times\left(-\frac{1}{2}\right)$
$= 8 + 6 - 9 - 7 = -2$

$, Y = 8\sin 0° + 4\sqrt{3}\sin 30°$
$+ 6\sqrt{3}\sin 150° + 14\sin 240°$
$= 8\times 0 + 4\sqrt{3}\times\frac{1}{2} + 6\sqrt{3}\times\frac{1}{2} + 14\times\left(-\frac{\sqrt{3}}{2}\right)$
$= -2\sqrt{3}$

$\therefore \vec{R} = -2\,\hat{i} - 2\sqrt{3}\,\hat{j}$

$\therefore R = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = 4$ newton

, $\tan\theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

$\because$ X and Y are negative $\quad \therefore \theta = 240°$

$\therefore$ The magnitude of the resultant is 4 newton and makes an angle of measure 240° with $\overrightarrow{OX}$

11 (a) **12** (b) **13** (c)

14 (a)

15 (b)

16

$\because$ Centre of 1st circle C_1 (1 , −3)

, the 2nd circle is $x^2 + y^2 - 2x + 6y + \frac{15}{4} = 0$

$\therefore$ Centre of 2nd circle $C_2 = (1 , -3)$

$\therefore$ They have same centre

$\therefore$ They are concentric circles.

$r_1 = \sqrt{(1)^2 + (-3)^2 - 1} = 3$ length unit.

, $r_2 = \sqrt{(1)^2 + (-3)^2 - \frac{15}{4}} = 2.5$ length unit.

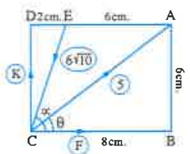
17 (b) **18** (b) **19** (d) **20** (d)

21 (d) **22** (c) **23** (b) **24** (b)

Model 4

1 (d) **2** (b) **3** (c)

4



From the figure :

CE = $2\sqrt{10}$ cm.

$\because$ The forces are in equilibrium

$\therefore X = 0 \quad , \quad y = 0$

$\therefore F\cos 0° + 5\cos\theta - 6\sqrt{10}\cos\alpha + k\cos 90° = 0$

$\therefore F + 5 \times \frac{8}{10} - 6\sqrt{10} \times \frac{2}{2\sqrt{10}} + k \times 0 = 0$

$\therefore$ F = 2 newton.

, $F\sin 0° + 5\sin\theta - 6\sqrt{10}\sin\alpha + k\sin 90° = 0$

$\therefore 0 + 5 \times \frac{6}{10} - 6\sqrt{10} \times \frac{6}{2\sqrt{10}} + k \times 1 = 0$

$\therefore$ k = 15 newton.

5 (d)

6 (d)

7

Radius of base = 6 units and its height = 8 units

(1) Volume of the cone

$= \frac{1}{3} \times \pi\,(6)^2 \times 8 = 96\,\pi$ cubic units

(2) Length of drawer $= \sqrt{6^2 + 8^2} = 10$ units

Total surface area $= \pi\, r\,(\ell + r)$

$= \pi \times 6\,(10 + 6) = 96\,\pi$ square units

8 (d) **9** (b)

10 (d)

11 (a)

12 (a)

13

Let the centre is $(x , 0) \in x$-axis

$\therefore$ The centre is equidistant from (1 , 3) , (2 , −4)

$\therefore \sqrt{(x-1)^2 + (0-3)^2} = \sqrt{(x-2)^2 + (0-(-4))^2}$

$\therefore \sqrt{x^2 - 2x + 1 + 9} = \sqrt{x^2 - 4x + 4 + 16}$

$\therefore x^2 - 2x + 10 = x^2 - 4x + 20$

$\therefore 2x = 10 \qquad \therefore x = 5$

$\therefore$ The centre is (5 , 0) $\quad \therefore r = \sqrt{(5-1)^2 + (0-3)^2} = 5$

$\therefore$ The equation of the circle is $(x-5)^2 + y^2 = 25$

14 (a) **15** (d) **16** (a)

17 (c) **18** (c) **19** (b)

20 (c) **21** (c) **22** (c)

23 (d) **24** (d)

Model 5

1 (d) **2** (a) **3** (d)

4 (b) **5** (c)

6

$\because$ The total surface area $= \pi\, r\,(L + r)$

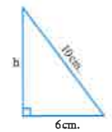
$96\,\pi = \pi \times r \times (10 + r)$

$\therefore r^2 + 10\,r - 96 = 0$

$\therefore r = -16$ (refused) or r = 6 cm.

$\therefore h = \sqrt{10^2 - 6^2} = 8$ cm.

$\therefore$ Volume of the cone $= \frac{1}{3}\,\pi\,(6)^2 \times 8 = 96\,\pi$ cm^3



7

From the figure Δ ABC represents a triangle

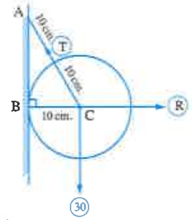
forces , AB $= \sqrt{20^2 - 10^2}$

$= 10\sqrt{3}$

$\therefore \frac{30}{10\sqrt{3}} = \frac{R}{10} = \frac{T}{20}$

$\therefore T = \frac{30 \times 20}{10\sqrt{3}} = 20\sqrt{3}$ gm.wt.

, $R = \frac{30 \times 10}{10\sqrt{3}} = 10\sqrt{3}$ gm.wt.



8 (b)

9 (c)

10

Consider $\overleftrightarrow{CB}$, $\overleftrightarrow{CD}$

axes $\overrightarrow{OX}$, $\overrightarrow{Oy}$

$\therefore X = 2\cos 0° + 7\sqrt{5}\cos\theta°$

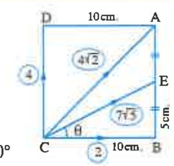
$+ 4\sqrt{2}\cos 45° + 4\cos 90°$

$= 2 + 7\sqrt{5} \times \frac{10}{5\sqrt{5}} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 0$

$\therefore$ X = 20 newton

, $Y = 2\sin 0° + 7\sqrt{5}\sin\theta + 4\sqrt{2}\sin 45° + 4\sin 90°$

$= 0 + 7\sqrt{5} \times \frac{5}{5\sqrt{5}} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 4$



$\therefore$ Y = 15 newton

$\therefore R = \sqrt{x^2 + y^2} = \sqrt{20^2 + 15^2} = 25$ newton

$\tan\theta = \frac{Y}{X} = \frac{15}{20} = \frac{3}{4}$

$\therefore \theta = 36° 52̀ 12̀$ with 1st force.

11 (b) **12** (d) **13** (b)

14 (b) **15** (c) **16** (d) **17** (d)

18 (d) **19** (c) **20** (c) **21** (b)

22 (d) **23** (b) **24** (d)

Model 6

1 (c) **2** (d) **3** (d) **4** (b) **5** (c)

6 (c)

7 (a)

8

$\because (AB)^2 = (BC)^2 + (AC)^2$

$\therefore m(\angle ACB) = 90°$

, $\therefore CD = \frac{1}{2}$ AB = 50 cm.

$\therefore$ CD = DB , $\qquad \therefore m(\angle B) = \theta_1$

, $\because$ CD = AD , $\qquad \therefore m(\angle A) = \theta_2$

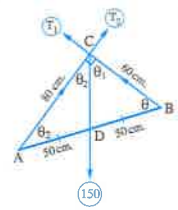
$\frac{T_1}{\sin(90° + \theta_1)} = \frac{T_2}{\sin(90° + \theta_2)}$

$= \frac{150}{\sin 90°}$

$\therefore \frac{T_1}{\cos\theta_1} = \frac{T_2}{\cos\theta_2} = \frac{150}{1}$

$\therefore \frac{T_1}{\frac{6}{10}} = \frac{T_2}{\frac{8}{10}} = 150$

$\therefore T_1 = 90$ gm.wt. , $T_2 = 120$ gm.wt.



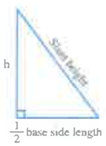
9 (d)

10

$\because$ The volume of the pyramid = 1296

$\therefore \frac{1}{3} \times (18)^2 \times h = 1296$

$\therefore$ h = 12 cm.



$\because$ Slant height $= \sqrt{h^2 + \left(\frac{1}{2}\text{side}\right)^2}$

$= \sqrt{12^2 + \left(\frac{18}{2}\right)^2} = 15$ cm.

$\therefore$ Lateral surfac area $= \frac{1}{2}$ perimeter of the base × S.h.

$= \frac{1}{2} \times (4 \times 18) \times 15 = 540$ cm^2

11 (a)

12 (d)

13 (c)

14 (c)

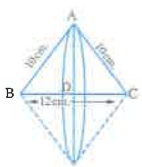
15 (c)

16

The solid which generated by rotation around $\overline{BC}$ is two cones have common base and are congruent.

$\because AD = \sqrt{10^2 - 6^2} = 8$ cm.

$\therefore$ The volume of the result solid



$= 2\left(\frac{1}{3}\pi r^2 h\right)$

$= 2 \times \frac{1}{3} \times \pi \times 8^2 \times 6$

$= 256\,\pi$ cm^2

17 (a) **18** (c) **19** (c) **20** (c)

21 (a) **22** (b) **23** (c) **24** (c)

Model 7

1 (c) **2** (a) **3** (a) **4** (b)

5 (c)

6 (b) **7** (a)

8

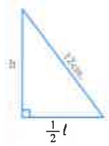
$\because$ Lateral surface area = 240

$\therefore \frac{1}{2}$ (perimeter of the base) × slant height = 240

If ℓ is side length of the base.

$\therefore \frac{1}{2} \times 4\,\ell \times 12 = 240$

$\therefore \ell = 10$ cm.



(1) $\because$ Height of pyramid

$= \sqrt{12^2 - 5^2} = \sqrt{119}$ cm.

(2) Volume of pyramid $= \frac{1}{3}$ × area of base × h

$= \frac{1}{3} \times (10)^2 \times \sqrt{119} = \frac{100}{3}\sqrt{119} = 363.6$ cm^3

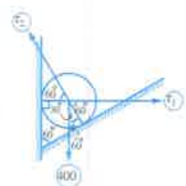
9 (c)

10

$\because$ The two planes are smooth

$\therefore r_1$ and r_2 are perpendicular to the two planes and passes through the centre of the sphere.

Applying lami's rale



$\therefore \frac{r_1}{\sin 150°} = \frac{r_2}{\sin 90°} = \frac{400}{\sin 120°}$

$\therefore r_1 = \frac{400 \times \sin 150°}{\sin 120°} = \frac{400\sqrt{3}}{3}$ kg.wt.

$\therefore r_2 = \frac{400 \times \sin 90°}{\sin 120} = \frac{800\sqrt{3}}{3}$ kg.wt.

11 (d) **12** (b)

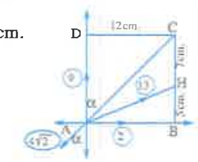
13

$\because$ Δ AHB is right-angled at B

$\therefore AH = \sqrt{(5)^2 + (12)^2} = 13$ cm.

$\therefore \sin(\angle BAH) = \frac{5}{13}$

, $\cos(\angle BAH) = \frac{12}{13}$



$\because \overline{AC}$ is a diagonal of the square ABCD

$\therefore \alpha = 45°$

$\therefore X = 2\cos 0° + 13\cos(\angle BAH) + 9\cos 90°$

$+ 4\sqrt{2}\cos 225°$

$= 2 \times 1 + 13 \times \frac{12}{13} + 9 \times 0 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$

, $Y = 2\sin 0° + 13\sin(\angle BAH) + 9\sin 90°$

$+ 4\sqrt{2}\sin 225°$

$= 2 \times 0 + 13 \times \frac{5}{13} + 9 \times 1 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$

$\therefore \vec{R} = 10\,\vec{i} + 10\,\vec{j}$

$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$ gm.wt.

, $\tan\theta = \frac{10}{10} = 1$ $\because X > 0$, $Y > 0$ $\therefore \theta = 45°$

$\therefore \vec{R}$ acts due to $\overrightarrow{AC}$

14 (c)

15 (b)

16 (d)

17 (b) **18** (b)

19 (c) **20** (c) **21** (c) **22** (b)

23 (d) **24** (c)

Model 8

1 (c) **2** (d) **3** (d) **4** (b) **5** (d)

6

$\because$ Volume of the pyramid $= \frac{1}{3}$ × base area × height

$\therefore 8\sqrt{3} = \frac{1}{3}$ × base area × 4

$\therefore$ Base area $= 6\sqrt{3}$ cm^2

$\therefore \frac{6}{4} \times x^2 \times \cot\frac{\pi}{6} = 6\sqrt{3}$

$\therefore x^2 = 4$ $\therefore x = 2$

$\therefore$ Side length of the hexogon = 2 cm.

$\therefore$ Base perimeter = 6 × 2 = 12 cm.

7 (c) **8** (a)

9 (d)

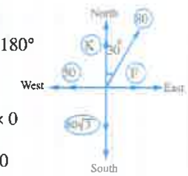
10 (d)

11

$\because X = F\cos 0° + 80\cos 60°$

$+ K\cos 90° + 50\cos 180°$

$+ 80\sqrt{3}\cos 270°$



$= F \times 1 + 80 \times \frac{1}{2} + K \times 0$

$+ 50 \times -1 + 80\sqrt{3} \times 0$

$= F - 10$

, $Y = F\sin 0° + 80\sin 60° + K\sin 90° + 50\sin 180°$

$+ 80\sqrt{3}\sin 270°$

$= F \times 0 + 80 \times \frac{\sqrt{3}}{2} + K \times 1 + 50 \times 0$

$+ 80\sqrt{3} \times -1 = K - 40\sqrt{3}$

$\therefore \vec{R} = (F - 10)\,\vec{i} + \left(K - 40\sqrt{3}\right)\vec{j}$ (1)

$\because$ R = 40 newton due to 60° North of East

$\therefore \vec{R} = 40\cos 60°\,\vec{i} + 40\sin 60°\,\vec{j}$

$= 20\,\vec{i} + 20\sqrt{3}\,\vec{j}$ (2)

From (1) and (2) : $\therefore$ F − 10 = 20

$\therefore$ F = 30 newton

, $K - 40\sqrt{3} = 20\sqrt{3}$ $\therefore K = 60\sqrt{3}$ newton

12 (d)

13 (a)

14 (b)

15 (b)

16

From circle C_1 :

$r_1 = \sqrt{k}$, $P_1(-2, -11)$ is its centre

From circle C_2 :

$r_2 = \sqrt{16} = 4$, $P_2(3, 1)$ is its centre

The distance between the two centres

$P_1P_2 = \sqrt{(3+2)^2 + (1+11)^2} = 13$ length unit.

In case the two circles are toucting externally

$\therefore P_1P_2 = r_1 + r_2$ $\therefore 13 = \sqrt{k} + 4$

$\therefore \sqrt{k} = 13 - 4 = 9$ $\therefore k = 9^2 = 81$

In case the two circles are touching internally

$\therefore P_1P_2 = |r_1 - r_2|$ $\therefore 13 = |\sqrt{k} - 4|$

$\therefore 13 = \sqrt{k} - 4$ i.e. $\sqrt{k} = 13 + 4 = 17$

, then k = 289

or $-13 = \sqrt{k} - 4$ i.e. $\sqrt{k} = -13 + 4 = -9$ refused

$\therefore k = 81$ or 289

17 (d) **18** (d)

19 (a) **20** (d) **21** (b) **22** (d)

23 (b) **24** (b)

Model 9

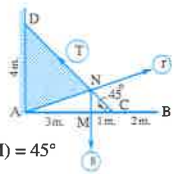
1 (c) **2** (c) **3** (b)

4 (d)

5 (a) **6** (c) **7** (d)

8 (c)

9



$\because AC = AD = 4$ metres

$\therefore m(\angle ACD) = 45°$

$\therefore CD = 4\sqrt{2}$ metres

From Δ MCN :

MN = 1 metre , m ($\angle$ NCM) = 45°

, m ($\angle$ NMC) = 90°

$\therefore NC = \sqrt{2}$ metres

$\therefore DN = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$ metres

From Δ AMN : AN $= \sqrt{10}$ metres (Pythagoras)

, $\because$ Since Δ AND is the triangle of forces

$\therefore \frac{r}{\sqrt{10}} = \frac{T}{3\sqrt{2}} = \frac{8}{4}$

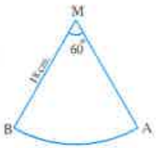
$\therefore r = 2\sqrt{10}$ kg.wt. , $T = 6\sqrt{2}$ kg.wt.

10 (d)

11 (b)

12

The length of the drawer of the cone = 18 cm.



, $\because$ the circumference of circle

of the cone base = the length of $\overset{\frown}{AB}$

$= r \times \theta^{rad} = 18 \times \frac{60° \times \pi}{180°} = 6\pi$

$\therefore 2\pi \grave{r} = 6\pi$ $\therefore \grave{r} = 3$ cm.

$\therefore h = \sqrt{\ell^2 - \grave{r}^2} = \sqrt{(18)^2 - 3^2} = 3\sqrt{35}$

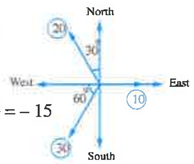
$\therefore$ Volume of the cone $= \frac{1}{3}\pi \grave{r}^2 h$

$= \frac{1}{3} \times \pi \times (3)^2 \times 3\sqrt{35}$

$\simeq 167.3$ cm^3.

13 (c)

14



$X = 10 \cos 0° + 20 \cos 120°$

$+ 30 \cos 240°$

$= 10 + 20 \times \frac{-1}{2} + 30 \times \frac{-1}{2} = -15$

$Y = 10 \sin 0° + 20 \sin 120°$

$+ 30 \sin 240°$

$= 0 + 20 \times \frac{\sqrt{3}}{2} + 30 \times \frac{-\sqrt{3}}{2} = -5\sqrt{3}$

$\therefore \vec{R} = -15\,\hat{i} - 5\sqrt{3}\,\hat{j}$

$\therefore R = \sqrt{(-15)^2 + \left(-5\sqrt{3}\right)^2} = 10\sqrt{3}$ newton.

, $\tan\theta = \frac{-5\sqrt{3}}{-15} = \frac{\sqrt{3}}{3}$ $\therefore \theta = 210°$

$\therefore$ Line of action of the resultant in direction 30° south of west.

15 (b) **16** (c) **17** (d) **18** (d)

19 (a) **20** (b) **21** (c) **22** (a)

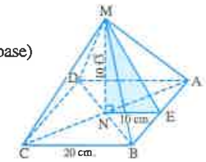
23 (b) **24** (b)

Model 10

1 (d) **2** (a) **3** (c) **4** (a) **5** (a)

6

The slant height $= \sqrt{\left(10\sqrt{3}\right)^2 + (10)^2} = 20$ cm.



(1) Lateral surface area

$= \frac{1}{2}$ (perimeter of the base)

$\times$ slant height

$= \frac{1}{2} \times (4 \times 20) \times 20$

$= 800$ cm^2.

(2) Volume $= \frac{1}{3}$ area of the base $\times$ h

$= \frac{1}{3} \times (20)^2 \times 10\sqrt{3} = \frac{4000\sqrt{3}}{3}$ cm^3.

7 (b)

8 (b)

9 (a)

10 (c)

11 (d)

12

$\because$ Volume of the cone $= \frac{1}{3} \times \pi (2)^2 \times 12 = 16\pi$ cm^3

$\therefore$ Volume of the dislodges water in the cylinder

$= 16\pi$ cm^3.

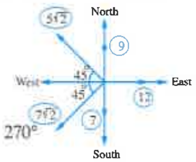
$\therefore 16\pi = \pi \grave{r}^2 \times 1$ $\therefore \grave{r}^2 = 16$

$\grave{r} = 4$ cm.

$\therefore$ The length of the diameter of the base of the vessel $= 4 \times 2 = 8$ cm.

13 (b)

14



$X = 12 \cos 0° + 9 \cos 90°$

$+ 5\sqrt{2} \cos 135° + 7\sqrt{2} \cos 225° + 7 \cos 270°$

$= 12 + 0 - 5 - 7 + 0 = 0$

, $Y = 12 \sin 0° + 9 \sin 90°$

$+ 5\sqrt{2} \sin 135°$

$+ 7\sqrt{2} \sin 225° + 7 \sin 270°$

$= 0 + 9 + 5 - 7 - 7 = 0$

$\therefore \vec{R} = \vec{0}$

$\therefore$ The system is in equilibrium.

15 (a) **16** (a)

17 (d) **18** (d)

19 (d) **20** (a)

21 (c) **22** (c)

23 (b) **24** (c)

77

Guide answers of multiple choice examinations

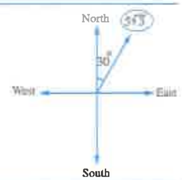
Model 1

1 (a)

2 (c)

Solution :

Component $= 5\sqrt{3}\sin 30^\circ = \frac{5\sqrt{3}}{2}$



3 (a)

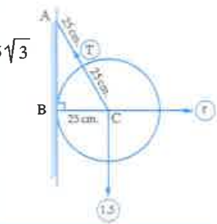
Solution :

$AB = \sqrt{(50)^2 - (25)^2} = 25\sqrt{3}$

$\therefore$ Δ ABC is the triangle of forces

$\therefore \frac{15}{25\sqrt{3}} = \frac{T}{50}$

$\therefore T = \sqrt{3}$ gm.wt



4 (c)

Solution :

$\overrightarrow{R} = \left(10\sqrt{2}\cos\frac{3\pi}{4}, 10\sqrt{2}\sin\frac{3\pi}{4}\right) = (-10, 10)$

, $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (-9 + a, 9 + b)$

$\therefore -9 + a = -10 \qquad \therefore a = -1$

, $9 + b = 10 \qquad \therefore b = 1 \qquad \therefore a + b = \text{zero}$

5 (b)

Solution :

$R = \sqrt{5^2 + 3^2 + 2 \times 5 \times 3 \cos 60} = 7$ newton.

6 (b)

7 (b)

Solution :

$\because \overrightarrow{R}$ is perpendicular to the force of magnitude 4 N

$\therefore 4 + F\cos 120^\circ = 0 \qquad \therefore F = 8$ newton

$\therefore R = \sqrt{4^2 + 8^2 + 2 \times 4 \times 8 \times \cos 120^\circ}$
$= 4\sqrt{3}$ newton.

8 (d)

Solution :

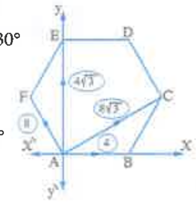
Let $\overrightarrow{AB}$ in direction of $\overrightarrow{OX}$

$\therefore X = 4\cos 0^\circ + 8\sqrt{3}\cos 30^\circ + 4\sqrt{3}\cos 90^\circ + 8\cos 120^\circ = 12$

, $Y = 4\sin 0^\circ + 8\sqrt{3}\sin 30^\circ + 4\sqrt{3}\sin 90^\circ + 8\sin 120^\circ = 12\sqrt{3}$

$\therefore \overrightarrow{R} = (12, 12\sqrt{3})$

$\therefore \|\overrightarrow{R}\| = \sqrt{(12)^2 + (12\sqrt{3})^2} = 24$ newton.



9 (c)

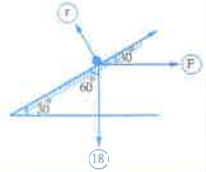
10 (a)

Solution :

Using Lami's rule

$\therefore \frac{18^\circ}{\sin 120^\circ} = \frac{F}{\sin 150^\circ}$

$\therefore F = 6\sqrt{3}$ newton.



11 (d)

Solution :

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (0, 0) = \overrightarrow{O}$

12 (d)

13 (a)

14 (b)

Solution :

$(x \quad y \quad 25)\begin{pmatrix} x \\ y \\ -4 \end{pmatrix} = (x^2 + y^2 - 100) = O$

$\therefore x^2 + y^2 = 100$

$\therefore r = 10$, diameter length $= 20$ length unit.

15 (d)

Solution :

$\because 2\pi r = 44 \qquad \therefore 2\left(\frac{22}{7}\right) r = 44 \qquad \therefore r = 7$

$\therefore v = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 15 = 770\ cm^3$.

16 (d)

Solution :

side length of the base $= 36 \div 4 = 9$

$\therefore$ base area $= 9 \times 9 = 81\ cm^2$.

$\therefore v = \frac{1}{3} \times 81 \times 10 = 270\ cm^3$.

17 (d)

18 (c)

Solution :

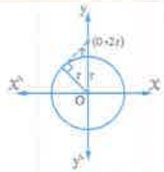
$v = \frac{1}{3}\ell^2 h$, $\hat{v} = \frac{1}{3}(2\ell)^2 h = 4\left(\frac{1}{3}\ell^2 h\right) = 4v$

19 (c)

Solution :

Length of tangent segment

$= \sqrt{(2r)^2 - r^2} = \sqrt{3}\, r$



20 (a)

Solution :

drawer $= \sqrt{8^2 + 6^2} = 10$ cm.

$\therefore$ lateral surface area $= \pi \times 6 \times 10 = 60\pi$

21 (c)

Solution :

with respect to the first circle

$M_1 = (1, -3)$

with respect to the second circle

$\because 4x^2 + 4y^2 - 8x + 24y - 60 = 0$

$\therefore x^2 + y^2 - 2x + 6y - 15 = 0$

$M_2 = (1, -3)$

$\therefore$ The two circle are concentric

Model 2

1 (a)

2 (d)

Solution :

$F_2 = \frac{12\sin 30^\circ}{\sin(30^\circ + 90^\circ)} = 4\sqrt{3}$ newton.

3 (a)

4 (c)

Solution :

$\overrightarrow{R} = \left(10\sqrt{2}\cos\frac{3\pi}{4}, 10\sqrt{2}\sin\frac{3\pi}{4}\right)$
$= (-10, 10)$

, $\because \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (a - 9, 9 + b)$

$\therefore a - 9 = -10 \qquad \therefore a = -1$

, $9 + b = 10 \qquad \therefore b = 1$

$\therefore a + b = -1 + 1 = 0$

5 (d)

Solution :

$\because F = 2F\cos\frac{\alpha}{2} \qquad \therefore \cos\frac{\alpha}{2} = \frac{1}{2}$

$\therefore \frac{\alpha}{2} = 60^\circ \qquad \therefore \alpha = 120^\circ$

6 (a)

Solution :

$\because$ D is the midpoint of $\overline{BC}$

$\therefore D = (5, 0) \qquad \therefore r = 5$, $M = (5, y)$

$\therefore MB = \sqrt{(5-2)^2 + (y-o)^2} = 5$

$\therefore 9 + y^2 = 25 \qquad \therefore y^2 = 16$

$\therefore y = 4$ (negative solution is refused)

$\therefore M = (5, 4)$

$\therefore$ equation of the circle : $(x - 5)^2 + (y - 4)^2 = 25$

7 (a) **8** (d)

9 (b)

Solution :

$\because \overrightarrow{R}$ is perpendicular to the force of magnitude 4 N.

$\therefore 4 + F\cos 120^\circ = 0$

$\therefore F = 8$ newton

$\therefore R = \sqrt{4^2 + 8^2 + 2 \times 4 \times 8 \times \cos 120^\circ} = 4\sqrt{3}$ N.

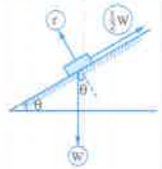
10 (a)

Solution :

$\because$ Using lami's rule

$\frac{W}{\sin 90^\circ} = \frac{\frac{1}{2}W}{\sin(180^\circ - \theta)}$

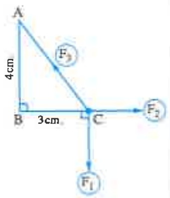
$\therefore \sin\theta = \frac{1}{2} \qquad \therefore \theta = 30^\circ$



11 (c)

12 (d)

Solution :



$AC = \sqrt{4^2 + 3^2} = 5$

$\therefore$ ABC is the triangle of forces

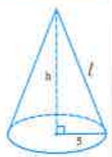
$\therefore \frac{F_1}{4} = \frac{F_2}{3} = \frac{F_3}{5}$

$\therefore F_1 : F_2 : F_3 = 4 : 3 : 5$

13 (c)

14 (c)

Solution :



$\because$ T.S.A $= \pi r \ell + \pi r^2$

$\therefore 90\pi = 5\pi \ell + 25\pi$

$\therefore 90 = 5\ell + 25 \qquad \therefore \ell = 13$ cm.

$\therefore h = \sqrt{(13)^2 - 5^2} = 12$ cm.

$\therefore v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 25 \times 12 = 100\pi$ cm^3.

15 (d)

16 (c)

Solution :

base side length $= \frac{40}{4} = 10$ cm.

$\therefore$ slant height $= \sqrt{5^2 + (12)^2} = 13$ cm.

$\therefore$ L.S.A $= \frac{1}{2}(40) \times 13 = 260$ cm^2.

17 (d)

Solution :

base side length $= \frac{36}{4} = 9$ cm.

$\therefore v = \frac{1}{3}$ base area $\times$ height $= \frac{1}{3} \times 9^2 \times 10 = 270$ cm^3.

18 (c)

Solution :

$(2F - 5)^2 + (F + 2)^2 = 45$

$\therefore 4F^2 + 25 - 20F + F^2 + 4F + 4 = 45$

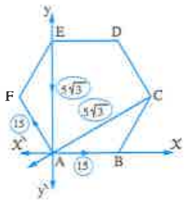
$\therefore 5F^2 - 16F - 16 = 0$

$\therefore F = 4$ newton , negative solutions refused

19 (c)

20 (d)

Solution :



$X = 15\cos 0° + 15\cos 120°$

$+ 5\sqrt{3}\cos 210°$

$+ 5\sqrt{3}\cos 270° = 0$

$, Y = 15\sin 0° + 15\sin 120°$

$+ 5\sqrt{3}\sin 210°$

$+ 5\sqrt{3}\sin 270° = 0$

$\therefore \vec{R} = (0, 0) \qquad \therefore \|\vec{R}\| = 0$

21 (b)

Solution :

$r = \sqrt{(-2-3)^2 + (5-2)^2} = \sqrt{34}$

$\therefore$ equation of the circle :

$(x + 2)^2 + (y - 5)^2 = 34$

$\therefore x^2 + y^2 + 4x - 10y - 5 = 0$

Model 3

1 (d)

Solution :

The resulted solid is a right cone of radius length of the base

$r = \sqrt{5^2 - 4^2} = 3$ cm. , height = 4 cm.

$\therefore v = \frac{1}{3}\pi(3^2) \times 4 = 12\pi$ cm^3.

2 (b)

Solution :

Let side length of the base $= \ell$

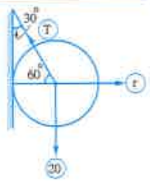
$\therefore 200 = \frac{1}{3}\ell^2 \times 6 \qquad \therefore \ell = 10$ cm.

$\therefore$ Slant height $= \sqrt{5^2 + 6^2} = \sqrt{61}$ cm.

$\therefore$ L.S.A. $= \frac{1}{2} \times 40 \times \sqrt{61} = 20\sqrt{61}$ cm^2.

3 (a)

Solution :



Using lami's rule

$\frac{20°}{\sin 120°} = \frac{r}{\sin 150°} = \frac{T}{\sin 90°}$

$\therefore \frac{T}{r} = \frac{\sin 90°}{\sin 150°} = 2 : 1$

4 (a)

5 (b)

Solution :

$\vec{R} = \vec{F_1} + \vec{F_2} = (3 + a, 2 + b) = (0, 0)$

$\therefore a = -3 \quad , \quad b = -2 \qquad \therefore a + b = -5$

6 (d)

7 (d) **8** (b) **9** (c)

10 (a)

Solution :

$R = \sqrt{3^2 + 5^2 + 2 \times 3 \times 5\cos 60°} = 7$ N

11 (d)

12 (c)

Solution :

$\because R \in [4, 12] \qquad \therefore F_1 - F_2 = 4 \quad (1)$

$, F_1 + F_2 = 12 \quad (2)$

adding (1) , (2) :

$\therefore 2F_1 = 16 \qquad \therefore F_1 = 8 , F_2 = 4$

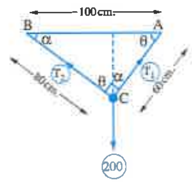
$\therefore$ The resultant = 12 when $\alpha = 0$

$\therefore$ (c) is not true

13 (c)

14 (c)

Solution :



Using lami's rule

$\therefore \frac{200}{\sin 90°} = \frac{T_1}{\sin\theta} = \frac{T_2}{\sin\alpha}$

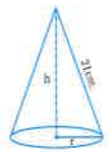
$\therefore 200 = \frac{T_1}{\frac{80}{100}} = \frac{T_2}{\frac{60}{100}}$

$\therefore T_1 = 160$ N $, T_2 = 120$ N

15 (b)

16 (b)

Solution :



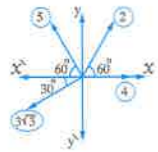
$\because 2\pi r = 44$ cm.

$\therefore r = 7$ cm.

$\therefore h = \sqrt{(21)^2 - 7^2} = 14\sqrt{2}$ cm.

17 (c)

Solution :



$X = 4\cos 0° + 2\cos 60°$

$+ 5\cos 120° + 3\sqrt{3}\cos 210°$

$= -2$

$, Y = 4\sin 0° + 2\sin 60°$

$+ 5\sin 120° + 3\sqrt{3}\sin 210 = 2\sqrt{3}$

$\therefore \vec{R} = \left(-2, 2\sqrt{3}\right)$

18 (b)

Solution :

centre of the circle = (0 , 0)

, radius length $= \sqrt{\left(\frac{3}{5} - 0\right)^2 + \left(\frac{-4}{5} - 0\right)^2} = 1$

$\therefore$ Equation of the circle : $x^2 + y^2 = 1$

$, \because 1^2 + 1^2 = 2 \neq 1$

$\therefore$ (1 , 1) does not lie on the circle

19 (d)

Solution :

$M = (0, 0) , r_1 = 2 , N = (3, 0) , r_2 = 3$

$\therefore MN = 3 , r_1 + r_2 = 5 , |r_1 - r_2| = 1$

$\therefore$ The two circles are intersecting.

20 (c)

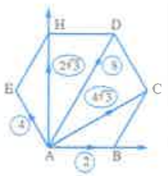
Solution :

$\because F = 2F\cos\frac{\alpha}{2} \qquad \therefore \cos\frac{\alpha}{2} = \frac{1}{2}$

$\therefore \frac{\alpha}{2} = 60° \qquad \therefore \alpha = 120°$

21 (c)

Solution :



$X = 2\cos 0° + 4\sqrt{3}\cos 30°$

$+ 8\cos 60° + 2\sqrt{3}\cos 90°$

$+ 4\cos 120° = 10$

$, Y = 2\sin 0° + 4\sqrt{3}\sin 30° + 8\sin 60°$

$+ 2\sqrt{3}\sin 90° + 4\sin 120° = 10\sqrt{3}$

$\therefore \vec{R} = \left(10, 10\sqrt{3}\right)$

$\therefore \vec{R} = \sqrt{(10)^2 + 10\sqrt{3}^{\,2}} = 20$

$, \tan\theta = \frac{10\sqrt{3}}{10} = \sqrt{3}$ $\therefore \theta = 60°$

$\therefore \vec{R}$ acts in direction of $\overrightarrow{AD}$

Model 4

1 (b)

2 (b)

Solution :

$\tan\theta = \frac{3\sqrt{2}\sin 135°}{6 + 3\sqrt{2}\cos 135°} = 1$ $\therefore \theta = 45°$

3 (c)

Solution :

$\because 6 = 2 \times 6\cos\frac{\alpha}{2}$ $\therefore \cos\frac{\alpha}{2} = \frac{1}{2}$

$\therefore \frac{\alpha}{2} = 60°$ $\therefore \alpha = 120°$

4 (c)

$\because 8 = 2F\cos\frac{\frac{\pi}{2}}{2}$ $\therefore F = 4\sqrt{2}$ N.

5 (c)

Solution :

$F_2 = \frac{10\sin 60°}{\sin((90° + 60°))} = 10\sqrt{3}$ N

6 (a)

7 (c)

Solution :

$F_1 = \frac{F\sin 45°}{\sin 75°}, F_2 = \frac{F\sin 30°}{\sin 75°}$

$\therefore F_1 + F_2 = \frac{\sin 45° + \sin 30°}{\sin 75°}$

$\therefore \frac{F_1 + F_2}{F} = \frac{\sin 45° + \sin 30°}{\sin 75°}$

8 (d)

Solution :

$\vec{R} = \vec{F_1} + \vec{F_2} = (12, -5)$

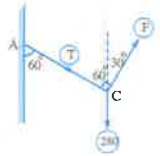
$\therefore \|\vec{R}\| = \sqrt{(12)^2 + (-5)^2} = 13$

9 (b)

10 (c)

Solution :

Using lami's rule



$\therefore \frac{T}{\sin 150°} = \frac{F}{\sin 120°} = \frac{280°}{\sin 90°}$

$\therefore T = 140$ gm.wt ,

$F = 140\sqrt{3}$ gm.wt

$\therefore \frac{F}{T} = \sqrt{3}$

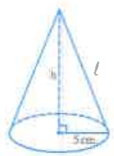
11 (c)

Solution :

$F = \sqrt{5^2 + 3^2 + 2 \times 3 \times 5\cos 60°} = 7$ newton.

12 (b)

Solution :



$\because \pi r \ell + \pi r^2 = 90\pi$

$\therefore 5\ell + 25 = 90$

$\therefore \ell = 13$

$\therefore h = \sqrt{(13)^2 - 5^2} = 12$ cm.

$\therefore v = \frac{1}{3} \times 5^2 \pi \times 12 = 100\pi$ cm^3.

13 (b)

Solution :

$\ell = 10$ cm.

$\therefore$ T.S.A. $= \pi \times 5 \times 10 + \pi \times 5^2 = 75\pi$ cm^2.

14 (b)

Solution :

side length of the base $= \frac{36}{4} = 9$

$\therefore v = \frac{1}{3} \times 9^2 \times 10 = 270$ cm^3.

15 (d)

Solution :

side length of the base $= \frac{40}{4} = 10$ cm.

$\therefore$ Slant height $= \sqrt{5^2 + (12)^2} = 13$ cm.

$\therefore$ T.S.A. $= \frac{1}{2}(40) \times 13 + (10)^2 = 360$ cm^2.

16 (d)

Solution :

$r^2 = 8$ $\therefore r = 2\sqrt{2}$

$\therefore$ Circumference $= 4\sqrt{2}\,\pi$

17 (a)

18 (b)

Solution :

$r = \sqrt{(3+2)^2 + (2-5)^2} = \sqrt{34}$ cm.

$\therefore$ The equation of the circle is $(x+2)^2 + (y-5)^2 = 34$

$\therefore x^2 + 4x + 4 + y^2 - 10y + 25 = 34$

$\therefore x^2 + y^2 + 4x - 10y - 5 = 0$

19 (c)

Solution :

$r = \sqrt{(-4-0)^2 + (3-0)^2} = 5$

$\therefore$ Equation of the circle is :

$(x+4)^2 + (y-3)^2 = 25$

20 (b) **21** (d)

Model 5

1 (c)

2 (c)

Solution :

$\because 12 = \frac{1}{3}$ base area $\times 4$ $\therefore$ base area $= 9$

$\therefore$ Side length = 3 cm.

3 (d)

Solution :

$F_1 = \frac{12\sin 30°}{\sin 75°} = 6\csc 75°$

4 (a)

5 (b)

Solution :

$\because 4 + F\cos 120° = 0$ $\therefore F = 8$ N.

6 (a)

Solution :

$\ell = \sqrt{6^2 + 8^2} = 10$ cm.

$\therefore$ L.S.A. $= \pi \times 6 \times 10$

$= 60\pi$ cm^2.

7 (c)

Solution :

$r = 5$ $\therefore$ circumference $= 10\pi$

8 (c)

Solution :

$\vec{R} = \left(10\sqrt{2}\cos\frac{3\pi}{4}, 10\sqrt{2}\sin\frac{3\pi}{4}\right)$

$= (-10, 10)$

$, \because \vec{F_1} + \vec{F_2} + \vec{F_3} = (-9 + a, 9 + b)$

$\therefore -9 + a = -10$ $\therefore a = -1$

$, 9 + b = 10$ $\therefore b = 1$ $\therefore a + b = 0$

9 (d)

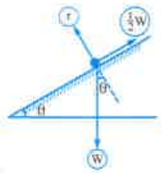
Solution :

edge length $= \frac{18}{6} = 3$ cm.

$\therefore$ Total area $= 4 \times \left(\frac{1}{2} \times 3 \times 3\sin 60°\right) = 9\sqrt{3}$

10 (a)

Solution :



Using lami's rule

$$\frac{\frac{1}{2}\,W}{\sin(180^\circ-\theta)}=\frac{W}{\sin 90^\circ}$$

$\therefore \sin\theta=\frac{1}{2}$ $\qquad \therefore \theta=30^\circ$

11 (c)

Solution :

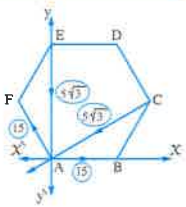
radius of the base = 3 cm.

, $\because 27\pi=\frac{1}{3}\pi(3^2)(h)$ $\qquad \therefore h=9$ cm.

12 (d)

13 (d)

Solution :



Let $\overrightarrow{AB}$ in direction of $\overrightarrow{OX}$

$\therefore X=15\cos 0+15\cos 120^\circ$

$+5\sqrt{3}\cos 210^\circ$

$+5\sqrt{3}\cos 270^\circ=0$

, $Y=15\sin 0+15\sin 120^\circ$

$+5\sqrt{3}\sin 210^\circ+5\sqrt{3}\sin 270^\circ=0$

$\therefore \vec{R}=(0\,,0)$ $\qquad \therefore \|\vec{R}\|=0$

14 (b)

15 (d)

16 (b)

Solution :

$8\sqrt{3}=\frac{1}{3}$ base area × 4

$\therefore$ base area $=6\sqrt{3}$

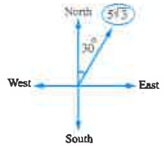
$\therefore \frac{6}{4}x^2\cot\frac{180}{6}=6\sqrt{3}$ $\qquad \therefore x^2=4$

$\therefore$ side length of hexagon = 2 cm.

$\therefore$ perimeter of base = 12 cm.

17 (c)

Solution :



Component in east direction

$=5\sqrt{3}\sin 30^\circ=\frac{5\sqrt{3}}{2}$

18 (c)

Solution :

M = (– 4 , 4) , r = 4

$\therefore$ The equation is $(x+4)^2+(y-4)^2=16$

$\therefore x^2+8x+16+y^2-8y+16=16$

$\therefore x^2+y^2+8x-8y+16=0$

19 (c)

Solution :

$\because 8=2F\cos\frac{90^\circ}{2}$ $\qquad \therefore F=4\sqrt{2}$

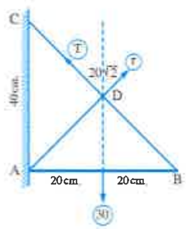
20 (c)

Solution :

$12=\frac{1}{3}\times 4\times h$ $\qquad \therefore h=9$ cm.

21 (c)

Solution :



The reaction bisects $\overline{BC}$

, Δ ACD is the triangle

of forces

$\therefore \frac{30}{20}=\frac{r}{10\sqrt{2}}=\frac{T}{10\sqrt{2}}$

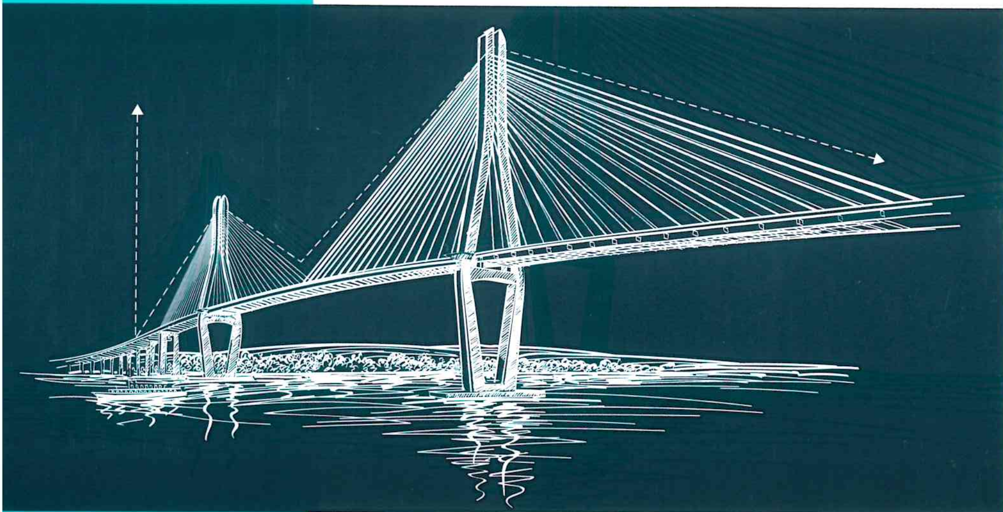
$\therefore r=15\sqrt{2}$

SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors



FIRST TERM

2

SEC.

2022

GUIDE ANSWERS

EL-MOASSER

Guide Answers of "Unit One"

Answers of accumulative exercise on vectors

1

(1) c	(2) b	(3) c	(4) c	(5) c
(6) b	(7) d	(8) d	(9) c	(10) c
(11) a	(12) c	(13) d	(14) b	(15) d
(16) c	(17) a	(18) a		

2

$18\overrightarrow{e}$, $45\overrightarrow{e}$, $-50\overrightarrow{e}$

3

$\overline{AC}$, $2\overline{DM}$ or $\overline{DB}$, $\overrightarrow{O}$, $\overline{AD}$, $\overline{MB}$

Exercise 1

First Multiple choice questions

(1) d	(2) c	(3) d	(4) c	(5) c
(6) b	(7) b	(8) c	(9) b	(10) c
(11) b	(12) b	(13) a	(14) a	(15) c
(16) d	(17) b	(18) b	(19) d	(20) d
(21) b	(22) c	(23) c	(24) c	(25) c
(26) d	(27) b	(28) a	(29) c	(30) d
(31) c	(32) a	(33) a	(34) b	(35) c
(36) b	(37) b	(38) c	(39) b	(40) d
(41) d	(42) c	(43) a	(44) a	(45) b
(46) c	(47) d	(48) d	(49) b	(50) c
(51) c				

Second Essay questions

1

$R = \sqrt{(8)^2 + (15)^2} = 17$ kg.wt. $\quad \tan\theta = \frac{15}{8}$

$\therefore \theta = 61^\circ\ 55'\ 39''$

2

Let $F_1 > F_2$ $\quad \therefore F_1 + F_2 = 17$, $F_1 - F_2 = 7$

adding $\quad \therefore 2F_1 = 24 \quad \therefore F_1 = 12$ kg.wt.

, $F_2 = 5$ kg.wt.

3

$\alpha = 30^\circ \times 2 = 60^\circ \qquad \therefore 4\sqrt{3} = 2F\cos 30^\circ$

$\therefore F = 4$ kg.wt.

4

$\therefore$ Let the two forces be F_1 , F_2 newton

$\therefore (50)^2 = F_1^2 + F_2^2 \qquad \therefore F_1^2 + F_2^2 = 2500 \quad (1)$

$\because \tan\theta = \frac{F_2}{F_1} \qquad \therefore \frac{F_2}{F_1} = \tan 30^\circ$

$\therefore \frac{F_2}{F_1} = \frac{1}{\sqrt{3}} \qquad \therefore F_1 = \sqrt{3}\,F_2 \quad (2)$

$\therefore F_1^2 = 3F_2^2 \quad (3)$

Substituting from (3) in (1) :

$\therefore 3F_2^2 + F_2^2 = 2500 \qquad \therefore 4F_2^2 = 2500$

$\therefore F_2^2 = \frac{2500}{4} = 625 \qquad \therefore F_2 = 25$ newton

Substituting in (2) : $\therefore F_1 = 25\sqrt{3}$ newton

$\therefore$ The magnitudes of the two forces are :

$25\sqrt{3}$, 25 newton.

5

$(26)^2 = (30)^2 + (16)^2 + 2 \times 30 \times 16\cos\alpha$

$\therefore \cos\alpha = -\frac{1}{2} \qquad \therefore \alpha = 120^\circ$

6

$\because$ The resultant is perpendicular to the first force

$\therefore 8 + 16\cos\alpha = 0$

$\therefore \cos\alpha = \frac{-8}{16} = -\frac{1}{2} \qquad \therefore \alpha = 120^\circ$

7

$\left(3\sqrt{7}\right)^2 = (9)^2 + (6)^2 + 2 \times 9 \times 6\cos\alpha$

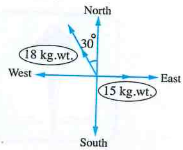
$\therefore \cos\alpha = -\frac{1}{2} \qquad \therefore \alpha = 120^\circ$

$\tan\theta = \frac{6\sin 120^\circ}{9 + 6\cos 120^\circ} = \frac{\sqrt{3}}{2} \qquad \therefore \theta \simeq 40^\circ\ 53'\ 36''$

8

$R = \sqrt{(15)^2 + (18)^2 + 2 \times 15 \times 18\cos 120^\circ}$

$= 3\sqrt{31}$ kg.wt.

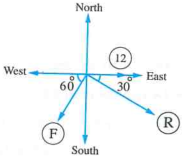


$\tan\theta = \frac{18\sin 120^\circ}{15 + 18\cos 120^\circ}$

$= \frac{3\sqrt{3}}{2}$

$\therefore \theta \simeq 68^\circ\ 56'\ 54''$

9



$\alpha = 120°$, $\theta = 30°$

$\tan 30° = \dfrac{F \sin 120°}{12 + F \cos 120°}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{F \times \frac{\sqrt{3}}{2}}{12 + F \times -\frac{1}{2}}$

$\therefore \frac{3}{2} F = 12 - \frac{1}{2} F$ $\qquad \therefore F = 6$ kg.wt.

, $R = \sqrt{(12)^2 + 6^2 + 2 \times 12 \times 6 \times \cos 120°}$

$\therefore R = 6\sqrt{3}$ kg.wt.

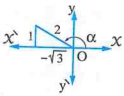
Another solution :

$\therefore$ The resultant is perpendicular to the second force.

$\therefore \cos 120 = \dfrac{-F}{12}$ $\qquad \therefore F = 6$ kg.wt.

$\therefore R = \sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ kg.wt.

10



Let F_1 be the smaller force and F_2 is the greater force.

, $\because$ the resultant is perpendicular to F_1

$\therefore F_1 + 30 \cos \alpha = 0$

$\therefore F_1 = 30 \times \dfrac{\sqrt{3}}{2} = 15\sqrt{3}$ kg.wt.

, $R = \sqrt{\left(15\sqrt{3}\right)^2 + (30)^2 + 2 \times 15\sqrt{3} \times 30 \times \left(-\frac{\sqrt{3}}{2}\right)}$

$= 15$ kg.wt.

11

(1) $R = \sqrt{\left(5\sqrt{2}\right)^2 + (5)^2 + 2 \times 5\sqrt{2} \times 5 \cos 45°}$

$\therefore R = 5\sqrt{5}$ newton

, $\tan \theta = \dfrac{5 \sin 45°}{5\sqrt{2} + 5 \cos 45°} = \dfrac{1}{3}$

$\theta \simeq 18° \, 26' \, 6''$

(2) $R = \sqrt{(3)^2 + \left(3\sqrt{2}\right)^2 + 2 \times 3 \times 3\sqrt{2} \times \cos 45°}$

$\therefore R = 3\sqrt{5}$ newton

, $\tan \theta = \dfrac{3\sqrt{2} \sin 45°}{3 + 3\sqrt{2} \cos 45°} = \dfrac{1}{2}$

$\theta \simeq 26° \, 33' \, 54''$

(3) $R = \sqrt{(150)^2 + (150)^2 + 2 \times 150 \times 150 \times \cos 120°}$

$\therefore R = 150$ newton

Another solution :

$R = 2 \times 150 \times \cos 60° = 150$ newton

, the resultant bisects the angle between the two forces.

12

$\left(4\sqrt{3}\right)^2 = F^2 + (4)^2 + 2 \times F \times 4 \cos 120°$

$\therefore F^2 - 4F - 32 = 0$ $\qquad \therefore (F + 4)(F - 8) = 0$

$\therefore F = 8$ newton

$\tan \theta = \dfrac{4 \sin 120°}{8 + 4 \cos 120°} = \dfrac{1}{\sqrt{3}}$ $\qquad \therefore \theta = 30°$

13

$\because$ The resultant is perpendicular to the first force

$\therefore \sqrt{3} F + 2F \cos \alpha = 0$ $\qquad \therefore \cos \alpha = -\dfrac{\sqrt{3}}{2}$

$\therefore \alpha = 150°$, when $F = 15$

$\therefore R = \sqrt{\left(15\sqrt{3}\right)^2 + (30)^2 + 2 \times 15\sqrt{3} \times 30 \cos 150°}$

$= 15$ newton

14

$\because$ The resultant is perpendicular to the second force

$\therefore R^2 = F_1^2 - F_2^2$ $\qquad \therefore 2 = 8 - F_2^2$

$\therefore F_2^2 = 6$ $\qquad \therefore F_2 = \sqrt{6}$ newton

$\because F_2 + F_1 \cos \alpha = 0$ $\qquad \therefore \sqrt{6} + 2\sqrt{2} \cos \alpha = 0$

$\therefore \cos \alpha = \dfrac{-\sqrt{6}}{2\sqrt{2}} = -\dfrac{\sqrt{3}}{2}$ $\qquad \therefore \alpha = 150°$

15

$\because \tan 30° = \dfrac{F \sin 120°}{16 + F \cos 120°}$ $\qquad \therefore \dfrac{1}{\sqrt{3}} = \dfrac{\frac{\sqrt{3}}{2} F}{16 - \frac{1}{2} F}$

$\therefore 16 - \frac{1}{2} F = \frac{3}{2} F$ $\qquad \therefore 2F = 16$

$\therefore F = 8$ kg.wt.

$\therefore R = \sqrt{(16)^2 + (8)^2 + 2 \times 16 \times 8 \cos 120°} = 8\sqrt{3}$ kg.wt.

16

The resultant of 1st and 2nd forces

$= \sqrt{(5)^2 + (10)^2 + 2 \times 5 \times 10 \cos 60°} = 5\sqrt{7}$ newton

$\therefore$ The maximum value of the resultant of the three forces $= 5\sqrt{7} + 4\sqrt{7} = 9\sqrt{7}$ newton

The minimum value of the resultant of the three forces $= 5\sqrt{7} - 4\sqrt{7} = \sqrt{7}$ newton

17

(1) $\because (3\,F)^2 = (2\,F)^2 + (3\,F)^2 + 2 \times 2\,F \times 3\,F \cos\theta$

$\therefore 9\,F^2 = 13\,F^2 + 12\,F^2 \cos\theta$

$\therefore \cos\theta = -\frac{1}{3}$ $\qquad \therefore \theta \simeq 109°\ 28'\ 16''$

(2) $\because$ The resultant $= F = 3\,F - 2\,F$

$\therefore$ The measure of the angle between the two forces $= 180°$

(3) $\because$ The resultant $= 5\,F = 2\,F + 3\,F$

$\therefore$ The measure of the angle between the two forces $=$ zero

(4) $\left(\sqrt{13}\,F\right)^2 = (2\,F)^2 + (3\,F)^2 + 2 \times 2\,F \times 3\,F \cos\theta$

$\therefore 13\,F^2 = 13\,F^2 + 12\,F^2 \cos\theta$

$\therefore \cos\theta =$ zero $\qquad \therefore \theta = 90°$

18

(1) $\because$ The direction of the resultant is perpendicular to the second force

$\therefore F + 2\cos 120° = 0$ $\qquad \therefore F = 1$ newton

(2) $\because \tan 45 = \frac{2\sin 120°}{F + 2\cos 120°}$ $\qquad \therefore \sqrt{3} = F - 1$

$\therefore F = \left(\sqrt{3} + 1\right)$ newton

19

$\because R \in [2\,,10]$

$\therefore$ The minimum value of $R = 2$ newton

$\therefore F_1 - F_2 = 2$ (1)

and the maximum value of $R = 10$ newton

$\therefore F_1 + F_2 = 10$ (2)

adding (1) and (2) : $\therefore 2\,F_1 = 12$

$\therefore F_1 = 6$ newton $\qquad \therefore F_2 = 4$ newton

$\therefore R = \sqrt{6^2 + 4^2 + 2 \times 6 \times 4\cos 120°} = 2\sqrt{7}$ newton

20

Let the two forces be F and $F + 3$

$\because$ The resultant is perpendicular to the smaller force.

$\therefore F + (F + 3)\cos\alpha = 0$ $\qquad \therefore \cos\alpha = \frac{-F}{F+3}$

$\because R = 3\sqrt{3}$

$\therefore 27 = F^2 + (F + 3)^2 + 2 \times F \times (F + 3) \times \frac{-F}{F+3}$

$\therefore 27 = F^2 + F^2 + 6\,F + 9 - 2\,F^2$ $\qquad \therefore F = 3$

$\therefore$ The two forces are 3 newton and 6 newton

$\cos\alpha = \frac{-3}{6} = -\frac{1}{2}$ $\qquad \therefore \alpha = 120°$

21

Let the two forces be F_1 and F_2

• **In the first case :** $10 = F_1^2 + F_2^2$ (1)

• **In the second case :**

$13 = F_1^2 + F_2^2 + 2\,F_1 \times F_2 \times \frac{1}{2}$

and from (1) : $\therefore 13 = 10 + F_1 F_2$ $\qquad \therefore F_1 F_2 = 3$

$\therefore F_2 = \frac{3}{F_1}$

substituting in (1) :

$\therefore 10 = F_1^2 + \frac{9}{F_1^2}$ $\qquad \therefore 10\,F_1^2 = F_1^4 + 9$

$\therefore F_1^4 - 10\,F_1^2 + 9 = 0$ $\qquad \therefore (F_1^2 - 9)(F_1^2 - 1) = 0$

$\therefore F_1^2 = 9$ $\quad \therefore F_1 = 3$ or $\quad \therefore F_1^2 = 1$ $\quad \therefore F_1 = 1$

$\therefore$ The two forces are 1 and 3 newton

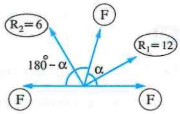
22

$R_1 = 2\,F\cos\frac{\alpha}{2} = 12$

$\therefore F\cos\frac{\alpha}{2} = 6$ (1)

$, R_2 = 2\,F\cos\left(\frac{180° - \alpha}{2}\right) = 6$

$\therefore F\sin\frac{\alpha}{2} = 3$ (2)



squaring the two equations and adding them

$\therefore F^2\cos^2\frac{\alpha}{2} + F^2\sin^2\frac{\alpha}{2} = 6^2 + 3^2$

$\therefore F^2\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right) = 45$

$\therefore F = \sqrt{45} = 3\sqrt{5}$ kg.wt.

23

• **In the first case :** $R^2 = F_1^2 + F_2^2 + 2\,F_1 F_2 \cos 120°$

$\therefore R^2 = F_1^2 + F_2^2 - F_1 F_2$ (1)

• **In the second case :**

The measure of the angle between the two forces $= 60°$

$\therefore 3\,R^2 = F_1^2 + F_2^2 + F_1 F_2$ (2)

Substituting from (1) in (2)

$\therefore 3\,(F_1^2 + F_2^2 - F_1 F_2) = F_1^2 + F_2^2 + F_1 F_2$

$\therefore 2\,F_1^2 + 2\,F_2^2 - 4\,F_1 F_2 = 0$

$\therefore F_1^2 - 2\,F_1 F_2 + F_2^2 = 0$ $\qquad \therefore (F_1 - F_2)^2 = 0$

$\therefore F_1 = F_2$

$\therefore \overrightarrow{R_1}$ in the first case makes an angle of measure $60°$ with $\overrightarrow{F_1}$

$\overrightarrow{R_2}$ in the second case makes an angle of measure $30°$ with $\overrightarrow{F_1}$ from the other side.

$\therefore$ The measure of the angle between the two resultants $= 90°$

24

$\overrightarrow{F_1} = (4 , 0°) , \overrightarrow{F_2} = (F , \alpha) , \overrightarrow{R} = (10 , 60°)$

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$\therefore (10 \cos 60° , 10 \sin 60°)$

$= (4 \cos 0° , 4 \sin 0°) + (F \cos \alpha , F \sin \alpha)$

$\therefore \left(5 , 5\sqrt{3}\right) = (4 , 0) + (F \cos \alpha , F \sin \alpha)$

$\therefore F \cos \alpha + 4 = 5 \qquad \therefore F \cos \alpha = 1 \qquad (1)$

$, F \sin \alpha = 5\sqrt{3} \qquad (2)$

, dividing (2) by (1) :

$\tan \alpha = 5\sqrt{3}$

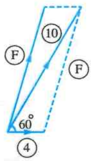
$\therefore \sin \alpha = \frac{5\sqrt{3}}{\sqrt{76}}$

$, \text{from (2)} : \therefore F \times \frac{5\sqrt{3}}{\sqrt{76}} = 5\sqrt{3}$

$\therefore F = \sqrt{76} = 2\sqrt{19}$ newton

Another solution :

Applying cosine rule :



$F^2 = (10)^2 + (4)^2 - 2 \times 4 \times 10 \times \cos 60°$

$= 76$

$\therefore F = 2\sqrt{19}$ newton

25

Let $F_1 > F_2 \quad \therefore F_1 - F_2 = 15 \quad \therefore F_1 = 15 + F_2 \quad (1)$

$(35)^2 = F_1^2 + F_2^2 + 2 F_1 \times F_2 \times -\frac{1}{2}$

substituting from (1)

$\therefore 1225 = 225 + F_2^2 + 30 F_2 + F_2^2 - 15 F_2 - F_2^2$

$\therefore F_2^2 + 15 F_2 - 1000 = 0 \quad \therefore (F_2 + 40)(F_2 - 25) = 0$

$\therefore F_2 = 25$

$\therefore$ The two forces are 40 and 25 newton

26

$F_1 + F_2 = 4 \qquad \therefore F_1 = 4 - F_2 \qquad (1)$

$13 = (F_1)^2 + (F_2)^2 + 2 F_1 F_2 \cos 60°$

Substituting from (1) :

$\therefore 13 = (4 - F_2)^2 + F_2^2 + (4 - F_2) \times F_2$

$\therefore 13 = 16 - 8 F_2 + F_2^2 + F_2^2 + 4 F_2 - F_2^2$

$\therefore F_2^2 - 4 F_2 + 3 = 0 \qquad \therefore (F_2 - 1)(F_2 - 3) = 0$

$\therefore F_2 = 1$ or 3

$\therefore$ The two forces are 1 and 3 newton

27

Let the two forces be F_1 and F_2 where $F_1 < F_2$

$\therefore F_1 + F_2 = 40$ then $F_1 = 40 - F_2 \qquad (1)$

$\because$ The resultant = 20 kg.wt.

$\therefore 400 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha \qquad (2)$

$\because$ The resultant is perpendicular to the smaller force

$\therefore F_1 + F_2 \cos \alpha = 0$

$\therefore \cos \alpha = \frac{-F_1}{F_2}$ substituting in (2) :

$\therefore 400 = F_1^2 + F_2^2 - 2 F_1 F_2 \times \frac{F_1}{F_2}$

$\therefore 400 = F_1^2 + F_2^2 - 2 F_1^2$

$\therefore 400 = F_2^2 - F_1^2$

Substituting from (1) : $\therefore 400 = F_2^2 - (40 - F_2)^2$

$\therefore 400 = F_2^2 - 1600 + 80 F_2 - F_2^2$

$\therefore 2000 = 80 F_2 \qquad \therefore F_2 = 25$ kg.wt.

Substituting in (1) :

$\therefore F_1 = 15$ kg.wt. $, \cos \alpha = \frac{-15}{25} = \frac{-3}{5}$

28

$F_1^2 + F_2^2 = 25 \qquad (1)$

$\because F_1^2 + F_2^2 + 2 F_1 F_2 \cos 120° = 13$

$\therefore F_1^2 + F_2^2 - F_1 F_2 = 13 \qquad (2)$

Subtracting (2) from (1) : $\therefore F_1 F_2 = 12$

$\therefore F_1 = \frac{12}{F_2}$

substituting in (1) : $\therefore \left(\frac{12}{F_2}\right)^2 + F_2^2 = 25$

$\therefore \frac{144}{F_2^2} + F_2^2 = 25 \qquad \therefore F_2^4 - 25 F_2^2 + 144 = 0$

$\because (F_2^2 - 9)(F_2^2 - 16) = 0$

$, \because F_1 > F_2 \qquad \therefore F_1 = 4$ kg.wt. $, F_2 = 3$ kg.wt.

29

$R_1 = 2 F \cos 60° = F \qquad R_2 = 2 (2 F) \cos 30° = 2\sqrt{3} F$

$\because R_2 - R_1 = 11 \qquad \therefore 2\sqrt{3} F - F = 11$

$\therefore F = \frac{11}{2\sqrt{3} - 1} \qquad \therefore F = 1 + 2\sqrt{3}$

30

First case :

$\left(\sqrt{5} F (m + 1)\right)^2 = F^2 + (2 F)^2 + 2 \times F \times 2 F \cos \alpha$

$\therefore 5 F^2 (m^2 + 2 m + 1) - 5 F^2 = 4 F^2 \cos \alpha$

$\therefore 5 F^2 m (m + 2) = 4 F^2 \cos \alpha \qquad (1)$

Second case :

$$\left[\sqrt{5}\,F\,(m-1)\right]^2 = F^2 + (2F)^2 + 2 \times F \times 2F \cos(90° - \alpha)$$

$$5F^2(m^2 - 2m + 1) - 5F^2 = 4F^2 \sin\alpha$$

$$\therefore 5F^2 m(m-2) = 4F^2 \sin\alpha \qquad (2)$$

by dividing (2) ÷ (1) :

$$\frac{5F^2 m(m-2)}{5F^2 m(m+2)} = \frac{4F^2 \sin\alpha}{4F^2 \cos\alpha} \qquad \therefore \tan\alpha = \frac{m-2}{m+2}$$

Third Higher skills

1

(1) d	(2) d	(3) b
(4) d	(5) a	(6) b
(7) c	(8) d	(9) a
(10) d	(11) c	

Instructions to solve 1 :

(1) $\because$ The maximum value $= F_1 + F_2$

, the minimum value $= F_1 - F_2$ where $F_1 > F_2$

$\therefore \frac{F_1 + F_2}{F_1 - F_2} = \frac{7}{3} \qquad \therefore 3F_1 + 3F_2 = 7F_1 - 7F_2$

$\therefore 4F_1 = 10F_2 \qquad \therefore \frac{F_1}{F_2} = \frac{10}{4} = \frac{5}{2}$

$\therefore$ The ratio between the two forces = 5 : 2

(2) $\because F_1 : F_2 : R = 4 : 3 : \sqrt{13}$

$\therefore F_1 = 4m \quad , \quad F_2 = 3m \quad , \quad R = \sqrt{13}\,m$

$\therefore \left(\sqrt{13}\,m\right)^2 = (4m)^2 + (3m)^2 + 2(4m)(3m)\cos\alpha$

$\therefore \cos\alpha = \frac{13m^2 - 16m^2 - 9m^2}{24m^2} = \frac{-1}{2}$

$\therefore$ The measure of the angle between the two forces $= \cos^{-1}\left(\frac{-1}{2}\right) = 120°$

(3) $\because$ The resultant $\perp F_1$

$\therefore F_1 + F_2 \cos\alpha = \text{zero} \qquad \therefore \cos\alpha = \frac{-F_1}{F_2}$

$\therefore$ The measure of the angle between the two forces $\cos^{-1}\left(\frac{-F_1}{F_2}\right)$

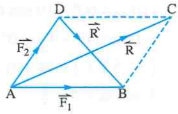
(4) $\because$ The measure of the angle between the two forces = 90°

, $\because$ the two forces are unequal

$\therefore$ The force inclined toward the greater force

$\therefore$ The measure of the angle θ between the greater force and the resultant must be less than 45°

(5) Representing the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ as two adjacent sides in parallelogram as in the opposite figure



$\therefore \overrightarrow{F_1} + \overrightarrow{F_2} = \overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \overrightarrow{R} \qquad (1)$

$, \overrightarrow{F_1} + (-\overrightarrow{F_2}) = \overrightarrow{AB} + (-\overrightarrow{AD}) = \overrightarrow{AB} - \overrightarrow{AD}$

$= \overrightarrow{DB} = \overrightarrow{R'} \qquad (2)$

, $\because \overrightarrow{R}$ and $\overrightarrow{R'}$ are represented by diagonals of parallelogram

$\therefore$ In case of $\overrightarrow{R} \perp \overrightarrow{R'}$, then ABCD is rhombus

$\therefore AB = AD \qquad \therefore F_1 = F_2$

(6) $R^2 = F^2 + (4)^2 + 2(F)(4)\cos 120°$

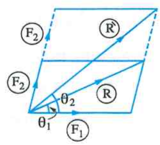
$\therefore R^2 = F^2 - 4F + 16$

$\therefore R^2 = (F-2)^2 + 12$

$\therefore R = \sqrt{(F-2)^2 + 12}$

$\therefore$ The smallest value of the resultant is when $F = 2$

(7) As F_2 increases as the resultant lean towards the greater force which is doubled and hence the angle between the resultant and the second force increases.



i.e. $\theta_2 > \theta_1$

The opposite figure show the idea.

(8) $\because F^2 = F^2 + 3F^2 + 2(F)\left(\sqrt{3}\,F\right)\cos\alpha$

$\therefore \cos\alpha = -\frac{\sqrt{3}}{2} \qquad \therefore \alpha = 150°$

$, \because \tan\theta_1 = \frac{\sqrt{3}\,F \sin 150°}{F + \sqrt{3}\,F \cos 150°} = -\sqrt{3}$

$\therefore \theta_1 = 120°$ and so $\theta_2 = 150° - 120° = 30°$

$\therefore \theta_1 = 4\,\theta_2$

(9) $\because 3 \le F_1 \le 12 \qquad \therefore 9 \le F_1^2 \le 144 \qquad (1)$

$, 4 \le F_2 \le 16 \qquad \therefore 16 \le F_2^2 \le 256 \qquad (2)$

By adding (1) , (2) :

$\therefore 25 \le F_1^2 + F_2^2 \le 400 \qquad \therefore 5 \le R \le 20$

(10) $\because 1 \le F_1 \le 9 \quad , \quad 3 \le F_2 \le 7$

$\therefore 4 \le F_1 + F_2 \le 16 \qquad (1)$

$\because 1 \le F_1 \le 9 \quad , \quad -7 \le -F_2 \le -3$

$\therefore -6 \le F_1 - F_2 \le 6 \qquad \therefore 0 \le |F_1 - F_2| \le 6 \quad (2)$

From (1) , (2) : $0 \le R \le 16$

(11) $\because 5 \le F_1 \le 20 \quad , \quad 12 \le F_2 \le 21$

$\therefore 17 \le F_1 + F_2 \le 41$

When $\theta = \frac{\pi}{2}$

$, (5)^2 + (12)^2 \le F_1^2 + F_2^2 \le (20)^2 + (21)^2$

$\therefore 169 \le F_1^2 + F_2^2 \le 841$

$\therefore 13 \le \sqrt{F_1^2 + F_2^2} \le 29$

$\therefore 13 \le R \le 41$ when $0 \le \theta \le \frac{\pi}{2}$

2

In the first case :

Let the two forces be F and 2 F and the measure of the angle between them = α

$\therefore \tan\theta = \frac{2\,F \sin\alpha}{F + 2\,F\cos\alpha} \; , \therefore \tan\theta = \frac{2\sin\alpha}{1 + 2\cos\alpha} \qquad (1)$

In the second case :

The small force = F + 4 and the great force = 4 F

$\therefore \tan\theta = \frac{4\,F\sin\alpha}{F + 4 + 4\,F\cos\alpha} \qquad (2)$

From (1) and (2) :

$\therefore \frac{2\sin\alpha}{1 + 2\cos\alpha} = \frac{4\,F\sin\alpha}{F + 4 + 4\,F\cos\alpha}$

$\therefore \frac{1}{1 + 2\cos\alpha} = \frac{2\,F}{F + 4 + 4\,F\cos\alpha}$

$\therefore F + 4 + 4\,F\cos\alpha = 2\,F + 4\,F\cos\alpha$

$\therefore F + 4 = 2\,F \qquad \therefore F = 4$ kg.wt.

$\therefore$ **In the first case :**

The magnitude of the first force is 4 kg.wt.

, the magnitude of the second force is 8 kg.wt.

$\therefore R_1 = \sqrt{16 + 64 + 2 \times 4 \times 8\cos\alpha} = 4\sqrt{5 + 4\cos\alpha}$

, in the second case :

The magnitude of the first force is 8 kg.wt.

, the magnitude of the second force is 16 kg.wt.

$, R_2 = \sqrt{64 + 256 + 2 \times 8 \times 16\cos\alpha} = 8\sqrt{5 + 4\cos\alpha}$

$\therefore \frac{R_1}{R_2} = \frac{4\sqrt{5 + 4\cos\alpha}}{8\sqrt{5 + 4\cos\alpha}} = \frac{1}{2}$

3

In the first case :

$R^2 = F_1^2 + F_2^2 + 2\,F_1\,F_2\cos\alpha \qquad (1)$

In the second case :

$3\,R^2 = F_1^2 + F_2^2 + 2\,F_1\,F_2\cos(180° - \alpha)$

$\therefore 3\,R^2 = F_1^2 + F_2^2 - 2\,F_1\,F_2\cos\alpha \qquad (2)$

multiplying (1) × 3 , then subtracting (2) from (1) :

$\therefore$ zero $= 2\,F_1^2 + 2\,F_2^2 + 8\,F_1\,F_2\cos\alpha$

$\therefore \cos\alpha = \frac{-F_1^2 - F_2^2}{4\,F_1\,F_2} \qquad (3)$

Let θ_1 is inclination angle of R on F_1

$\therefore \tan\theta_1 = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$

Let θ_2 is the inclination angle of $R\sqrt{3}$ on F_1

$\therefore \tan\theta_2 = \frac{F_2\sin(180° - \alpha)}{F_1 + F_2\cos(180° - \alpha)}$

$\tan\theta_2 = \frac{F_2\sin\alpha}{F_1 - F_2\cos\alpha}$

$\because \theta_1 + \theta_2 = 90° \qquad \therefore \tan\theta_1 = \cot\theta_2$

$\therefore \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha} = \frac{F_1 - F_2\cos\alpha}{F_2\sin\alpha}$

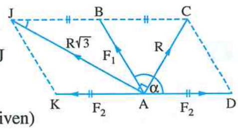
$\therefore F_2^2\sin^2\alpha = F_1^2 - F_2^2\cos^2\alpha$

$\therefore F_2^2(\sin^2\alpha + \cos^2\alpha) = F_1^2$

$\therefore F_2^2 = F_1^2 \qquad \therefore F_2 = F_1$

Substituting in (3) : $\therefore \cos\alpha = \frac{-2\,F_1^2}{4\,F_1^2} = -\frac{1}{2}$

$\therefore \alpha = 120°$



Another solution :

$\because AD = AK = BC = BJ$

In Δ ACJ :

$\because m(\angle CAJ) = 90°$ (given)

and $\overline{AB}$ is a median from the vertex of the right angle

$\therefore AB = \frac{1}{2}JC = BJ = BC \qquad \therefore F_1 = F_2$

$\tan(\angle CJA) = \frac{R}{R\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore m(\angle CJA) = 30° \qquad \therefore m(\angle ACB) = 60°$

$\therefore \Delta$ ABC is equilateral triangle

$\therefore m(\angle ACB) = m(\angle CAB) = m(\angle CAD) = 60°$

$\therefore \alpha = 120°$

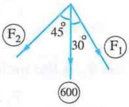
Exercise 2

First Multiple choice questions

(1) c	(2) d	(3) d	(4) d	(5) c
(6) a	(7) c	(8) b	(9) b	(10) b
(11) a	(12) c	(13) b	(14) a	(15) d
(16) c	(17) c	(18) c	(19) c	(20) a
(21) a	(22) b	(23) c	(24) b	(25) b
(26) b	(27) c	(28) c	(29) c	(30) d

Second Essay questions

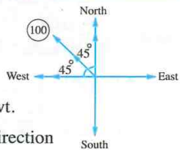
1



$F_1 = \frac{600 \sin 45°}{\sin 75°} \simeq 439.23$ gm.wt.

, $F_2 = \frac{600 \sin 30°}{\sin 75°} \simeq 310.68$ gm.wt.

2



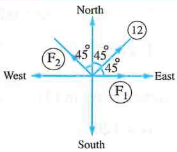
The component in the North direction $= 100 \sin 45°$

$= 50\sqrt{2}$ gm.wt.

The component in the West direction

$= 100 \cos 45° = 50\sqrt{2}$ gm.wt.

3

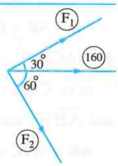


$F_1 = \frac{12 \sin 90°}{\sin 135°} = 12\sqrt{2}$ kg.wt.

$F_2 = \frac{12 \sin 45°}{\sin 135°}$

$= 12$ kg.wt.

4



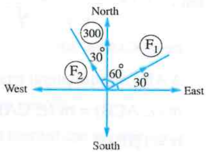
$F_1 = 160 \cos 30°$

$= 80\sqrt{3}$ gm.wt.

$F_2 = 160 \sin 30°$

$= 80$ gm.wt.

5



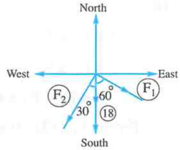
$F_1 = 300 \cos 60°$

$= 150$ dyne

$F_2 = 300 \sin 60°$

$= 150\sqrt{3}$ dyne

6



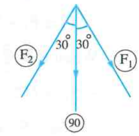
$F_1 = 18 \cos 60°$

$= 9$ newton

$F_2 = 18 \sin 60°$

$= 9\sqrt{3}$ newton

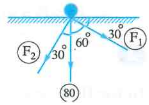
7



$F_1 = F_2 = \frac{90 \sin 30°}{\sin 60°}$

$= 30\sqrt{3}$ newton

8

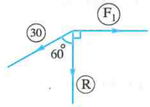


$F_1 = 80 \cos 60°$

$= 40$ newton

$F_2 = 80 \cos 30° = 40\sqrt{3}$ newton

9



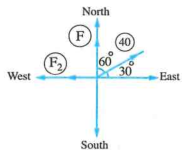
$\because \tan \alpha = -\frac{1}{\sqrt{3}}$ $\therefore \alpha = 150°$

$\because 30 = \frac{R \sin 90°}{\sin 150°}$

$\therefore R = \frac{30 \sin 150°}{\sin 90°} = 15$ newton

$F_1 = \frac{15 \sin 60°}{\sin 150°} = 15\sqrt{3}$ newton

10



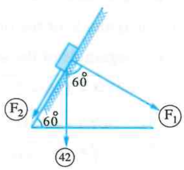
$\because F_1 = \frac{F \sin 90°}{\sin 150°}$

$\therefore F = \frac{40 \sin 150°}{\sin 90°}$

$= 20$ newton

$F_2 = \frac{20 \sin 60°}{\sin 150°} = 20\sqrt{3}$ newton

11



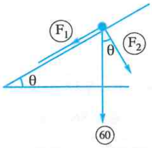
$F_1 = 42 \cos 60°$

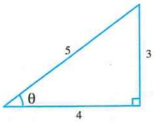
$= 21$ newton

$F_2 = 42 \sin 60°$

$= 21\sqrt{3}$ newton

12





$F_1 = 60 \sin \theta = 60 \times \frac{3}{5} = 36$ newton

$F_2 = 60 \cos \theta = 60 \times \frac{4}{5} = 48$ newton

13

$F_1 = \dfrac{120 \sin 48^\circ}{\sin (48^\circ + 90^\circ)} \simeq 133.27$ gm.wt.

$F_2 = \dfrac{120 \sin 90^\circ}{\sin (48^\circ + 90^\circ)} \simeq 179.34$ gm.wt.

14

$(30)^2 = \left(15\sqrt{3}\right)^2 + (F_2)^2$

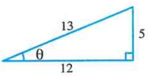
$F_2 = 15$ newton

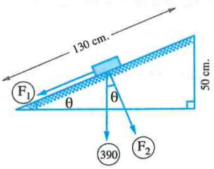
15

$w_1 = w_2 = \dfrac{20 \sin 85^\circ}{\sin (85^\circ + 85^\circ)} \simeq 114.74$ newton

when the angle with the horizontal decreases less than 5° the magnitude of the component of the weight in the direction of the two rods increases till it became unlimited when the rods are horizontal.

16





$F_1 = 390 \sin \theta = 390 \times \frac{5}{13} = 150$ gm.wt.

$F_2 = 390 \cos \theta = 390 \times \frac{12}{13} = 360$ gm.wt.

17

T_1 in direction of $\overrightarrow{AB}$

$\therefore T_1 = \dfrac{5000 \times \sin 30^\circ}{\sin 75^\circ} = 2588.2$ N

, T_2 in direction of $\overrightarrow{AC}$

$\therefore T_2 = \dfrac{5000 \times \sin 45^\circ}{\sin 75^\circ} = 3660.3$ N

Exercise 3

Important remark :

We will solve the problems of this exercise using the polar angles, but it is possible use the resolution of the forces into two perpendicular directions.

First Multiple choice questions

(1) c	(2) b	(3) c	(4) d	(5) d
(6) a	(7) b	(8) c	(9) a	(10) c
(11) d	(12) c	(13) d	(14) d	
(15) First : b		Second : c	(16) b	(17) c
(18) d	(19) a	(20) a	(21) c	(22) b
(23) a	(24) b			

Second Essay questions

1

(1) $X = 27 \cos 0^\circ + 18 \cos 90^\circ + 12\sqrt{2} \cos 135^\circ + 15\sqrt{2} \cos 225^\circ + 9 \cos 270^\circ$

$= 27 \times 1 + 18 \times 0 + 12\sqrt{2} \times \frac{-1}{\sqrt{2}} + 15 \times \frac{-1}{\sqrt{2}} + 9 \times 0 =$ zero

, $Y = 27 \sin 0^\circ + 18 \sin 90^\circ + 12\sqrt{2} \sin 135^\circ + 15\sqrt{2} \sin 225^\circ + 9 \sin 270^\circ$

$= 27 \times 0 + 18 \times 1 + 12\sqrt{2} \times \frac{1}{\sqrt{2}} + 15\sqrt{2} \times \frac{-1}{\sqrt{2}} + 9 \times -1 = 6$

$\therefore \overrightarrow{R} = 6\,\overrightarrow{j}$ $\qquad \therefore R = 6$ newton

$\therefore$ The magnitude of $\overrightarrow{R} = 6$ newton and acts in the direction of $\overrightarrow{OY}$

(2) $X = 4 \cos 60^\circ + 3\sqrt{3} \cos 270^\circ + 2\sqrt{3} \cos 330^\circ$

$= 4 \times \frac{1}{2} + 3\sqrt{3} \times 0 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 5$

, $Y = 4 \sin 60^\circ + 3\sqrt{3} \sin 270^\circ + 2\sqrt{3} \sin 330^\circ$

$= 4 \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times -1 + 2\sqrt{3} \times \frac{-1}{2} = -2\sqrt{3}$

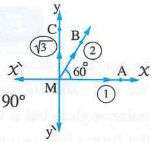
$\therefore \overrightarrow{R} = 5\,\overrightarrow{i} - 2\sqrt{3}\,\overrightarrow{j}$

$\therefore R = \sqrt{(5)^2 + \left(-2\sqrt{3}\right)^2} = \sqrt{37}$ newton

, $\tan \theta = \dfrac{-2\sqrt{3}}{5}$ $\qquad \therefore \theta \simeq 325^\circ\ 17'$

$\therefore$ The magnitude of $\overrightarrow{R} = \sqrt{37}$ newton and makes an angle of measure $325^\circ\ 17'$ with $\overrightarrow{OX}$

2



Consider $\overrightarrow{OX}$ is the direction of the first force.

$X = 1 \times \cos 0° + 2 \cos 60° + \sqrt{3} \cos 90°$

$= 1 \times 1 + 2 \times \frac{1}{2} + \sqrt{3} \times 0 = 2$

$Y = 1 \times \sin 0° + 2 \times \sin 60° + \sqrt{3} \sin 90°$

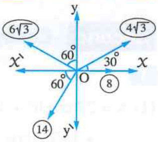
$= 1 \times 0 + 2 \times \frac{\sqrt{3}}{2} + \sqrt{3} \times 1 = 2\sqrt{3}$

$\therefore \vec{R} = 2\hat{i} + 2\sqrt{3}\,\hat{j}$, $R = \sqrt{(2)^2 + \left(2\sqrt{3}\right)^2} = 4$ newton

, $\tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$ $\qquad \therefore \theta = 60°$

$\therefore$ The magnitude of $\vec{R} = 4$ newton and its direction is $\overrightarrow{MB}$

3



Suppose $\overrightarrow{OX}$ is the direction of the first force

$X = 8 \cos 0° + 4\sqrt{3} \cos 30°$

$+ 6\sqrt{3} \cos 150° + 14 \cos 240°$

$= 8 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 6\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) + 14 \times \left(-\frac{1}{2}\right)$

$= 8 + 6 - 9 - 7 = -2$

, $Y = 8 \sin 0° + 4\sqrt{3} \sin 30°$

$+ 6\sqrt{3} \sin 150° + 14 \sin 240°$

$= 8 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 6\sqrt{3} \times \frac{1}{2} + 14 \times \left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$\therefore \vec{R} = -2\,\hat{i} - 2\sqrt{3}\,\hat{j}$

$\therefore R = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = 4$ newton

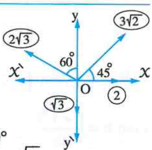
, $\tan \theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

$\because$ X and Y are negative $\qquad \therefore \theta = 240°$

$\therefore$ The magnitude of the resultant is 4 newton and makes an angle of measure 240° with $\overrightarrow{OX}$

i.e. In direction of 4th force

4



Consider $\overrightarrow{OX}$ be the direction of first force

$X = 2 \cos 0° + 3\sqrt{2} \cos 45°$

$+ 2\sqrt{3} \cos 150° + \sqrt{3} \cos 270°$

$= 2 \times 1 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times -\frac{\sqrt{3}}{2} + \sqrt{3} \times 0 = 2$

, $Y = 2 \sin 0° + 3\sqrt{2} \sin 45° + 2\sqrt{3} \sin 150° + \sqrt{3} \sin 270°$

$= 2 \times 0 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \frac{1}{2} + \sqrt{3} \times -1 = 3$

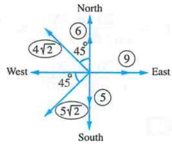
$\because \vec{R} = 2\hat{i} + 3\hat{j}$, $R = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$ newton

, $\tan \theta = \frac{3}{2}$ $\qquad \therefore \theta \simeq 56° 19'$

$\therefore$ The magnitude of $\vec{R} = \sqrt{13}$ netwon and makes an angle of measure $56° 19'$ with $\overrightarrow{OX}$

i.e. Between 2nd and 3rd forces and makes an angle of measure $11° 19'$ with 2nd force

5



$X = 9 \cos 0° + 6 \cos 90°$

$+ 4\sqrt{2} \cos 135°$

$+ 5\sqrt{2} \cos 225°$

$+ 5 \cos 270°$

$= 9 \times 1 + 6 \times 0 + 4\sqrt{2} \times -\frac{1}{\sqrt{2}} + 5\sqrt{2} \times -\frac{1}{\sqrt{2}}$

$+ 5 \times 0 =$ zero

, $Y = 9 \sin 0° + 6 \sin 90° + 4\sqrt{2} \sin 135° + 5\sqrt{2} \sin 225°$

$+ 5 \sin 270°$

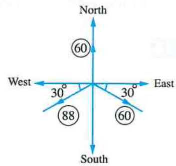
$= 9 \times 0 + 6 \times 1 + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 5\sqrt{2} \times -\frac{1}{\sqrt{2}} + 5 \times -1$

$= 0$

$\therefore \vec{R} = \vec{O}$

$\therefore$ The forces are in equilibrium.

6



$X = 60 \cos 90° + 88 \cos 210°$

$+ 60 \cos 330°$

$= 60 \times 0 + 88 \times -\frac{\sqrt{3}}{2} + 60$

$\times \frac{\sqrt{3}}{2} = -14\sqrt{3}$

$Y = 60 \sin 90° + 88 \sin 210° + 60 \sin 330°$

$= 60 \times 1 + 88 \times -\frac{1}{2} + 60 \times -\frac{1}{2} = -14$

$\vec{R} = -14\sqrt{3}\,\hat{i} - 14\,\hat{j}$

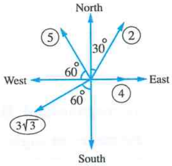
$\therefore R = \sqrt{\left(-14\sqrt{3}\right)^2 + (-14)^2} = 28$ gm.wt.

$\tan \theta = \frac{-14}{-14\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\because$ X and Y are both negative. $\qquad \therefore \theta = 210°$

$\therefore$ The magnitude of $\vec{R} = 28$ gm.wt. and acts in the direction 30° South of the West.

7



$\because X = 4\cos 0° + 2\cos 60° + 5\cos 120° + 3\sqrt{3}\cos 210°$

$= 4 \times 1 + 2 \times \frac{1}{2} + 5 \times -\frac{1}{2} + 3\sqrt{3} \times -\frac{\sqrt{3}}{2} = -2$

, $Y = 4\sin 0° + 2\sin 60° + 5\sin 120° + 3\sqrt{3}\sin 210°$

$= 4 \times 0 + 2 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times -\frac{1}{2} = 2\sqrt{3}$

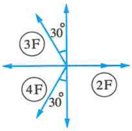
$\vec{R} = -2\,\vec{i} + 2\sqrt{3}\,\vec{j}$

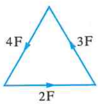
$\therefore R = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2} = 4$ newton

, $\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\because X < 0$, $Y > 0$ $\qquad \therefore \theta = 120°$

8





Suppose $\overrightarrow{OX}$ in the direction of the 1st force.

$X = 2F\cos 0° + 3F\cos 120° + 4F\cos 240°$

$= 2F \times 1 + 3F \times -\frac{1}{2} + 4F \times -\frac{1}{2} = -\frac{3}{2}F$

, $Y = 2F\sin 0° + 3F\sin 120° + 4F\sin 240°$

$= 2F \times 0 + 3F \times \frac{\sqrt{3}}{2} + 4F \times -\frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{2}F$

$\therefore \vec{R} = -\frac{3}{2}F\,\vec{i} - \frac{\sqrt{3}}{2}F\,\vec{j}$

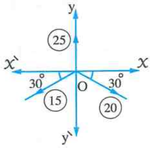
$\therefore R = \sqrt{\left(-\frac{3}{2}F\right)^2 + \left(-\frac{\sqrt{3}}{2}F\right)^2} = \sqrt{3}\,F$ newton

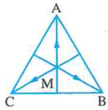
$\tan\theta = \frac{-\sqrt{3}}{2} \times \frac{-2}{3} = \frac{\sqrt{3}}{3}$ $\qquad \therefore \theta = 210°$

$\therefore$ The magnitude of the resultant is $\sqrt{3}\,F$ newton and its direction makes an angle of measure 210° with $\overrightarrow{OX}$

i.e. Between the forces 3 F , 4 F and perpendicular to the force 3 F

9





Let $\overrightarrow{OY}$ is the direction of the 3rd force

$\therefore X = 25\cos 90° + 15\cos 210° + 20\cos 330°$

$= 25 \times 0 + 15 \times -\frac{\sqrt{3}}{2} + 20 \times \frac{\sqrt{3}}{2} = \frac{5}{2}\sqrt{3}$

$Y = 25\sin 90° + 15\sin 210° + 20\sin 330°$

$= 25 \times 1 + 15 \times -\frac{1}{2} + 20 \times -\frac{1}{2} = \frac{15}{2}$

$\therefore \vec{R} = \frac{5}{2}\sqrt{3}\,\vec{i} + \frac{15}{2}\,\vec{j}$

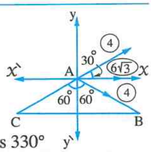
$\therefore R = \sqrt{\left(2.5\sqrt{3}\right)^2 + (7.5)^2} = 5\sqrt{3}$ newton

, $\tan\theta = \frac{7.5}{2.5\sqrt{3}} = \sqrt{3}$ $\qquad \therefore \theta = 60°$

$\therefore$ The resultant $= 5\sqrt{3}$ newton in magnitude and makes an angle of measure 60° with $\overrightarrow{OX}$

i.e. Between $\overrightarrow{MA}$, $\overrightarrow{MB}$ making an angle of measure 30° with $\overrightarrow{MA}$

10



Let $\overleftrightarrow{YY}$ is the axis of symmetry of Δ ABC and the point A Coincides the origin O

$X = 6\sqrt{3}\cos 0° + 4\cos 30° + 4\cos 330°$

$= 6\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2} + 4 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

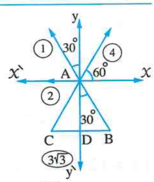
, $Y = 6\sqrt{3}\sin 0° + 4\sin 30° + 4\sin 330°$

$= 6\sqrt{3} \times 0 + 4 \times \frac{1}{2} + 4 \times -\frac{1}{2} =$ zero

$\therefore \vec{R} = 10\sqrt{3}\,\vec{i}$

$\therefore R = 10\sqrt{3}$ newton and acts due to $\overrightarrow{OX}$ *i.e.* $\overrightarrow{CB}$

11



Let $\overleftrightarrow{YY}$ is the symmetric axis of Δ ABC , point A coincides on the origin O

$\therefore X = 4\cos 60° + 1 \times \cos 120° + 2\cos 180° + 3\sqrt{3}\cos 270°$

$= 4 \times \frac{1}{2} + 1 \times \left(-\frac{1}{2}\right) + 2 \times -1 + 3\sqrt{3} \times 0 = -\frac{1}{2}$

$, Y = 4\sin 60° + 1 \times \sin 120° + 2\sin 180° + 3\sqrt{3}\sin 270°$

$= 4 \times \frac{\sqrt{3}}{2} + 1 \times \frac{\sqrt{3}}{2} + 2 \times 0 + 3\sqrt{3} \times -1 = -\frac{\sqrt{3}}{2}$

$\therefore \overrightarrow{R} = -\frac{1}{2}\overrightarrow{i} - \frac{\sqrt{3}}{2}\overrightarrow{j}$

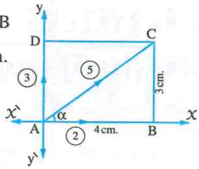
$\therefore R = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = 1$ newton

$\tan\theta = -\frac{\sqrt{3}}{2} \times -2 = \sqrt{3}$

$\because X < 0, Y < 0 \qquad \therefore \theta = 240°$

$\therefore$ The magnitude of $\overrightarrow{R} = 1$ newton and acts due to $\overrightarrow{AC}$

12



$\because \Delta ABC$ is right-angled at B

$\therefore AC = \sqrt{(3)^2 + (4)^2} = 5$ cm.

$\therefore \cos\alpha = \frac{4}{5}, \sin\alpha = \frac{3}{5}$

$X = 2\cos 0° + 5\cos\alpha + 3\cos 90°$

$= 2 \times 1 + 5 \times \frac{4}{5} + 3 \times \text{zero} = 6$

$, Y = 2\sin 0° + 5\sin\alpha + 3\sin 90°$

$= 2 \times 0 + 5 \times \frac{3}{5} + 3 \times 1 = 6$

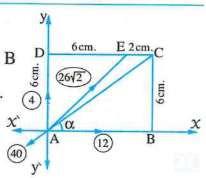
$\therefore \overrightarrow{R} = 6\overrightarrow{i} + 6\overrightarrow{j}$

$\therefore R = \sqrt{(6)^2 + (6)^2} = 6\sqrt{2}$ kg.wt.

$, \tan\theta = \frac{6}{6} = 1 \qquad \therefore \theta = 45°$

$\therefore$ The resultant is of magnitude $6\sqrt{2}$ kg.wt. and makes an angle of measure 45° with $\overrightarrow{AB}$

13



$\because \Delta ABC$ is right-angled at B

$\therefore AC = \sqrt{8^2 + 6^2} = 10$ cm.

$\therefore \sin\alpha = \frac{6}{10} = \frac{3}{5}$

$, \cos\alpha = \frac{4}{5}$

$\therefore X = 12\cos 0° + 26\sqrt{2}\cos 45° + 4\cos 90° + 40\cos(180° + \alpha)$

$= 12 \times 1 + 26\sqrt{2} \times \frac{1}{\sqrt{2}} + 4 \times 0 + 40 \times \frac{-4}{5} = 6$

$, Y = 12\sin 0° + 26\sqrt{2}\sin 45° + 4\sin 90° + 40\sin(180° + \alpha)$

$= 12 \times 0 + 26\sqrt{2} \times \frac{1}{\sqrt{2}} + 4 \times 1 + 40 \times \frac{-3}{5} = 6$

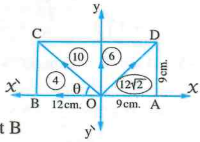
$\therefore \overrightarrow{R} = 6\overrightarrow{i} + 6\overrightarrow{j}$

$\therefore R = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ newton

$, \tan\theta = 1 \qquad \therefore \theta = 45°$

$\therefore$ The magnitude of $\overrightarrow{R} = 6\sqrt{2}$ newton and makes an angle of measure 45° with $\overrightarrow{AB}$

14



$\because \Delta ADO$ is an isosceles and right-angled at A

$\therefore m(\angle AOD) = 45°$

$\because \Delta OBC$ is right-angled at B

$\therefore OC = \sqrt{(12)^2 + (9)^2} = 15$ cm.

$\therefore \cos\theta = \frac{12}{15} = \frac{4}{5}, \sin\theta = \frac{9}{15} = \frac{3}{5}$

$\therefore X = 12\sqrt{2} \times \cos 45° + 6\cos 90° + 10\cos(180° - \theta) + 4\cos 180°$

$= 12\sqrt{2} \times \frac{1}{\sqrt{2}} + 6 \times \text{zero} + 10 \times -\frac{4}{5} + 4 \times -1$

$=$ zero

$, Y = 12\sqrt{2}\sin 45° + 6\sin 90° + 10\sin(180° - \theta) + 4\sin 180°$

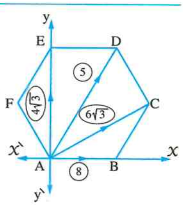
$= 12\sqrt{2} \times \frac{1}{\sqrt{2}} + 6 \times 1 + 10 \times \frac{3}{5} + 4 \times 0 = 24$

$\therefore \overrightarrow{R} = 24\overrightarrow{j} \qquad \therefore R = 24$ kg.wt.

The magnitude of the resultant = 24 kg.wt. and acts in the direction $\overrightarrow{OY}$

i.e. In the direction of $\overrightarrow{BC}$

15



$X = 8\cos 0° + 6\sqrt{3}\cos 30° + 5\cos 60° + 4\sqrt{3}\cos 90°$

$= 8 \times 1 + 6\sqrt{3} \times \frac{\sqrt{3}}{2} + 5 \times \frac{1}{2} + 4\sqrt{3} \times 0 = \frac{39}{2}$

$, Y = 8\sin 0° + 6\sqrt{3}\sin 30° + 5\sin 60° + 4\sqrt{3}\sin 90°$

$= 8 \times 0 + 6\sqrt{3} \times \frac{1}{2} + 5 \times \frac{\sqrt{3}}{2} + 4\sqrt{3} \times 1 = \frac{19\sqrt{3}}{2}$

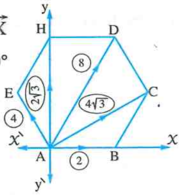
$\therefore \overrightarrow{R} = \frac{39}{2}\overrightarrow{i} + \frac{19\sqrt{3}}{2}\overrightarrow{j}$

$\therefore R = \sqrt{\left(\frac{39}{2}\right)^2 + \left(\frac{19\sqrt{3}}{2}\right)^2} = \sqrt{651}$ newton

$\tan\theta = \frac{19}{39}\sqrt{3} \qquad \therefore \theta \simeq 40° 9'$

i.e. Resultant direction is between $\overrightarrow{AC}$ and $\overrightarrow{AD}$ making an angle of measure $40° 9'$ with $\overrightarrow{AB}$

16



Let $\overrightarrow{AB}$ in the direction of $\overrightarrow{OX}$

$\therefore X = 2\cos 0° + 4\sqrt{3}\cos 30° + 8\cos 60° + 2\sqrt{3}\cos 90° + 4\cos 120°$

$= 2\times 1 + 4\sqrt{3}\times\frac{\sqrt{3}}{2} + 8\times\frac{1}{2} + 2\sqrt{3}\times 0 + 4\times\frac{-1}{2} = 10$

$, Y = 2\sin 0° + 4\sqrt{3}\sin 30° + 8\sin 60° + 2\sqrt{3}\sin 90° + 4\sin 120°$

$= 2\times 0 + 4\sqrt{3}\times\frac{1}{2} + 8\times\frac{\sqrt{3}}{2} + 2\sqrt{3}\times 1 + 4\times\frac{\sqrt{3}}{2}$

$= 10\sqrt{3}$

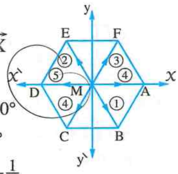
$\therefore \vec{R} = 10\,\vec{i} + 10\sqrt{3}\,\vec{j}$

$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20$ kg.wt.

$, \tan\theta = \frac{10\sqrt{3}}{2} = \sqrt{3}$ $\qquad \therefore \theta = 60°$

$\therefore$ The magnitude of $\vec{R} = 20$ kg.wt. and makes an angle of measure $60°$ with $\overrightarrow{AB}$

17



Let $\overrightarrow{OA}$ in the direction of $\overrightarrow{OX}$

$\therefore X = 4\cos 0° + 3\cos 60° + 2\cos 120° + 5\cos 180° + 4\cos 240° + \cos 300°$

$= 4\times 1 + 3\times\frac{1}{2} + 2\times -\frac{1}{2} + 5\times -1 + 4\times -\frac{1}{2} + \frac{1}{2} = -2$

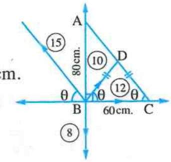
$, Y = 4\sin 0° + 3\sin 60° + 2\sin 120° + 5\sin 180° + 4\sin 240° + \sin 300° = 4\times 0 + 3\times\frac{\sqrt{3}}{2} + 2\times\frac{\sqrt{3}}{2} + 5\times 0 + 4\times -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} =$ zero

$\therefore \vec{R} = -2\,\vec{i}$

$\therefore$ The magnitude of the resultant = 2 gm.wt. and acts due to $\overrightarrow{OX`}$

i.e. In the direction of $\overrightarrow{OD}$

18



$\therefore$ In Δ ABC which is right-angled at B

$\therefore AC = \sqrt{(60)^2 + (80)^2} = 100$ cm.

$\therefore \sin\theta = \frac{80}{100} = \frac{4}{5}$

$, \cos\theta = \frac{60}{100} = \frac{3}{5}$ $, \tan\theta = \frac{4}{3}$

$\therefore X = 12\cos 0° + 10\cos\theta + 15\cos(180° - \theta) + 8\cos 270°$

$= 12\times 1 + 10\times\frac{3}{5} + 15\times\frac{-3}{5} + 8\times 0 = 9$

$, Y = 12\sin 0° + 10\sin\theta + 15\sin(180° - \theta) + 8\sin 270°$

$= 12\times 0 + 10\times\frac{4}{5} + 15\times\frac{4}{5} + 8\times -1 = 12$

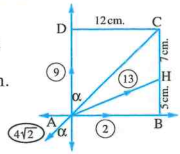
$\therefore \vec{R} = 9\,\vec{i} + 12\,\vec{j}$

$\therefore R = \sqrt{(9)^2 + (12)^2} = 15$ newton $\quad , \tan\alpha = \frac{12}{9} = \frac{4}{3}$

$\because \tan\alpha = \tan\theta , X > 0 , Y > 0$

$\therefore$ The resultant acts in the direction of $\overrightarrow{BD}$

19



$\because \Delta$ AHB is right-angled at B

$\therefore AH = \sqrt{(5)^2 + (12)^2} = 13$ cm.

$\therefore \sin(\angle BAH) = \frac{5}{13}$

$, \cos(\angle BAH) = \frac{12}{13}$

$\because \overline{AC}$ is a diagonal of the square ABCD $\quad \therefore \alpha = 45°$

$\therefore X = 2\cos 0° + 13\cos(\angle BAH) + 9\cos 90° + 4\sqrt{2}\cos 225°$

$= 2\times 1 + 13\times\frac{12}{13} + 9\times 0 + 4\sqrt{2}\times\frac{-1}{\sqrt{2}} = 10$

$, Y = 2\sin 0° + 13\sin(\angle BAH) + 9\sin 90° + 4\sqrt{2}\sin 225°$

$= 2\times 0 + 13\times\frac{5}{13} + 9\times 1 + 4\sqrt{2}\times\frac{-1}{\sqrt{2}} = 10$

$\therefore \vec{R} = 10\,\vec{i} + 10\,\vec{j}$

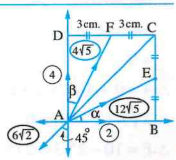
$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$ gm.wt.

$, \tan\theta = \frac{10}{10} = 1$

$\because X > 0 , Y > 0$ $\qquad \therefore \theta = 45°$

$\therefore \vec{R}$ acts due to $\overrightarrow{AC}$

20



$\because \overline{AC}$ is a diagonal in the square

$\therefore m(\angle CAD) = 45°$

$, \because \Delta$ ABE is right-angled at B

$\therefore AE = \sqrt{(6)^2 + (3)^2} = 3\sqrt{5}$ cm.

$\therefore \sin \alpha = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}}$, $\cos \alpha = \frac{6}{3\sqrt{2}} = \frac{2}{\sqrt{5}}$

, in Δ AFD which is right-angled at D

$AF = \sqrt{(3)^2 + (6)^2} = 3\sqrt{5}$ cm.

$\therefore \sin \beta = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}}$, $\cos \beta = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}}$

$\therefore X = 2 \cos 0° + 12\sqrt{5} \cos \alpha + 4\sqrt{5} \cos (90° - \beta)$

$+ 4 \cos 90° + 6\sqrt{2} \cos 225°$

$= 2 \times 1 + 12\sqrt{5} \times \frac{2}{\sqrt{5}} + 4\sqrt{5} \times \frac{1}{\sqrt{5}}$

$+ 4 \times 0 + 6\sqrt{2} \times \frac{-1}{\sqrt{2}} = 24$

, $Y = 2 \sin 0° + 12\sqrt{5} \sin \alpha + 4\sqrt{5} \sin (90° - \alpha)$

$+ 4 \sin 90° + 6\sqrt{2} \sin 225°$

$= 2 \times 0 + 12\sqrt{5} \times \frac{1}{\sqrt{5}} + 4\sqrt{5} \times \frac{2}{\sqrt{5}} + 4 \times 1$

$+ 6\sqrt{2} \times -\frac{1}{\sqrt{2}} = 18$

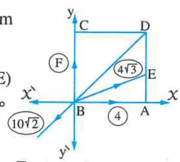
$\therefore \vec{R} = 24\,\vec{i} + 18\,\vec{j}$

$\because R = \sqrt{(24)^2 + (18)^2} = 30$ kg.wt. , $\tan \theta = \frac{18}{24} = \frac{3}{4}$

$\because X > 0$, $Y > 0$ $\qquad \therefore \theta \simeq 36° \, 52' \, 12''$

$\therefore$ The magnitude of resultant is 30 kg.wt. and makes an angle of measure $36° \, 52' \, 12''$ with $\overrightarrow{AB}$

21



$\because$ The forces are in equilibrium

$\therefore X = 0$, $Y = 0$

$\therefore 4 \cos 0° + 4\sqrt{3} \cos (\angle ABE)$

$+ F \cos 90° + 10\sqrt{2} \cos 225°$

$= 0$

$\therefore 4 \times 1 + 4\sqrt{3} \cos (\angle ABE) + F \times$ zero

$+ 10\sqrt{2} \times \frac{-1}{\sqrt{2}} = 0$

$\therefore 4 + 4\sqrt{3} \cos (\angle ABE) - 10 = 0$

$\therefore \cos (\angle ABE) = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore m (\angle ABE) = 30°$

, $4 \sin 0° + 4\sqrt{3} \sin 30° + F \sin 90° + 10\sqrt{2} \sin 225°$

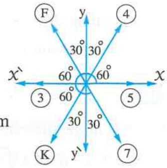
$= 0$

$\therefore 4 \times 0 + 4\sqrt{3} \times \frac{1}{2} + F \times 1 + 10\sqrt{2} \times \frac{-1}{\sqrt{2}} = 0$

$\therefore 2\sqrt{3} + F - 10 = 0$

$\therefore F = 10 - 2\sqrt{3} = 2 (5 - \sqrt{3})$ kg.wt.

22



$\because$ Let the force of magnitude 5 kg.wt acts in the direction of $\overrightarrow{OX}$

, $\because$ the forces are in equilibrium

$\therefore X = Y = 0$

$\therefore X = 5 \cos 0° + 4 \cos 60° + F \cos 120°$

$+ 3 \cos 180° + K \cos 240° + 7 \cos 300 = 0$

$\therefore 5 \times 1 + 4 \times \frac{1}{2} + F \times \frac{-1}{2} + 3 \times -1 + K \times \frac{-1}{2} + 7 \times \frac{1}{2} = 0$

$\therefore F + K = 15$ (1)

, $Y = 5 \sin 0° + 4 \sin 60° + F \sin 120° + 3 \sin 180°$

$+ K \sin 240° + 7 \sin 300 = 0$

$\therefore 5 \times 0 + 4 \times \frac{\sqrt{3}}{2} + F \times \frac{\sqrt{3}}{2} + 3 \times 0 + K \times \frac{-\sqrt{3}}{2}$

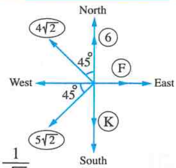
$+ 7 \times \frac{-\sqrt{3}}{2} = 0$

$\therefore F - K = 3$ (2)

by solving the two equations (1) , (2)

$\therefore F = 9$ kg.wt. $K = 6$ kg.wt.

23



$\because X = F \cos 0° + 6 \cos 90°$

$+ 4\sqrt{2} \cos 135°$

$+ 5\sqrt{2} \cos 225°$

$+ K \cos 270°$

$= F \times 1 + 6 \times 0 + 4\sqrt{2} \times -\frac{1}{\sqrt{2}}$

$+ 5\sqrt{2} \times -\frac{1}{\sqrt{2}} + K \times 0$

$= F - 9$

, $Y = F \sin 0° + 6 \sin 90° + 4\sqrt{2} \times \sin 135°$

$+ 5\sqrt{2} \sin 225° + K \sin 270°$

$= F \times 0 + 6 \times 1 + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 5\sqrt{2} \times -\frac{1}{\sqrt{2}} + K \times -1$

$= 5 - K$

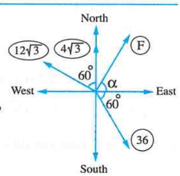
$\therefore \vec{R} = (F - 9)\,\vec{i} + (5 - K)\,\vec{j}$

$\because$ The resultant = 2 newton in the direction of the North

$\therefore F - 9 = 0$ $\qquad \therefore F = 9$ newton

, $5 - K = 2$ $\qquad \therefore K = 3$ newton

24



Let the measure of the polar angle of $\vec{F}$ be α

$\therefore X = F \cos \alpha + 4\sqrt{3} \cos 90°$

$+ 12\sqrt{3} \cos 150°$

$+ 36 \cos 300°$

$= F \cos \alpha + 4\sqrt{3} \times 0 + 12\sqrt{3} \times -\frac{\sqrt{3}}{2} + 36 \times \frac{1}{2}$

$= F \cos \alpha$

$, Y = F \sin \alpha + 4\sqrt{3} \sin 90° + 12\sqrt{3} \sin 150° + 36 \sin 300°$

$= F \sin \alpha + 4\sqrt{3} \times 1 + 12\sqrt{3} \times \frac{1}{2} + 36 \times -\frac{\sqrt{3}}{2}$

$= F \sin \alpha - 8\sqrt{3}$

$\therefore \vec{R} = F \cos \alpha \, \vec{i} + \left(F \sin \alpha - 8\sqrt{3}\right) \vec{j}$

$\because$ The magnitude of the resultant = 8 gm.wt. due to east

$\therefore F \cos \alpha = 8$ (1)

$, F \sin \alpha = 8\sqrt{3}$ (2)

Dividing (2) by (1) : $\therefore \tan \alpha = \frac{8\sqrt{3}}{8} = \sqrt{3}$

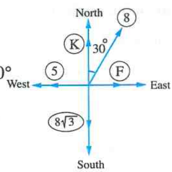
$\therefore \cos \alpha > 0$ and $\sin \alpha > 0$

$\therefore \alpha$ lies in the first quadrant $\therefore \alpha = 60°$

substituting in (1) : $\therefore F \cos 60° = 8$

$\therefore F = 16$ gm.wt. and its direction is 60° North of East

25



$\because X = F \cos 0° + 8 \cos 60° + K \cos 90° + 5 \cos 180° + 8\sqrt{3} \cos 270°$

$= F \times 1 + 8 \times \frac{1}{2} + K \times 0 + 5 \times -1 + 8\sqrt{3} \times 0$

$= F - 1$

$, Y = F \sin 0° + 8 \sin 60° + K \sin 90° + 5 \sin 180° + 8\sqrt{3} \sin 270°$

$= F \times 0 + 8 \times \frac{\sqrt{3}}{2} + K \times 1 + 5 \times 0 + 8\sqrt{3} \times -1$

$= K - 4\sqrt{3}$

$\therefore \vec{R} = (F - 1)\vec{i} + \left(K - 4\sqrt{3}\right)\vec{j}$ (1)

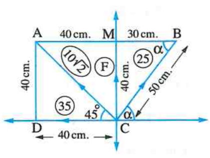
$\because R = 4$ newton due to 60° North of East

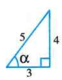
$\therefore \vec{R} = 4 \cos 60° \, \vec{i} + 4 \sin 60° \, \vec{j} = 2\vec{i} + 2\sqrt{3}\,\vec{j}$ (2)

From (1) and (2) : $\therefore F - 1 = 2$ $\quad\therefore F = 3$ newton

$, K - 4\sqrt{3} = 2\sqrt{3}$ $\quad\therefore K = 6\sqrt{3}$ newton

26





$\because X = 25 \cos \alpha + F \cos 90° + 10\sqrt{2} \cos 135° + 35 \cos 180°$

$= 25 \times \frac{3}{5} + F \times 0 + 10\sqrt{2} \times \frac{-1}{\sqrt{2}} + 35 \times -1$

$= 15 - 10 - 35 = -30$

$, Y = 25 \sin \alpha + F \sin 90° + 10\sqrt{2} \sin 135° + 35 \sin 180°$

$= 25 \times \frac{4}{5} + F \times 1 + 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 20 + F + 10$

$= 30 + F$

$\because R^2 = X^2 + Y^2$ $\quad\therefore (50)^2 = (-30)^2 + (30 + F)^2$

$\therefore 2500 = 900 + 900 + 60 F + F^2$

$\therefore 2500 = 1800 + 60 F + F^2$

$\therefore F^2 + 60 F - 700 = 0$

$\therefore (F + 70)(F - 10) = 0$ $\quad\therefore F = 10$ gm.wt.

27

(1) $\because X = F \cos 0° + K \cos 120° + 5\sqrt{3} \cos 210° + 7\sqrt{3} \cos 330° = 0$

$\therefore F \times 1 + K \times \frac{-1}{2} + 5\sqrt{3} \times \frac{-\sqrt{3}}{2} + 7\sqrt{3} \times \frac{\sqrt{3}}{2} = 0$

$\therefore F - \frac{1}{2} K + 3 = 0$ (1)

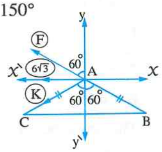
$, Y = F \sin 0° + K \sin 120° + 5\sqrt{3} \sin 210° + 7\sqrt{3} \sin 330° = 0$

$\therefore F \times 0 + K \times \frac{\sqrt{3}}{2} + 5\sqrt{3} \times \frac{-1}{2} + 7\sqrt{3} \times \frac{-1}{2} = 0$

$\therefore \frac{\sqrt{3}}{2} K = 6\sqrt{3}$ $\quad\therefore K = 12$ newton

, from (1) : $\therefore F = 3$ newton

(2) $\because X = 6\sqrt{3} \cos 0° + F \cos 150° + K \cos 210° = 0$



$\therefore 6\sqrt{3} \times 1 + F \times \frac{-\sqrt{3}}{2} + K \times \frac{-\sqrt{3}}{2} = 0$

$\therefore \frac{\sqrt{3}}{2} F + \frac{\sqrt{3}}{2} K = 6\sqrt{3}$ (1)

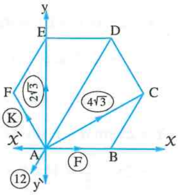
$, \because Y = 6\sqrt{3} \sin 0° + F \sin 150° + K \sin 210° = 0$

$\therefore 6\sqrt{3} \times 0 + F \times \frac{1}{2} + K \times \frac{-1}{2} = 0$

$\therefore \frac{1}{2} F - \frac{1}{2} K = 0$ (2)

From (1) , (2) : $\therefore F = K = 6$ newton

(3) $\because X = F\cos 0° + 4\sqrt{3}\cos 30° + 2\sqrt{3}\cos 90° + K\cos 120° + 12\cos 240° = 0$



$\therefore F \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 0 + K \times \frac{-1}{2} + 12 \times \frac{-1}{2} = 0$

$\therefore F - \frac{1}{2}K = 0$ (1)

$, Y = F\sin 0° + 4\sqrt{3}\sin 30° + 2\sqrt{3}\sin 90° + K\sin 120° + 12\sin 240° = 0$

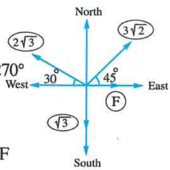
$\therefore F \times 0 + 4\sqrt{3} \times \frac{1}{2} + 2\sqrt{3} \times 1 + K \times \frac{\sqrt{3}}{2} + 12 \times \frac{-\sqrt{3}}{2} = 0$

$\therefore \frac{\sqrt{3}}{2}K = 2\sqrt{3}$ $\therefore K = 4$ newton

From (1) : $\therefore F = 2$ newton

28

$\because X = F\cos 0° + 3\sqrt{2}\cos 45° + 2\sqrt{3}\cos 150° + \sqrt{3}\cos 270°$



$= F \times 1 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \frac{-\sqrt{3}}{2} + \sqrt{3} \times 0 = F$

$, Y = F\sin 0° + 3\sqrt{2}\sin 45° + 2\sqrt{3}\sin 150° + \sqrt{3}\sin 270°$

$= F \times 0 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \frac{1}{2} + \sqrt{3} \times -1 = 3$

$\therefore \overrightarrow{R} = F\overrightarrow{i} + 3\overrightarrow{j}$ $\therefore \left(3\sqrt{2}\right)^2 = F^2 + 3^2$

$\therefore F^2 = 9$ $\therefore F = 3$ newton

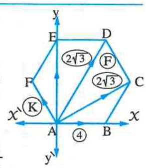
$\therefore \overrightarrow{R} = 3\overrightarrow{i} + 3\overrightarrow{j}$ $\therefore \tan\theta = \frac{3}{3} = 1$

$\therefore \theta = 45°$

$\therefore$ The angle between the line of action of $\overrightarrow{R}$ and the first force is of measure 45°

29

$\because X = 4\cos 0° + 2\sqrt{3}\cos 30° + F\cos 60° + 2\sqrt{3}\cos 90° + K\cos 120°$



$= 4 \times 1 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} + F \times \frac{1}{2} + 2\sqrt{3} \times 0 + K \times -\frac{1}{2} = 7 + \frac{1}{2}F - \frac{1}{2}K$

$, Y = 4\sin 0° + 2\sqrt{3}\sin 30° + F\sin 60° + 2\sqrt{3}\sin 90° + K\sin 120°$

$= 4 \times 0 + 2\sqrt{3} \times \frac{1}{2} + F \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + K \times \frac{\sqrt{3}}{2}$

$= 3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K$

$\therefore \overrightarrow{R} = \left(7 + \frac{1}{2}F - \frac{1}{2}K\right)\overrightarrow{i} + \left(3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K\right)\overrightarrow{j}$ (1)

$\because$ The resultant = 20 kg.wt. in the direction $\overrightarrow{AD}$

$\therefore \overrightarrow{R} = 20\cos 60°\,\overrightarrow{i} + 20\sin 60°\,\overrightarrow{j} = 10\overrightarrow{i} + 10\sqrt{3}\,\overrightarrow{j}$ (2)

From (1) and (2) : $\therefore 7 + \frac{1}{2}F - \frac{1}{2}K = 10$

$\therefore F - K = 6$ (3)

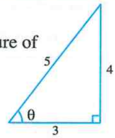
$, 3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K = 10\sqrt{3}$ $\therefore F + K = 14$ (4)

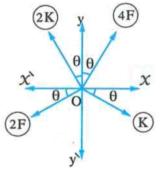
Adding (3) and (4) : $\therefore 2F = 20$

$\therefore F = 10$ kg.wt. , then $K = 4$ kg.wt.

30

$\because \theta$ is the measure of an acute angle ,





$\sin\theta = \frac{4}{5}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore X = 4F\cos(90° - \theta) + 2K\cos(90° + \theta) + 2F\cos(180° + \theta) + K\cos(360° - \theta)$

$= 4F\sin\theta - 2K\sin\theta - 2F\cos\theta + K\cos\theta$

$= 4F \times \frac{4}{5} - 2K \times \frac{4}{5} - 2F \times \frac{3}{5} + K \times \frac{3}{5}$

$= 2F - K$

$, Y = 4F\sin(90° - \theta) + 2K\sin(90° + \theta) + 2F\sin(180° + \theta) + K\sin(360° - \theta)$

$= 4F\cos\theta + 2K\cos\theta - 2F\sin\theta - K\sin\theta$

$= 4F \times \frac{3}{5} + 2K \times \frac{3}{5} - 2F \times \frac{4}{5} - K \times \frac{4}{5}$

$= \frac{4}{5}F + \frac{2}{5}K$

$\therefore \overrightarrow{R} = (2F - K)\overrightarrow{i} + \left(\frac{4}{5}F + \frac{2}{5}K\right)\overrightarrow{j}$ (1)

$\because \overrightarrow{R} = 8\sqrt{2}\cos 135°\,\overrightarrow{i} + 8\sqrt{2}\sin 135°\,\overrightarrow{j} = -8\overrightarrow{i} + 8\overrightarrow{j}$ (2)

From (1) and (2) : $\therefore 2F - K = -8$ (3)

$, \frac{4}{5}F + \frac{2}{5}K = 8$ *i.e.* $2F + K = 20$ (4)

Adding (3) and (4) : $\therefore 4F = 12$ $\therefore F = 3$ newton

and from (3) : $K = 14$ newton

31

$\because \overrightarrow{R} = (5 + a - 14)\,\overrightarrow{i} + (3 + 6 + b)\,\overrightarrow{j}$

$= (a - 9)\,\overrightarrow{i} + (9 + b)\,\overrightarrow{j}$ (1)

$\because \overrightarrow{R} = \left(10\sqrt{2}\,,\, 135°\right)$

$\therefore \overrightarrow{R} = 10\sqrt{2}\cos 135°\,\overrightarrow{i} + 10\sqrt{2}\sin 135°\,\overrightarrow{j}$

$= -10\,\overrightarrow{i} + 10\,\overrightarrow{j}$ (2)

From (1) and (2) : $\therefore a - 9 = -10$ $\therefore a = -1$

, $9 + b = 10$ $\therefore b = 1$

Exercise 4

First Multiple choice questions

(1) b	(2) b	(3) b	(4) c	(5) c
(6) c	(7) b	(8) c	(9) c	(10) d
(11) c	(12) a	(13) c	(14) b	(15) b
(16) a	(17) b	(18) a	(19) d	(20) d
(21) d	(22) d	(23) b	(24) c	(25) a
(26) c	(27) d	(28) b	(29) a	(30) b
(31) b	(32) d	(33) a	(34) b	(35) c
(36) b				

Second Essay questions

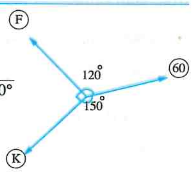
1

Applying the triangle of forces rule

$\therefore \frac{F_1}{3} = \frac{F_2}{4} = \frac{75}{5}$ $F_1 = \frac{3 \times 75}{5} = 45$ newton

$F_2 = \frac{4 \times 75}{5} = 60$ newton

2



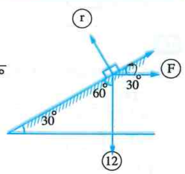
Applying lami's rule

$\therefore \frac{60}{\sin 90°} = \frac{K}{\sin 120°} = \frac{F}{\sin 150°}$

$\therefore F = 30$ newton

, $K = 30\sqrt{3}$ newton

3



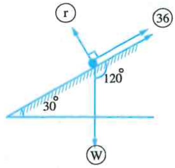
Applying lami's rule

$\therefore \frac{F}{\sin 150°} = \frac{r}{\sin 90°} = \frac{12}{\sin 120°}$

$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{1} = \frac{12}{\frac{\sqrt{3}}{2}}$

$\therefore F = \frac{12 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 4\sqrt{3}$ kg.wt. , $r = \frac{12}{\frac{\sqrt{3}}{2}} = 8\sqrt{3}$ kg.wt.

4



Applying lami's rule.

$\frac{36}{\sin 150°} = \frac{W}{\sin 90°} = \frac{r}{\sin 120°}$

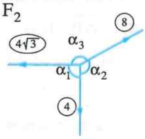
$\therefore W = \frac{36 \sin 90°}{\sin 150°} = 72$ newton

, $r = \frac{36 \sin 120°}{\sin 150°} = 36\sqrt{3}$ newton

5

$\because$ The three forces are in equilibrium.

$\therefore$ The resultant of the two forces $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ equals in magnitude F_1



$\therefore (8)^2 = \left(4\sqrt{3}\right)^2 + (4)^2 + 2 \times 4\sqrt{3} \times 4 \cos \alpha_1$

$\therefore 64 = 64 + 32\sqrt{3}\cos \alpha_1$

$\therefore \cos \alpha_1 = \text{zero}$ $\therefore \alpha_1 = 90°$

The resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_3}$ equals F_2 in magnitude.

$\therefore \left(4\sqrt{3}\right)^2 = (8)^2 + (4)^2 + 2 \times 8 \times 4 \cos \alpha_2$

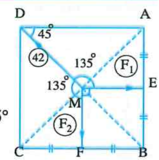
$\therefore 48 = 80 + 64 \cos \alpha_2$ $\therefore \cos \alpha_2 = -\frac{1}{2}$

$\therefore \alpha_2 = 120°$

$\therefore \alpha_3 = 360° - (90° + 120°) = 150°$

$\therefore$ The measures of the angles between forces are 90° , 120° , 150°

6



From the figure

$\therefore m(\angle EMF) = 90°$

, $m(\angle DME) = m(\angle DMF) = 135°$

Applying lami's rule

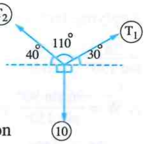
$\therefore \frac{F_1}{\sin 135°} = \frac{F_2}{\sin 135°} = \frac{42}{\sin 90°}$

$\therefore \frac{F_1}{\frac{1}{\sqrt{2}}} = \frac{F_2}{\frac{1}{\sqrt{2}}} = \frac{42}{1}$

$\therefore F_1 = F_2 = 42 \times \frac{1}{\sqrt{2}} = 21\sqrt{2}$ gm.wt.

7

From the figure and applying lami's rule.



$$\therefore \frac{T_1}{\sin 130°} = \frac{T_2}{\sin 120°} = \frac{10}{\sin 110°}$$

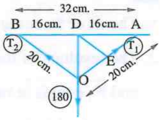
$$\therefore T_1 = \frac{10 \times \sin 130°}{\sin 110°} \simeq 8.15 \text{ newton}$$

$$, T_2 = \frac{10 \times \sin 120°}{\sin 110°} \simeq 9.216 \text{ newton}$$

8

From the figure

$\overline{OD} \perp \overline{AB}$, D is the midpoint of $\overline{AB}$



$\therefore (OD)^2 = (20)^2 - (16)^2 = 144 \qquad \therefore OD = 12$ cm.

$\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \,//\, \overline{OB}$

$\therefore$ E is the midpoint of $\overline{AO}$, $DE = \frac{1}{2}$ OB

$\therefore$ EO = 10 cm., DE = 10 cm.

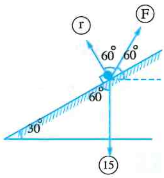
$\therefore \Delta$ DOE is the triangle of forces

$$\therefore \frac{180}{DO} = \frac{T_1}{OE} = \frac{T_2}{ED} \qquad \therefore \frac{180}{12} = \frac{T_1}{10} = \frac{T_2}{10}$$

$$\therefore T_1 = T_2 = \frac{180 \times 10}{12} = 150 \text{ kg.wt.}$$

9

Let the angle between the inclined plane and the horizontal measure θ



$\therefore \sin\theta = \frac{1}{2} \qquad \therefore \theta = 30°$

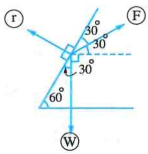
Applying lami's rule

$$\therefore \frac{F}{\sin 150°} = \frac{r}{\sin 150°} = \frac{15}{\sin 60°}$$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{15}{\frac{\sqrt{3}}{2}} \qquad \therefore F = r = \frac{15 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 5\sqrt{3} \text{ kg.wt.}$$

10

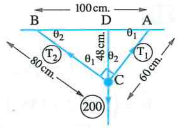
Let the measure of the angle between the plane and the horizontal be θ



$, \because \cos\theta = \frac{1}{2} \qquad \therefore \theta = 60°$

$$\therefore \frac{F}{\sin 120°} = \frac{r}{\sin 120°} = \frac{W}{\sin 120°} \qquad \therefore F = r = W$$

11



$\because (60)^2 + (80)^2 = (100)^2$

$\therefore \Delta$ ACB is right-angled at C

From lami's rule

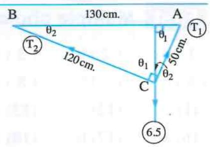
$$\therefore \frac{200}{\sin 90°} = \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2}$$

$$\because \sin\theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}, \sin\theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{200}{1} = \frac{T_1}{\frac{4}{5}} = \frac{T_2}{\frac{3}{5}}$$

$$\therefore T_1 = 200 \times \frac{4}{5} = 160 \text{ gm.wt. } T_2 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$

12



$AB = \sqrt{(50)^2 + (120)^2}$

$= 130$ cm.

From lami's rule

$$\therefore \frac{6.5}{\sin 90°} = \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2}$$

$$\therefore \sin\theta_1 = \frac{BC}{AB} = \frac{120}{130} = \frac{12}{13}, \sin\theta_2 = \frac{AC}{AB} = \frac{50}{130} = \frac{5}{13}$$

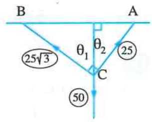
$$\therefore \frac{6.5}{1} = \frac{T_1}{\frac{12}{13}} = \frac{T_2}{\frac{5}{13}}$$

$$\therefore T_1 = 6.5 \times \frac{12}{13} = 6 \text{ newton}$$

$$, T_2 = 6.5 \times \frac{5}{13} = 2.5 \text{ newton}$$

13

Suppose that the inclination angles of the two strings to the vertical be of measures θ_1 and θ_2 from lami's rule.



$$\therefore \frac{50}{\sin 90°} = \frac{25}{\sin\theta_1} = \frac{25\sqrt{3}}{\sin\theta_2}$$

$$\therefore \sin\theta_1 = \frac{25 \sin 90°}{50} = \frac{1}{2}$$

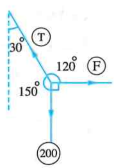
$$\therefore \theta_1 = 30°, \sin\theta_2 = \frac{25\sqrt{3} \times \sin 90°}{50} = \frac{\sqrt{3}}{2}$$

$\therefore \theta_2 = 60°$

$\therefore$ The two angles of inclination of the strings to the vertical are of measure 30° and 60°

14

From the figure and using lami's rule.



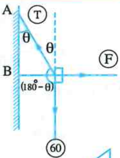
$$\therefore \frac{200}{\sin 120°} = \frac{F}{\sin 150°} = \frac{T}{\sin 90°}$$

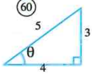
$$\therefore \frac{200}{\frac{\sqrt{3}}{2}} = \frac{F}{\frac{1}{2}} = \frac{T}{1}$$

$\therefore F = \dfrac{200 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{200\sqrt{3}}{3}$ gm.wt.

$T = \dfrac{200 \times 1}{\frac{\sqrt{3}}{2}} = \dfrac{400\sqrt{3}}{3}$ gm.wt.

15





From lami's rule.

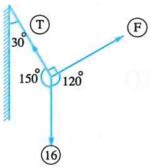
$\therefore \dfrac{60}{\sin(90° + \theta)} = \dfrac{F}{\sin(180° - \theta)} = \dfrac{T}{\sin 90}$

$\therefore \dfrac{60}{\cos\theta} = \dfrac{F}{\sin\theta} = \dfrac{T}{1}$

$\because \dfrac{60}{\frac{4}{5}} = \dfrac{F}{\frac{3}{5}} = \dfrac{T}{1}$

$\therefore F = \dfrac{60 \times \frac{3}{5}}{\frac{4}{5}} = 45$ gm.wt. , $T = \dfrac{60}{\frac{4}{5}} = 75$ gm.wt.

16



Applying lami's rule.

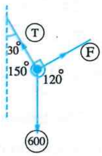
$\therefore \dfrac{F}{\sin 150°} = \dfrac{T}{\sin 120°} = \dfrac{16}{\sin 90°}$

$\therefore \dfrac{F}{\frac{1}{2}} = \dfrac{T}{\frac{\sqrt{3}}{2}} = 16$

$\therefore F = 8$ newton

, $T = 8\sqrt{3}$ newton

17



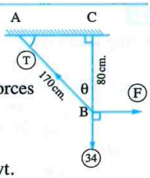
From the figure and using lami's rule.

$\therefore \dfrac{600}{\sin 90°} = \dfrac{F}{\sin 150°} = \dfrac{T}{\sin 120°}$

$\therefore \dfrac{600}{1} = \dfrac{F}{\frac{1}{2}} = \dfrac{T}{\frac{\sqrt{3}}{2}}$

$\therefore F = 300$ gm.wt. $\qquad T = 300\sqrt{3}$ gm.wt.

18



(1) $\because AC = \sqrt{(170)^2 - (80)^2} = 150$

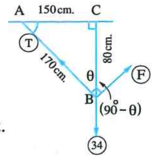
$\therefore \Delta ACB$ is the triangle of forces

$\therefore \dfrac{F}{150} = \dfrac{T}{170} = \dfrac{34}{80}$

$\therefore F = \dfrac{34 \times 150}{80} = 63.75$ gm.wt.

$T = \dfrac{34 \times 170}{80} = 72.25$ gm.wt.

(2) From Lami's rule :



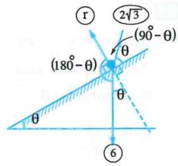
$\dfrac{34}{\sin 90°} = \dfrac{F}{\sin\theta} = \dfrac{T}{\sin(90° - \theta)}$

$\therefore \dfrac{34}{1} = \dfrac{F}{\frac{150}{170}} = \dfrac{T}{\frac{80}{170}}$

$\therefore F = 34 \times \dfrac{150}{170} = 30$ gm.wt.

$T = 34 \times \dfrac{80}{170} = 16$ gm.wt.

19



Applying lami's rule.

$\dfrac{r}{\sin(90° + 2\theta)} = \dfrac{2\sqrt{3}}{\sin(180° - \theta)}$

$= \dfrac{6}{\sin(90° - \theta)}$

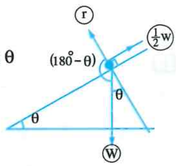
$\therefore \dfrac{r}{\cos 2\theta} = \dfrac{2\sqrt{3}}{\sin\theta} = \dfrac{6}{\cos\theta}$

$\because \dfrac{2\sqrt{3}}{\sin\theta} = \dfrac{6}{\cos\theta} \qquad \therefore \dfrac{\sin\theta}{\cos\theta} = \dfrac{2\sqrt{3}}{6} = \dfrac{\sqrt{3}}{3}$

$\therefore \tan\theta = \dfrac{1}{\sqrt{3}} \qquad \therefore \theta = 30°$

$\therefore \dfrac{r}{\cos 60°} = \dfrac{2\sqrt{3}}{\sin 30°} \qquad \therefore r = 2\sqrt{3}$ newton

20



Let the inclination angle of the plane to the horizontal is θ

$\therefore \dfrac{W}{\sin 90°} = \dfrac{\frac{1}{2}W}{\sin(180° - \theta)}$

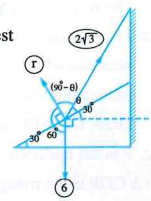
$= \dfrac{r}{\sin(90° + \theta)}$

$\therefore \dfrac{W}{1} = \dfrac{\frac{1}{2}W}{\sin\theta} = \dfrac{r}{\cos\theta} \qquad \therefore \sin\theta = \dfrac{\frac{1}{2}W}{W} = \dfrac{1}{2}$

$\therefore \theta = 30° \qquad \therefore \dfrac{W}{1} = \dfrac{r}{\cos 30°}$

$\therefore r = W\cos 30° = W \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}\,W}{2}$

21



Suppose the string make an angle measures θ with the line of greatest slope of the plane upwards.

$\therefore \dfrac{2\sqrt{3}}{\sin 150°} = \dfrac{6}{\sin(90° - \theta)}$

$= \dfrac{r}{\sin(120° + \theta)}$

$\therefore \dfrac{2\sqrt{3}}{\frac{1}{2}} = \dfrac{6}{\cos\theta} = \dfrac{r}{\sin(120° + \theta)}$

$\therefore \cos\theta = \dfrac{6 \times \frac{1}{2}}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ $\qquad \therefore \theta = 30°$

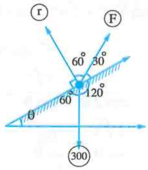
$\therefore$ The string makes an angle of measure 30° with the plane.

$\because \dfrac{2\sqrt{3}}{\frac{1}{2}} = \dfrac{r}{\sin(120° + 30°)}$

$\therefore r = \dfrac{2\sqrt{3}\sin 150°}{\frac{1}{2}} = \dfrac{2\sqrt{3} \times \frac{1}{2}}{\frac{1}{2}} = 2\sqrt{3}$ kg.wt.

22

Let the angle between the inclined plane and the horizontal be θ



$\therefore \tan\theta = \dfrac{1}{\sqrt{3}}$ $\qquad \therefore \theta = 30°$

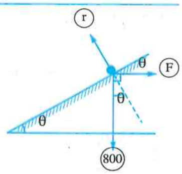
$\therefore \dfrac{F}{\sin 150°} = \dfrac{r}{\sin 150°} = \dfrac{300}{\sin 60°}$

$\therefore \dfrac{F}{\frac{1}{2}} = \dfrac{r}{\frac{1}{2}} = \dfrac{300}{\frac{\sqrt{3}}{2}}$

$\therefore F = r = \dfrac{300 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 100\sqrt{3}$ gm.wt.

23

Applying lami's rule.



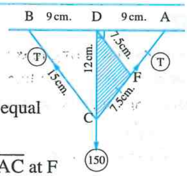
$\therefore \dfrac{800}{\sin(90° + \theta)} = \dfrac{r}{\sin 90°} = \dfrac{F}{\sin(180° - \theta)}$

$\therefore \dfrac{800}{\cos\theta} = \dfrac{r}{1} = \dfrac{F}{\sin\theta}$

$\therefore r = 800 \div \cos\theta = 800 \times \dfrac{10}{8} = 1000$ gm.wt.

, $F = 800 \times \sin\theta \div \cos\theta = 800 \times \dfrac{6}{8} = 600$ gm.wt.

24



$\because$ The ring is smooth.

$\therefore$ The tensions in the two branches of the string are equal in magnitudes.

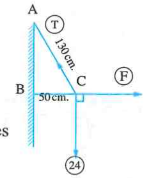
Draw $\overline{DF}$ // $\overline{BC}$ and cuts $\overline{AC}$ at F

$\therefore$ F is the midpoint of $\overline{AC}$

, Δ CDF is the triangle of forces.

$\therefore \dfrac{150}{12} = \dfrac{T}{7.5} = \dfrac{T}{7.5}$ $\qquad \therefore T = \dfrac{7.5 \times 150}{12} = 93.75$ gm.wt.

25



(1) $\because AB = \sqrt{(130)^2 - (50)^2}$

$= 120$ cm.

From the figure we get

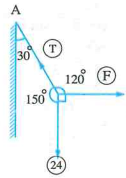
Δ ABC is the triangle of forces

$\therefore \dfrac{T}{130} = \dfrac{F}{50} = \dfrac{24}{120}$

$\therefore T = \dfrac{24 \times 130}{120} = 26$ newton

$\therefore F = \dfrac{24 \times 50}{120} = 10$ newton

(2) Using lami's rule :



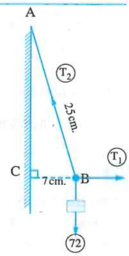
$\therefore \dfrac{T}{\sin 90°} = \dfrac{F}{\sin 150°} = \dfrac{24}{\sin 120°}$

$\therefore \dfrac{T}{1} = \dfrac{F}{\frac{1}{2}} = \dfrac{24}{\frac{\sqrt{3}}{2}}$

$\therefore T = \dfrac{24}{\frac{\sqrt{3}}{2}} = 16\sqrt{3}$ newton

$F = \dfrac{24 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 8\sqrt{3}$ newton

26



$AC = \sqrt{(25)^2 - (7)^2} = 24$ cm.

$\therefore \Delta$ ABC is the triangle of forces

$\therefore \dfrac{72}{AC} = \dfrac{T_1}{CB} = \dfrac{T_2}{BA}$

$\therefore \dfrac{72}{24} = \dfrac{T_1}{7} = \dfrac{T_2}{25}$

$\therefore T_1 = \dfrac{7 \times 72}{24} = 21$ gm.wt.

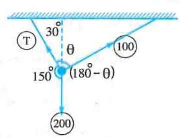
$T_2 = \dfrac{25 \times 72}{24} = 75$ gm.wt.

$\therefore$ The tension in the horizontal string = 21 gm.wt.

, the tension in each part of the first string 75 and 72 gm.wt.

27

Using lami's rule



$\therefore \dfrac{100}{\sin 150°} = \dfrac{T}{\sin(180° - \theta)} = \dfrac{200}{\sin(30° + \theta)}$

$\therefore \dfrac{100}{\frac{1}{2}} = \dfrac{T}{\sin\theta} = \dfrac{200}{\sin(30° + \theta)}$

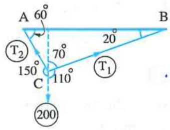
$\therefore \sin(30° + \theta) = \dfrac{200 \times \frac{1}{2}}{100} = 1$ $\qquad \therefore 30° + \theta = 90°$

$\therefore \theta = 60°$

$T = \dfrac{100 \sin 60°}{\frac{1}{2}} = 100\sqrt{3}$ gm.wt.

28

Applying lami's rule :



$\dfrac{T_1}{\sin 150°} = \dfrac{200}{\sin 100°}$

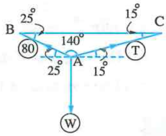
$= \dfrac{T_2}{\sin 110°}$

$\therefore T_1 = \dfrac{200 \sin 150°}{\sin 100°} \simeq 102$ newton

$, T_2 = \dfrac{200 \sin 110°}{\sin 100°} \simeq 191$ newton.

29

From the figure and applying lami's rule :

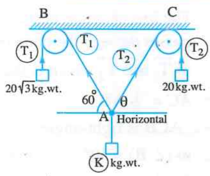


$\therefore \dfrac{T}{\sin 115°} = \dfrac{W}{\sin 140°}$

$= \dfrac{80}{\sin 105°}$

$\therefore T \simeq 75$ newton , $W \simeq 53$ newton.

30



$T_1 = 20\sqrt{3}$ kg.wt.

$, T_2 = 20$ kg.wt.

Applying lami's rule :

$\therefore \dfrac{20}{\sin 150°} = \dfrac{20\sqrt{3}}{\sin (90° + \theta)}$

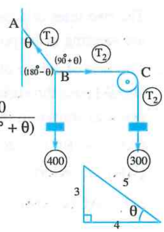
$= \dfrac{K}{\sin (120° - \theta)}$

$\therefore \cos \theta = \dfrac{20\sqrt{3} \times \sin 150°}{20} = \dfrac{\sqrt{3}}{2}$ $\qquad \therefore \theta = 30°$

$, K = \dfrac{20 \times \sin 90°}{\sin 150°} = 40$ kg.wt.

Third Higher skills

1



$T_2 = 300$ gm.wt.

using lami's rule.

$\therefore \dfrac{T_1}{\sin 90°} = \dfrac{300}{\sin (180° - \theta)} = \dfrac{400}{\sin (90° + \theta)}$

$\therefore \dfrac{T_1}{1} = \dfrac{300}{\sin \theta} = \dfrac{400}{\cos \theta}$

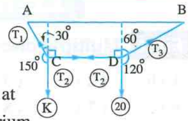
$\therefore \dfrac{\sin \theta}{\cos \theta} = \dfrac{300}{400} = \dfrac{3}{4}$

$\therefore \tan \theta = \dfrac{3}{4}$ $\qquad \therefore \theta \simeq 36° \, 52'$

$\therefore$ The angle of inclination of $\overline{AB}$ to the vertical measures $36° \, 52'$

$\therefore \dfrac{T_1}{1} = \dfrac{300}{\frac{3}{5}}$ $\qquad \therefore T_1 = 500$ gm.wt.

2



$\because$ The forces whose magnitudes are T_1 , T_2 and K which are meeting at the point C are in equilibrium.

$\therefore \dfrac{T_1}{\sin 90°} = \dfrac{T_2}{\sin 150°} = \dfrac{K}{\sin 120°}$ $\qquad \therefore \dfrac{T_1}{1} = \dfrac{T_2}{\frac{1}{2}} = \dfrac{K}{\frac{\sqrt{3}}{2}}$

, $\because$ the forces whose magnitudes are T_2 , T_3 and 20 which are meeting at the point D are in equilibrium

$\therefore \dfrac{20}{\sin 150°} = \dfrac{T_2}{\sin 120°} = \dfrac{T_3}{\sin 90°}$ $\qquad \therefore \dfrac{20}{\frac{1}{2}} = \dfrac{T_2}{\frac{\sqrt{3}}{2}} = \dfrac{T_3}{1}$

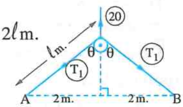
$\therefore T_2 = \dfrac{20 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 20\sqrt{3}$ gm.wt.

$T_3 = \dfrac{20 \times 1}{\frac{1}{2}} = 40$ gm.wt.

Substituting in (1) : $\dfrac{T_1}{1} = \dfrac{20\sqrt{3}}{\frac{1}{2}} = \dfrac{K}{\frac{\sqrt{3}}{2}}$

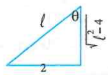
$\therefore T_1 = 40\sqrt{3}$ gm.wt. , $K = \dfrac{20\sqrt{3} \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 60$ gm.wt.

3



Let the length of the string is 2ℓ m.

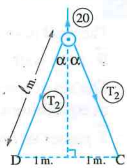
- When fixing the string at the position A , B



$\therefore 20 = 2\, T_1 \cos \theta$

$\therefore T_1 = \dfrac{10}{\cos \theta} = 10 \times \dfrac{\ell}{\sqrt{\ell^2 - 4}}$ (1)

- When fixing the string at the position C , D



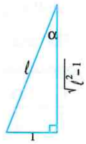
$\therefore 20 = 2\, T_2 \cos \alpha$

$\therefore T_2 = \dfrac{10}{\cos \alpha} = 10 \times \dfrac{\ell}{\sqrt{\ell^2 - 1}}$ (2)

From (1) and (2) :

$\therefore T_1 > T_2$

i.e. The tension in the string when fixing at A , B is greater than the tension when fixing at C , D



Exercise 5

First Multiple choice questions

(1) b	(2) b	(3) a	(4) c	(5) b
(6) c	(7) c	(8) d		
(9) First : d		Second : a		
(10) b	(11) b	(12) c		

Second Essay questions

1

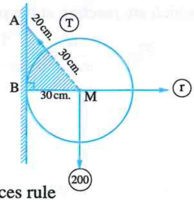
Δ MAB is the triangle of forces where :

MA = 30 + 20 = 50 cm.

BM = 30 cm.

$\therefore$ AB = 40 cm.

(pythagoras theorem)



Applying the triangle of forces rule

$\therefore \frac{r}{BM} = \frac{T}{MA} = \frac{200}{AB}$ $\qquad \therefore \frac{r}{30} = \frac{T}{50} = \frac{200}{40}$

$\therefore r = 200 \times \frac{30}{40} = 150$ gm.wt.

$T = 200 \times \frac{50}{40} = 250$ gm.wt.

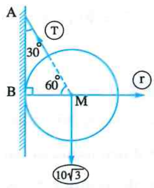
2

$\because$ The wall is smooth

$\therefore \vec{r} \perp$ the wall

$\because$ The set of forces are in equilibrium.

$\therefore \vec{T}$ passes through the point M



Applying lami's rule

$\therefore \frac{T}{\sin 90°} = \frac{10\sqrt{3}}{\sin 120°} = \frac{r}{\sin 150°}$

$\therefore T = 20$ gm.wt. $\qquad r = 10$ gm.wt.

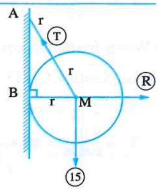
3

$\because$ The wall is smooth

$\therefore \vec{R} \perp$ the wall

$\because$ The set of forces are in equilibrium.

$\therefore \vec{T}$ passes through the point M



$\therefore \Delta$ ABM is the triangle of forces where AM = 2 r , MB = r , AB = $\sqrt{3}$ r

applying the triangle of forces rule

$\therefore \frac{T}{2r} = \frac{R}{r} = \frac{15}{\sqrt{3}r}$ $\qquad \therefore T = 10\sqrt{3}$ newton

$R = 5\sqrt{3}$ newton

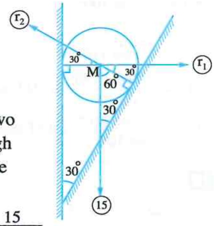
$\therefore$ The pressure on the wall $= 5\sqrt{3}$ newton

4

Since the two planes are smooth

$\therefore r_1$ and r_2 are perpendicular to the two planes and pass through the center of the sphere

Applying lami's rule



$\therefore \frac{r_1}{\sin 120°} = \frac{r_2}{\sin 90°} = \frac{15}{\sin 150°}$

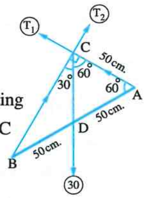
$\therefore r_1$ (The reaction of the vertical plane) $= 15\sqrt{3}$ kg.wt.

r_2 (The reaction of the inclined plane) = 30 kg.wt.

5

$\because$ The set of forces are in equilibrium.

$\therefore$ The line of action of the weight passes through the point of meeting of $\overrightarrow{T_1}$ and $\overrightarrow{T_2}$ which is the point C



$\because AC = \frac{1}{2} AB$

, $\angle$ ACB is right-angle

$\therefore m(\angle B) = 30°$ $\qquad \therefore m(\angle A) = m(\angle ACD) = 60°$

$\therefore$ Applying lami's rule

$\therefore \frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{30}{\sin 90°}$

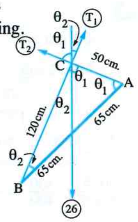
$\therefore T_1 = 30 \times \frac{1}{2} = 15$ kg.wt.

$T_2 = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}$ kg.wt.

6

$\because$ The two lines of action of tensions are meeting at the point of suspending.

$\therefore$ The line of action of the weight should pass through the same point as shown in the figure



$\because (AC)^2 + (BC)^2 = 16900$

$(AB)^2 = 16900$

$\therefore m(\angle ACB) = 90°$

$\therefore \sin\theta_1 = \frac{12}{13}$, $\sin\theta_2 = \frac{5}{13}$

Applying lami's theorem

$\therefore \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{26}{\sin 90^\circ}$ $\quad \therefore \frac{T_1}{\frac{12}{13}} = \frac{T_2}{\frac{5}{13}} = 26$

$\therefore T_1 = 24$ newton $\quad T_2 = 10$ newton

7

$\because$ The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point E

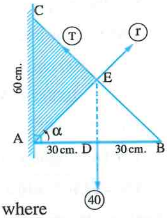
$\because$ D is the midpoint of $\overline{AB}$

, $\overline{DE} \mathbin{//} \overline{AC}$

$\therefore$ E is the midpoint of $\overline{BC}$

$BC = 60\sqrt{2}$ cm.

(Pythagoras theorem)



Δ AEC is the triangle of forces where

$AE = \frac{1}{2}\, BC = 30\sqrt{2}$ cm. , $EC = 30\sqrt{2}$ cm.

, $AC = 60$ cm.

$\therefore \frac{r}{30\sqrt{2}} = \frac{T}{30\sqrt{2}} = \frac{40}{60}$ $\quad \therefore r = T = 20\sqrt{2}$ newton

8

$\because$ The set of forces are in equilibrium.

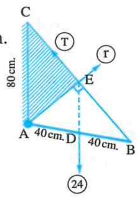
$\therefore \vec{r}$ passes through the point E

$\because \overline{DE} \mathbin{//} \overline{AC}$

$\therefore$ E is the midpoint of $\overline{BC}$

$\therefore \overline{AE} \perp \overline{BC}$

, $EC = 40\sqrt{3}$ $\quad AC = 80$ cm.



$\therefore AE = 40$ cm. (Pythagoras theorem)

Δ AEC is the triangle of forces

$\therefore \frac{r}{40} = \frac{T}{40\sqrt{3}} = \frac{24}{80}$ $\quad \therefore r = 40 \times \frac{24}{80} = 12$ kg.wt.

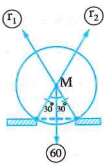
$T = 40\sqrt{3} \times \frac{24}{80} = 12\sqrt{3}$ kg.wt.

9

$\because$ The rod is a tangent to the sphere.

$\therefore$ The reaction is perpendicular to the rod

$\therefore \vec{r_1}$ and $\vec{r_2}$ are passing through the centre of the sphere



Applying lami's rule

$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 150^\circ} = \frac{60}{\sin 60^\circ}$

$\therefore r_1 = r_2 = 20\sqrt{3}$ newton

$\therefore P_1 = P_2 = 20\sqrt{3}$ newton

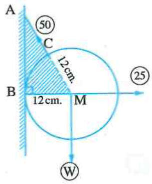
10

Δ MAB is the triangle of forces

$\therefore \frac{50}{MA} = \frac{25}{BM} = \frac{W}{AB}$

$\therefore \frac{MA}{MB} = \frac{50}{25} = 2$

$\therefore MA = 2 \times 12 = 24$



The length of the string

$\overline{CA} = 24 - 12 = 12$ cm.

From Δ MAB which is right-angled at B

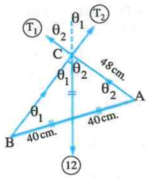
$\therefore AB = 12\sqrt{3}$ $\quad \therefore \frac{25}{12} = \frac{W}{12\sqrt{3}}$

$\therefore W = 25\sqrt{3}$ newton

11

$\because$ The set of forces are in equilibrium.

$\therefore$ The line of action of the weight passes through the intersection point of $\vec{T_1}$ and $\vec{T_2}$ which is (C)



$\because$ The two strings are perpendicular

$\therefore \Delta$ ABC is right-angled.

$\therefore$ The length of the other string = 64 cm.

$\therefore \sin\theta_1 = \frac{48}{80} = \frac{3}{5}$, $\sin\theta_2 = \frac{64}{80} = \frac{4}{5}$

Applying lami's rule

$\therefore \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{12}{\sin 90^\circ}$ $\quad \therefore \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}} = 12$

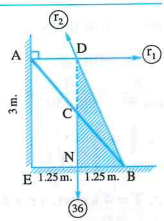
$\therefore T_1 = 7.2$ newton

$T_2 = 9.6$ newton

12

$\because$ The reaction of the wall (r_1) is perpendicular to it , the weight of the ladder acts vertically downward and they are meeting at D

$\therefore$ The reaction of the ground $(\vec{r_2})$ should pass through (D)



$DN = AE = 3$ metres , $BN = 1.25$ metre

From Δ DNB which are right angled at N

, then $BD = 3.25$ metres (Pythagoras)

$\because \Delta$ NBD is the triangle of forces , then applying the rule of the triangle of forces.

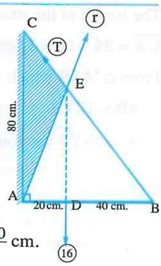
$\therefore \frac{r_1}{NB} = \frac{r_2}{BD} = \frac{36}{DN}$

$\therefore \frac{r_1}{1.25} = \frac{r_2}{3.25} = \frac{36}{3}$

$\therefore r_1 = 15$ kg.wt.

, $r_2 = 39$ kg.wt.

13



$\because$ The set of forces are in equilibrium.

$\therefore \overleftarrow{r}$ passes through the point E

$BC = 100$ cm.

(Pythagoras theorem)

$\frac{BD}{BA} = \frac{BE}{BC} = \frac{ED}{CA}$

$\therefore \frac{2}{3} = \frac{BE}{100} = \frac{ED}{80}$ $\quad \therefore BE = \frac{200}{3}$ cm.

, $ED = \frac{160}{3}$ cm.

From Δ ADE : $AE = \frac{20\sqrt{73}}{3}$ cm. (Pythagoras)

$\therefore \Delta$ AEC is the triangle of forces

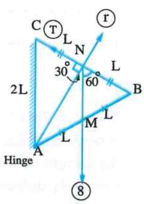
Applying the triangle of forces rule.

$\therefore \frac{r}{\frac{20\sqrt{73}}{3}} = \frac{T}{\frac{100}{3}} = \frac{16}{80}$

$\therefore r = \frac{4}{3}\sqrt{73}$ kg.wt.

$T = \frac{20}{3} = 6\frac{2}{3}$ kg.wt.

14



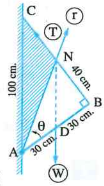
$\because$ The rod is in equilibrium under the action of three forces meeting at the point N using lami's we get

$\frac{8}{\sin 90°} = \frac{T}{\sin 150°} = \frac{r}{\sin 120°}$

$\therefore \frac{8}{1} = \frac{T}{\frac{1}{2}} = \frac{r}{\frac{\sqrt{3}}{2}}$

$\therefore T = 4$ kg.wt. , $r = 4\sqrt{3}$ kg.wt.

15



$\because$ The set of forces are in equilibrium.

$\therefore$ The weight and the tension are meeting at the point (N)

$\because$ The line of action of r should pass through N

$\because \overline{DN} // \overline{AC}$, D is the midpoint of $\overline{AB}$

$\therefore$ N is the midpoint of $\overline{BC}$

$\therefore \Delta$ ANC is the triangle of forces where $NC = 40$ cm.

, $CA = 100$ cm.

$\because (AC)^2 = 10000$, $(AB)^2 + (BC)^2 = 10000$

$\therefore m(\angle B) = 90°$

$\therefore \Delta$ NBA is right-angled.

$\therefore AN = \sqrt{(40)^2 + (60)^2} = 20\sqrt{13}$ cm.

Applying the triangle of forces rule we get

$\frac{r}{20\sqrt{13}} = \frac{T}{40} = \frac{W}{100}$ $\quad \therefore T = \frac{2}{5} W$ kg.wt.

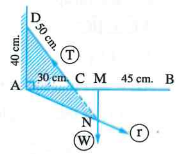
$r = \frac{\sqrt{13}}{5} W$ kg.wt.

Supposing that θ is the measure of the angle between $\overleftarrow{r}$ and the rod

$\therefore$ In Δ NBA which is right-angled at B

, then $\tan\theta = \frac{40}{60} = \frac{2}{3}$ $\quad \therefore \theta \simeq 33° 41'$

16



In Δ ADC :

$AD = \sqrt{(50)^2 - (30)^2} = 40$ cm.

, $\because$ the set of forces are in equilibrium.

$\therefore \overleftarrow{r}$ passes through N

$\Delta CAD \sim \Delta CMN$ (because of the equality of measures of their corresponding angles)

$\therefore \frac{CA}{CM} = \frac{AD}{MN} = \frac{CD}{CN}$ $\quad \therefore \frac{30}{15} = \frac{40}{MN} = \frac{50}{CN}$

$\therefore MN = 20$ cm. , $CN = 25$ cm.

$\therefore DN = 50 + 25 = 75$ cm.

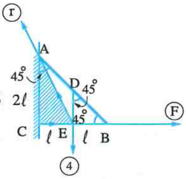
From Δ AMN we get $AN = 5\sqrt{97}$ cm. (Pythagoras)

$\because \Delta$ AND is the triangle of forces.

$\therefore \frac{T}{75} = \frac{r}{5\sqrt{97}} = \frac{W}{40}$

$\therefore T = \frac{15}{8} W$ kg.wt. $\quad r = \frac{\sqrt{97}}{8} W$ kg.wt.

17



$\because$ The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point E

suppose that AC = 2ℓ

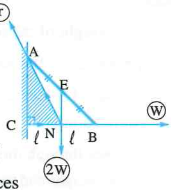
$\therefore$ BC = 2ℓ $\quad \therefore$ EC = ℓ

AE = $\sqrt{5}\,\ell$ (Pythagoras)

$\because \Delta$ AEC is the triangle of forces and applying the triangle of forces rule

$\therefore \frac{4}{2\ell} = \frac{r}{\sqrt{5}\ell} = \frac{F}{\ell}$ $\quad \therefore$ F = 2 kg.wt. , r = $2\sqrt{5}$ kg.wt.

18



Let the weight of the rod be (2 W)

$\because$ The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point N

$\therefore \Delta$ ANC is the triangle of forces

Applying the rule of the triangle of forces

$\therefore \frac{W}{\ell} = \frac{2W}{AC} = \frac{r}{AN}$ $\quad \therefore$ AC = 2ℓ

$\because$ E is the midpoint of $\overline{AB}$, $\overline{EN} // \overline{AC}$

$\therefore$ N is the midpoint of $\overline{BC}$ $\quad \therefore$ BC = 2ℓ

$\therefore$ m ($\angle$ BAC) = 45°

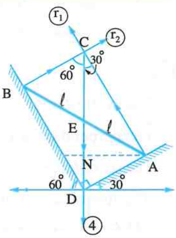
$\therefore$ The rod inclines to the vertical at an angle of measure 45°

From Δ ANC : $\therefore$ AN = $\sqrt{5}\,\ell$ (Pythagoras)

$\therefore \frac{W}{\ell} = \frac{r}{\sqrt{5}\ell}$ $\quad \therefore r = \sqrt{5}$ W

$\therefore$ The reaction = $\frac{\sqrt{5}}{2}$ × (The weight of the rod)

19



$\because$ The set of forces are in equilibrium.

$\therefore$ The line of action of the weight passes through the point of meeting of the two reactions at (C)

$\because$ The angle between the two planes is right and each of the two reactions is perpendicular to the plane going out it.

$\therefore$ ADBC is a rectangle. $\quad \therefore$ m ($\angle$ ACB) = 90°

Applying lami's rule

$\therefore \frac{4}{\sin 90°} = \frac{r_1}{\sin 120°} = \frac{r_2}{\sin 150°}$

$\therefore r_1 = 2\sqrt{3}$ newton , $r_2 = 2$ newton

$\therefore P_1 = 2\sqrt{3}$ newton , $P_2 = 2$ newton

$\because$ EA = ED (properties of the rectangle)

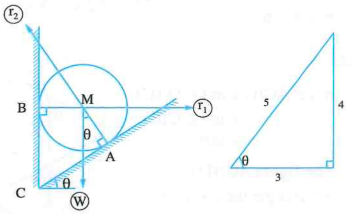
, m ($\angle$ ADE) = 60°

$\therefore \Delta$ AED is an equilateral triangle.

$\therefore$ m ($\angle$ EAD) = 60° $\quad \therefore$ m ($\angle$ EAN) = 30°

$\therefore$ The rod inclines to the horizontal at an angle of measure 30°

20



$\because \vec{r_1}$ is perpendicular to $\overline{BC}$

$\vec{r_2}$ is perpendicular to $\overline{CA}$

They are meeting at M where the weight of the sphere acts

Applying lami's rule we get

$\frac{r_1}{\sin(180° - \theta)} = \frac{r_2}{\sin 90°} = \frac{W}{\sin(90° + \theta)}$

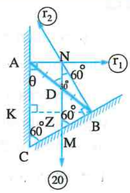
$\therefore \frac{r_1}{\sin\theta} = r_2 = \frac{W}{\cos\theta}$ $\quad \therefore \frac{r_1}{\frac{4}{5}} = r_2 = \frac{W}{\frac{3}{5}}$

$\therefore r_1 = \frac{4}{3}$ W kg.wt. , $r_2 = \frac{5}{3}$ W kg.wt.

$\therefore$ The pressure on the wall = $\frac{4}{3}$ W kg.wt.

, the pressure on the inclined plane = $\frac{5}{3}$ W kg.wt.

21



$\because$ The set of forces are in equilibrium and $\vec{r_1}$ and $\vec{r_2}$ are meeting at N

$\therefore$ The weight of the rod passes through N

$\because \overline{NM} // \overline{AC}$

$\therefore$ m ($\angle$ NMB) = m ($\angle$ C) = 60°

$\therefore$ In Δ NMB : m ($\angle$ B) = 90° , m ($\angle$ M) = 60°

$\therefore$ m ($\angle$ MNB) = 30°

Applying lami's rule we get

$\therefore \dfrac{r_1}{\sin 150°} = \dfrac{r_2}{\sin 90°} = \dfrac{20}{\sin 120°} \quad \therefore \dfrac{r_1}{\frac{1}{2}} = \dfrac{r_2}{1} = \dfrac{20}{\frac{\sqrt{3}}{2}}$

$\therefore r_1 = \dfrac{20\sqrt{3}}{3}$ kg.wt. , $r_2 = \dfrac{40\sqrt{3}}{3}$ kg.wt.

Drawing $\overline{BK} \perp \overline{AC}$ $\quad \therefore \overline{BZ} \perp \overline{NM}$

Supposing that : BZ = ℓ $\quad \therefore$ BN = 2 ℓ , NZ = $\sqrt{3}\,\ell$

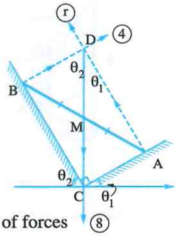
$\because \Delta$ BZD $\equiv \Delta$ AND $\quad \therefore$ ND = $\dfrac{\sqrt{3}}{2}\ell$

AN = BZ = ℓ $\quad \therefore$ AK = $\ell\sqrt{3}$, BK = 2 ℓ

$\therefore \tan(\angle BAK) = \tan\theta = \dfrac{BK}{AK} = \dfrac{2\ell}{\ell\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$\therefore \theta \simeq 49°\ 6'$

22



$\because$ m ($\angle$ ACB) = m ($\angle$ DAC)
= m ($\angle$ CBD)
= 90°

$\therefore$ The figure ACBD is a rectangle the rod is in equilibrium under the action of three forces , and D is the meeting point of forces

Assuming that the angle of inclination of the plane on which the end A rests is θ_1 and the angle of inclination of the plane on which the end B rests is θ_2 and using lami's rule

$\therefore \dfrac{r}{\sin(90° + \theta_1)} = \dfrac{4}{\sin(90° + \theta_2)} = \dfrac{8}{\sin 90°}$

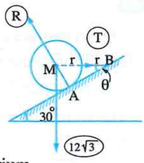
$\therefore \dfrac{r}{\cos\theta_1} = \dfrac{4}{\cos\theta_2} = 8 \quad \therefore \cos\theta_2 = \dfrac{1}{2}$

$\therefore \theta_2 = 60° \quad \therefore \theta_1 = 30°$

$\therefore$ r = 8 cos 30° = $4\sqrt{3}$ newton

$\therefore$ The pressure on the plane at A = $4\sqrt{3}$ newton

23



$\because$ The inclined plane is smooth

$\therefore$ The reaction of the plane at A is perpendicular to the plane

$\therefore$ It passes through the centre of the sphere (M)

$\because$ The set of forces are in equilibrium

$\therefore$ The tension in the string passes through the point M

In Δ MAB : m ($\angle$ A) = 90° , AM = $\frac{1}{2}$ BM

$\therefore \theta = 30°$

$\therefore$ m ($\angle$ MBA) = the measure of the angle of inclination of the plane to the horizontal.

$\therefore$ The string $\overline{MB}$ is horizontal

Applying lami's rule we get

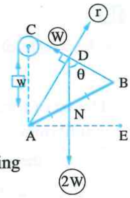
$\dfrac{R}{\sin 90°} = \dfrac{T}{\sin 150°} = \dfrac{12\sqrt{3}}{\sin 120°} \quad \therefore \dfrac{R}{1} = \dfrac{T}{\frac{1}{2}} = \dfrac{12\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\therefore$ R = 2 T = 24

$\therefore$ Tension in the string T = 12 kg.wt.

The reaction of the plane R = 24 kg.wt.

24



Let the weight of the rod = 2 W

$\therefore$ the weight of the body = W

$\because$ The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point of meeting of the weight and the tension in the string (D)

$\because$ N is the midpoint of $\overline{AB}$, $\overline{ND} \parallel \overline{AC}$

$\therefore$ D is the midpoint of $\overline{BC}$

$\because$ AB = AC $\quad \therefore \overline{AD} \perp \overline{BC}$

Applying lami's rule

$\therefore \dfrac{r}{\sin(180° - \theta)} = \dfrac{2W}{\sin 90°} = \dfrac{W}{\sin(90° + \theta)}$

$\therefore 2W = \dfrac{W}{\cos\theta} \quad \therefore \cos\theta = \dfrac{1}{2}$

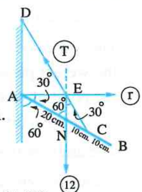
$\therefore \theta = 60° \quad \therefore$ m ($\angle$ BCA) = 60° , AB = AC

$\therefore \Delta$ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ BAC) = 60° $\therefore$ m ($\angle$ BAE) = 30°

$\therefore$ The measure of the inclination angle of the rod to the horizontal = 30°

25



$\because$ The wall is smooth.

$\therefore \vec{r}$ is perpendicular to the wall

$\because$ The set of forces are in equilibrium.

$\therefore$ The line of action of tension passes through the point of meeting the reaction and the weight (E)

EN = $\frac{1}{2}$ AN = 10 cm.

$\therefore \angle$ ENA is an exterior angle of the isosceles triangle ENC where EN = NC = 10 cm.

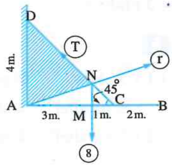
$\therefore m(\angle CEN) = 30^\circ$

Applying lami's rule

$\therefore \frac{r}{\sin 150^\circ} = \frac{T}{\sin 90^\circ} = \frac{12}{\sin 120^\circ}$

$\therefore r = 4\sqrt{3}$ newton , $T = 8\sqrt{3}$ newton

26



$\because$ AC = AD = 4 metres

$\therefore m(\angle ACD) = 45^\circ$

$\therefore CD = 4\sqrt{2}$ metres

From Δ MCN

MN = 1 metre , NC = $\sqrt{2}$ metres

DN = $4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$ metres

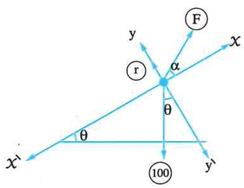
From Δ AMN : AN = $\sqrt{10}$ metres (Pythagoras)

Since Δ AND is the triangle of forces

$\therefore \frac{r}{\sqrt{10}} = \frac{T}{3\sqrt{2}} = \frac{8}{4}$

$\therefore r = 2\sqrt{10}$ kg.wt. , $T = 6\sqrt{2}$ kg.wt.

27



$\because$ The body is in equilibrium $\therefore X = 0 , Y = 0$

$\therefore F\cos\alpha + r\cos 90^\circ + 100\cos(270^\circ - \theta) = 0$

$\therefore F \times \frac{12}{13} + r \times 0 + 100 \times (-\sin\theta) = 0$

$\therefore \frac{12}{13} F - 100 \times \frac{3}{5} = 0$

$\therefore F = 65$ newton

, $F\sin\alpha^\circ + r\sin 90^\circ$

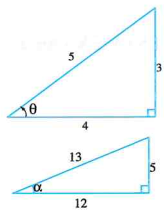
$+ 100\sin(270^\circ - \theta) = 0$

$\therefore 65 \times \frac{5}{13} + r \times 1$

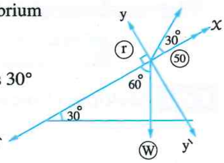
$+ 100 \times -\cos\theta = 0$

$\therefore 25 + r - 100 \times \frac{4}{5} = 0$

$\therefore r = 55$ newton



28



$\because$ The body is in equilibrium

$\therefore X = 0 , Y = 0$

$\therefore 50\cos 0^\circ + 20\sqrt{3}\cos 30^\circ$

$+ r\cos 90^\circ$

$+ W\cos 240^\circ = 0$

$\therefore 50 \times 1 + 20\sqrt{3} \times \frac{\sqrt{3}}{2} + r \times 0 + W \times -\cos 60^\circ = 0$

$\therefore 50 + 30 + 0 - \frac{1}{2} W = 0$

$\therefore W = 160$ newton

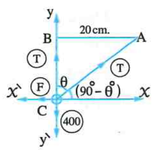
, $50\sin 0^\circ + 20\sqrt{3}\sin 30^\circ + r\sin 90^\circ + W\sin 240^\circ = 0$

$\therefore 50 \times 0 + 20\sqrt{3} \times \frac{1}{2} + r \times 1 - W\sin 60^\circ = 0$

$0 + 10\sqrt{3} + r - 160 \times \frac{\sqrt{3}}{2} = 0$

$\therefore r = 70\sqrt{3}$ newton

29



Δ ABC in which

$m(\angle B) = 90^\circ$, AC + CB = 40

$\therefore$ AC = 40 – CB

$\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore (40 - BC)^2 = (20)^2 + (BC)^2$

$\therefore 1600 - 80\,BC + (BC)^2 = 400 + (BC)^2$

$\therefore 80\,BC = 1200$

$\therefore$ BC = 15 cm. , AC = 25 cm.

$\because$ The ring is smooth

$\therefore$ The tension in $\overline{CA}$ = the tension in $\overline{CB}$ = T

$\because$ The ring is in equilibrium $\therefore X = 0 , Y = 0$

$\therefore T\cos(90^\circ - \theta) + T\cos 90^\circ + F\cos 180^\circ$

$+ 400\cos 270^\circ = 0$

$\therefore T\sin\theta + T \times 0 + F \times -1 + 400 \times 0 = 0$

$\therefore T \times \frac{20}{25} - F = 0$ $\qquad \therefore 4T - 5F = 0$ (1)

, $T\sin(90^\circ - \theta) + T\sin 90^\circ + F\sin 180^\circ$

$+ 400\sin 270^\circ = 0$

$\therefore T\cos\theta^\circ + T + F \times 0 + 400 \times -1 = 0$

$\therefore T \times \frac{15}{25} + T - 400 = 0$ $\qquad \therefore \frac{8}{5} T = 400$

$\therefore T = 250$ gm.wt.

substituting in (1) :

$\therefore 1000 - 5F = 0$ $\qquad \therefore F = 200$ gm.wt.

Guide Answers of "Unit Two"

Exercise 6

First Multiple choice questions

(1) d	(2) a	(3) d	(4) a	(5) d
(6) d	(7) c	(8) a	(9) d	(10) d
(11) a	(12) b	(13) a	(14) b	(15) b
(16) d	(17) c	(18) c	(19) a	(20) c
(21) c	(22) b	(23) c	(24) d	(25) d
(26) d	(27) a	(28) d	(29) d	(30) b
(31) b	(32) a	(33) c		

(34) First : c Second : a
Third : b Fourth : d

(35) First : a Second : b
Third : c Fourth : c

(36) First : b Second : d
Third : c Fourth : d

(37) First : a Second : b
Third : c Fourth : a
Fifth : b

(38) First : a Second : d
Third : c

(39) First : b Second : d
Third : d Fourth : c
Fifth : d

(40) First : a Second : b
Third : c Fourth : c

Second Essay questions

1

(1) 8 line segments.
(2) $\overleftrightarrow{AB}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{AM}$
(3) 5 planes
(4) The planes : ABCD , ABM , ADM

2

(1) $\overleftrightarrow{AB}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{AA}$
(2) $\overleftrightarrow{AB}$
(3) ABB'A' , ABCD , ADD'A'
(4) ABB'A' , ABCD , ABC'D'

3

(1) $\overleftrightarrow{AD}$, $\overleftrightarrow{AA'}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{BB'}$
(2) $\overleftrightarrow{A'B'}$, $\overleftrightarrow{D'C'}$, $\overleftrightarrow{DC}$
(3) $\overleftrightarrow{A'D'}$, $\overleftrightarrow{B'C'}$, $\overleftrightarrow{DD'}$, $\overleftrightarrow{CC'}$

4

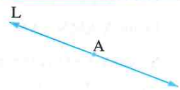
(1) infinite (2) infinite
(3) infinite (4) only one plane

5

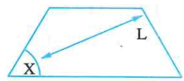
(1) ① skew ② parallel ③ skew
④ parallel ⑤ skew ⑥ intersecting
(2) ① parallel ② intersecting ③ intersecting
(3) $\sqrt{(6\sqrt{2})^2 + (6)^2} = 6\sqrt{3}$ cm.

6

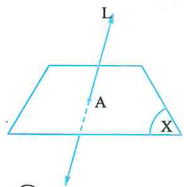
(1) $A \in L$



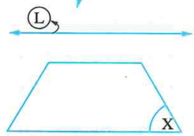
(2) $L \subset X$



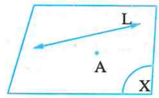
(3) $L \cap X = \{A\}$



(4) $L \parallel X$



(5) $A \in X$, $A \notin L$, $L \subset X$



Exercise 7

First Multiple choice questions

(1) c	(2) b	(3) b	(4) b	(5) b
(6) d	(7) b	(8) c	(9) d	(10) b
(11) c	(12) b	(13) a	(14) b	(15) b
(16) b	(17) c	(18) c	(19) b	(20) b
(21) c	(22) d	(23) a	(24) c	(25) d
(26) a	(27) b	(28) a	(29) a	(30) d
(31) b	(32) c	(33) c	(34) d	(35) a
(36) b	(37) d	(38) b	(39) c	(40) a
(41) a	(42) c	(43) b	(44) c	(45) a
(46) c	(47) c	(48) d	(49) c	(50) c
(51) d	(52) b			

Second Essay questions

1 (1) 5 (2) 6 (3) 5

(4) 10 (5) 6

, $\because$ number of faces + number of vertices

$= 6 + 6 = 12$

, number of edges + 2 = 10 + 2 = 12

$\therefore$ Achieve Euler's rule

2 $\because$ ABCD is a square of side length 10 cm.

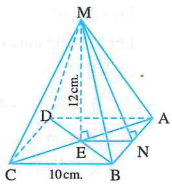
$\therefore$ NE = 5 cm.

, $\because$ the pyramid is regular

$\therefore \overline{ME} \perp \overline{NE}$

$\therefore$ The slant height (MN)

$= \sqrt{5^2 + 12^2} = 13$ cm.



3 $\because$ The pyramid is regular

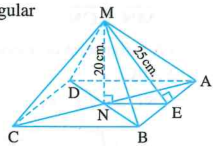
$\overline{MN} \perp \overline{NE}$

$\therefore NE = \sqrt{25^2 - 20^2}$

= 15 cm.

$\therefore$ BC = 30 cm.

$\therefore$ The length of the base side = 30 cm.



4 $\because$ The pyramid is regular

$\therefore \overline{MN} \perp \overline{AN}$

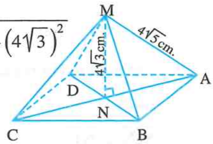
$\therefore AN = \sqrt{(4\sqrt{5})^2 - (4\sqrt{3})^2}$

$= 4\sqrt{2}$ cm.

$\therefore AC = 8\sqrt{2}$ cm.

(The square diagonal)

$\therefore$ The square side length $= \frac{8\sqrt{2}}{\sqrt{2}} = 8$ cm.



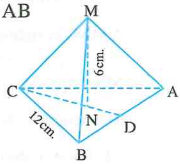
5 Let D be the midpoint of $\overline{AB}$

, $\because \Delta$ ABC is equilateral

$\therefore \overline{CD} \perp \overline{AB}$

$\therefore CD = \sqrt{(12)^2 - (6)^2}$

$= 6\sqrt{3}$ cm.



, $\because$ N is the intersection point of the medians of Δ ABC

$\therefore NC = \frac{2}{3} \times 6\sqrt{3} = 4\sqrt{3}$ cm.

$\therefore \Delta$ MNC is a right-angled at N

$\therefore MC = \sqrt{(6)^2 + (4\sqrt{3})^2} = 2\sqrt{21}$ cm.

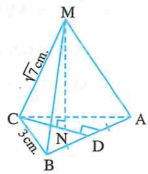
6 Let D be the midpoint of $\overline{AB}$

, Δ ABC is equilateral

$\therefore \overline{CD} \perp \overline{AB}$

$\therefore CD = \sqrt{(3)^2 - (1.5)^2}$

$= \frac{3}{2}\sqrt{3}$ cm.



, $\because$ the pyramid is regular

$\therefore \overline{MN} \perp \overline{NC}$

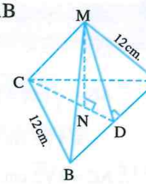
, N is the intersection point of the medians of Δ ABC $\quad \therefore CN = \sqrt{3}$ cm.

, $\because \Delta$ MNC is right-angled at N

$\therefore MN = \sqrt{(\sqrt{7})^2 - (\sqrt{3})^2} = 2$ cm.

$\therefore$ The pyramid height = 2 cm.

7 Let D is the midpoint of $\overline{AB}$



Δ ABC is an equilateral

$\therefore$ CD = $6\sqrt{3}$ cm.

$\because$ The pyramid of regular faces.

$\therefore$ $\overline{MN} \perp \overline{NC}$

, N is the intersection point of medians of Δ ABC

CN = $4\sqrt{3}$ cm.

$\therefore$ Δ MNC is right-angled at N

$\therefore$ MN $= \sqrt{(12)^2 - (4\sqrt{3})^2} = 4\sqrt{6}$ cm.

$\therefore$ The height of the pyramid = $4\sqrt{6}$ cm.

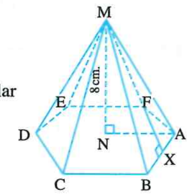
, $\because$ D is the midpoint of $\overline{AB}$ in the equilateral Δ ABC

$\therefore$ $\overline{MD} \perp \overline{AD}$

$\therefore$ MD $= \sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ cm.

$\therefore$ The slant height = $6\sqrt{3}$ cm.

8 The base side length



$= \frac{24\sqrt{3}}{6} = 4\sqrt{3}$ cm.

, $\because$ the pyramid is regular

$\therefore$ $\overline{MN} \perp \overline{AN}$

, N is the geometrical centre of the base

$\therefore$ AN = The base side length = $4\sqrt{3}$ cm.

$\therefore$ AM $= \sqrt{(8)^2 + (4\sqrt{3})^2} = 4\sqrt{7}$ cm.

$\therefore$ The length of lateral edge = $4\sqrt{7}$ cm.

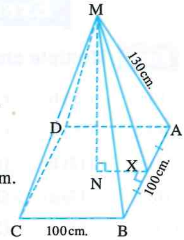
Let X is the midpoint of $\overline{AB}$

, $\because$ MA = MB $\quad\therefore$ $\overline{MX} \perp \overline{AB}$

$\therefore$ MX $= \sqrt{(4\sqrt{7})^2 - (2\sqrt{3})^2} = 10$ cm.

$\therefore$ The length of slant height = 10 cm.

9 Let X is the midpoint of $\overline{AB}$



, $\because$ MA = MB

$\therefore$ $\overline{MX} \perp \overline{AB}$

$\therefore$ MX $= \sqrt{(130)^2 - (50)^2}$

= 120 cm.

$\therefore$ The slant height = 120 cm.

$\therefore$ The height of the lateral face = 120 cm.

, $\because$ the pyramid is regular

$\therefore$ $\overline{MN} \perp \overline{NX}$

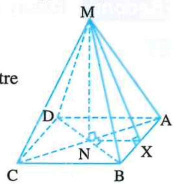
, $\because$ N is the geometrical centre of the base

$\therefore$ NX = 50 cm.

$\therefore$ MN $= \sqrt{(120)^2 - (50)^2} = 10\sqrt{119}$ cm.

$\therefore$ The height = $10\sqrt{119}$ cm.

10 Fig. (1) : (regular quadrilateral pyramid)



$\because$ The pyramid is regular

$\therefore$ $\overline{MN} \perp \overline{XN}$

, N is the geometrical centre of the base

$\therefore$ XN = 5 cm.

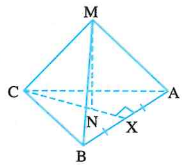
$\therefore$ MN $= \sqrt{(13)^2 - (5)^2}$

= 12 cm.

$\therefore$ The height of the pyramid = 12 cm.

Fig (2) : (triangular regular faces pyramid)

Let X is the midpoint of $\overline{AB}$



, $\because$ Δ ABC is equilateral

$\therefore$ $\overline{CX} \perp \overline{AB}$

$\therefore$ CX $= \sqrt{(6)^2 - (3)^2}$

$= 3\sqrt{3}$ cm.

, $\because$ the pyramid of regular faces

$\therefore$ $\overline{MN} \perp \overline{CN}$

, N is the point of intersection of the medians of Δ ABC

$\therefore$ NC = $2\sqrt{3}$ cm.

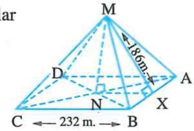
, MN $= \sqrt{(6)^2 - (2\sqrt{3})^2} = 2\sqrt{6}$ cm.

$\therefore$ The height = $2\sqrt{6}$ cm.

11 $\because$ The pyramid is regular

$\therefore \overline{MN} \perp \overline{XN}$

, N is the geometrical centre of the base



$\therefore$ X N = 116 cm.

$\therefore MN = \sqrt{(186)^2 - (116)^2} \simeq 145.4$ m.

$\therefore$ The height $\simeq$ 145.4 m.

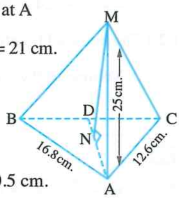
---

12 $\because \Delta$ ABC is right-angled at A

$\therefore BC = \sqrt{(12.6)^2 + (16.8)^2} = 21$ cm.

, $\because \overline{AD}$ is a median drawn from A

$\therefore AD = \frac{1}{2}$ the length of the hypotenuse = 10.5 cm.



$\because$ The pyramid is right.

$\therefore \overline{MN} \perp \overline{NA}$

, N is the intersection point of the medians.

$\therefore AN = \frac{2}{3} \times 10.5 = 7$ cm.

, $\because \Delta$ MNA is right-angled at N

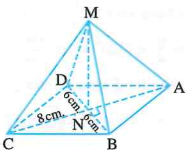
$\therefore$ The height (MN) $= \sqrt{(25)^2 - 7^2} = 24$ cm.

---

13 $\because$ The pyramid is right

$\therefore \overline{MN} \perp \overline{AC}$

, $\overline{MN} \perp \overline{BD}$



$\therefore$ MB = MD

$= \sqrt{(10)^2 + (6)^2}$

$= 2\sqrt{34}$ cm.

, $MA = MC = \sqrt{(10)^2 + (8)^2} = 2\sqrt{41}$ cm.

14

$\because$ The base area of the pyramid

$= \frac{1}{2} \times 18 \times 18 \times \sin 60°$

$= 81\sqrt{3}$ cm^2.

$\therefore$ The volume of the pyramid $= \frac{1}{3} \times 81\sqrt{3} \times 12$

$= 324\sqrt{3}$ cm^3.

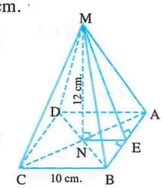
15

(1) $\because$ The height of the regular pyramid is MN

$\therefore \overline{MN} \perp \overline{NE}$, $NE = \frac{1}{2}$ BC = 5 cm.

$\therefore ME = \sqrt{(12)^2 + (5)^2} = 13$ cm.

$\therefore$ The slant height = 13 cm.



(2) Volume of the pyramid

$= \frac{1}{3}$ base area $\times$ the height

$= \frac{1}{3} \times (10)^2 \times 12 = 400$ cm^3.

(3) The total area

= the lateral area + base area

$= \frac{1}{2} \times (4 \times 10) \times 13 + (10 \times 10)$

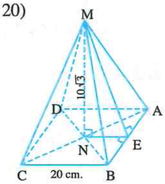
$= 260 + 100 = 360$ cm^2.

16

The slant height $= \sqrt{(10)^2 + \left(10\sqrt{3}\right)^2} = 20$ cm.

(1) The lateral area $= \frac{1}{2} \times (4 \times 20)$

$\times 20 = 800$ cm^2.



(2) Volume of the pyramid

$= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3} \times (20)^2$

$\times 10\sqrt{3} = \frac{4000}{3}\sqrt{3}$ cm^3.

17

Base of the regular pyramid is a square whose diagonal $24\sqrt{2}$ cm.

$\therefore$ The side length = 24 cm.

The lateral area $= \frac{1}{2}$ base perimeter $\times$ the slant height

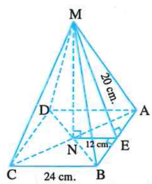
$= \frac{1}{2} \times (4 \times 24) \times 20 = 960$ cm^2.

$\therefore$ The total area = lateral area + base area

$= 960 + (24 \times 24) = 1536$ cm^2.

Height of the pyramid

$= \sqrt{(20)^2 - (12)^2} = 16$ cm.



Volume of the pyramid

$= \frac{1}{3}$ base area $\times$ height

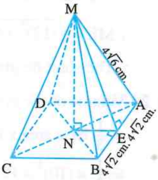
$= \frac{1}{3} \times (24)^2 \times 16 = 3072$ cm^3.

18

(1) $\because$ MABCD is a right pyramid of a square base

$\therefore$ The pyramid is regular

, the slant height



$= \sqrt{\left(4\sqrt{6}\right)^2 - \left(4\sqrt{2}\right)^2}$

$= 8$ cm.

The lateral area

$= \frac{1}{2} \times$ base perimeter

$\times$ slant height $= \frac{1}{2}$

$\times \left(4 \times 8\sqrt{2}\right) \times 8 = 128\sqrt{2}$ cm^2.

(2) $\because$ EN $= \frac{1}{2}$ BC $= 4\sqrt{2}$ cm.

$\therefore$ In Δ MEN :

Height of the pyramid MN

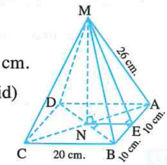
$= \sqrt{(8)^2 - \left(4\sqrt{2}\right)^2} = 4\sqrt{2}$ cm.

$\therefore$ Volume of the pyramid

$= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times \left(8\sqrt{2}\right)^2 \times 4\sqrt{2} = \frac{512}{3}\sqrt{2}$ cm^3.

19



(1) In Δ MAE :

ME $= \sqrt{(26)^2 - (10)^2} = 24$ cm.

(slant height of the pyramid)

(2) In Δ MEN :

MN $= \sqrt{(24)^2 - (10)^2}$

$= 2\sqrt{119}$ cm. (height of the pyramid)

(3) The lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (4 \times 20) \times 24 = 960$ cm^2.

(4) Volume of the pyramid $= \frac{1}{3}$ base area

$\times$ height $= \frac{1}{3} \times (20)^2 \times 2\sqrt{119} = \frac{800}{3}\sqrt{119}$ cm^2.

20

$\because$ The pyramid is a triangular regular faces pyramid

$\therefore 2\,\ell^2 = 3\,h^2$ $\qquad$ $\therefore 2 \times (12)^2 = 3 \times h^2$

$\therefore h = 4\sqrt{6}$ cm.

Volume of the pyramid $= \frac{1}{3} \times \left(\frac{1}{2}\ell^2 \times \frac{\sqrt{3}}{2}\right) \times h$

$= \frac{1}{3} \times \frac{1}{2} \times (12)^2 \times \frac{\sqrt{3}}{2} \times 4\sqrt{6} = 144\sqrt{2}$ cm^3.

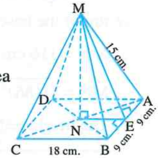
Its total area $= \ell^2\sqrt{3} = (12)^2\sqrt{3} = 144\sqrt{3}$ cm^2.

21

(1) $\because$ MABCD is a right pyramid with a square base

$\therefore$ The pyramid is a regular pyramid

, its slant height



$= \sqrt{(15)^2 - 9^2} = 12$ cm.

$\therefore$ Its total area = lateral area

+ base area $= \frac{1}{2}$

$\times (18 \times 4) \times 12 + 18^2$

$= 432 + 324 = 756$ cm^2.

(2) The pyramid height $= \sqrt{(12)^2 - 9^2} = 3\sqrt{7}$ cm.

$\therefore$ Volume of the pyramid $= \frac{1}{3} \times (18)^2 \times 3\sqrt{7}$

$= 324\sqrt{7}$ cm^3.

22

Base area $= \frac{n}{4} \times x^2 \times \cot\frac{\pi}{n}$

$= \frac{5}{4} \times (16)^2 \times \cot\frac{180°}{5} = 440.44$ cm^2.

Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 440.44 \times 12 \simeq 1761.8$ cm^3.

23

The lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (6 \times 12) \times 10\sqrt{3} = 360\sqrt{3}$ cm^2.

, base area $= \frac{n}{4} x^2 \cot\frac{\pi}{n}$

$= \frac{6}{4} \times (12)^2 \times \cot\frac{180°}{6} = 216\sqrt{3}$ cm^2.

, the total area = lateral area + base area

$= 360\sqrt{3} + 216\sqrt{3} = 576\sqrt{3}$ cm^2.

24

(1) Base area $= \frac{1}{2} \times (6)^2 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ cm^2.

The lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (3 \times 6) \times 10 = 90$ cm^2.

The total area $= (90 + 9\sqrt{3}) \simeq 105.6$ cm^2.

(2) Base area $= (12)^2 = 144$ cm^2.

The lateral area $= \frac{1}{2} \times (12 \times 4) \times 15 = 360$ cm^2.

The total area = lateral area + base area

$= 360 + 144 = 504$ cm^2.

(3) Base area $= (20)^2 = 400 \text{ cm}^2$.

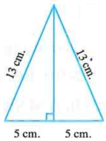
The slant height $= \sqrt{(24)^2 + (10)^2} = 26$ cm.

The lateral area $= \frac{1}{2} \times (20 \times 4) \times 26 = 1040 \text{ cm}^2$.

The total area $= 1040 + 400 = 1440 \text{ cm}^2$.

(4) Base area $= \frac{6}{4} \times 10^2 \times \cot \frac{\pi}{6}$

$= 150\sqrt{3} \simeq 259.8 \text{ cm}^2$.



The slant height $= \sqrt{(13)^2 - 5^2}$

$= 12$ cm.

The lateral area

$= \frac{1}{2} \times (6 \times 10) \times 12 = 360 \text{ cm}^2$.

The total area $= 360 + 259.8 = 619.8 \text{ cm}^2$.

25

(1) The volume $= \frac{1}{3} \times (10)^2 \times 21 = 700 \text{ cm}^3$.

(2) The volume $= \frac{1}{3} \times \left(\frac{6}{4} \times 8^2 \times \cot \frac{\pi}{6}\right) \times 14$

$= 448\sqrt{3} \text{ cm}^3$.

(3) Base area $= \frac{1}{2} \times 6^2 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2$.

The volume $= \frac{1}{3} \times (9\sqrt{3}) \times 12 = 36\sqrt{3} \text{ cm}^3$.

(4) AE $= \sqrt{(17)^2 - (15)^2}$

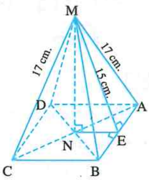
$= 8$ cm.

$\therefore$ (AB) The side length

$= 16$ cm. , EN $= 8$ cm.

, height of the pyramid

$= \sqrt{(15)^2 - 8^2} = \sqrt{161}$ cm.



Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times (16)^2 \times \sqrt{161} = \frac{256}{3}\sqrt{161} \simeq 1082.8 \text{ cm}^3$.

26

Volume of the quadrilateral pyramid

$= \frac{1}{3} \times \left(\frac{1}{2} \times 4 \times 8\right) \times 12 = 64 \text{ cm}^3$.

Volume of the cube $= (4)^3 = 64 \text{ cm}^3$.

$\therefore$ The two volumes are equal.

27

Base perimeter $= 5 + 6 + 7 = 18$ cm.

$\therefore$ Half base perimeter (S) $= 9$ cm.

Base area $= \sqrt{S(S - AB)(S - BC)(S - CA)}$

$= \sqrt{9 \times (9 - 5) \times (9 - 6) \times (9 - 7)}$

$= 6\sqrt{6} \text{ cm}^2$.

The volume $= \frac{1}{3} \times 6\sqrt{6} \times 15 = 30\sqrt{6} \text{ cm}^3$.

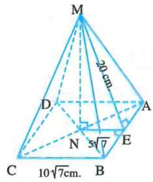
28

The base of the regular pyramid is a square whose area $= 700 \text{ cm}^2$.

$\therefore$ The side length $= 10\sqrt{7}$ cm.

The height $= \sqrt{(20)^2 - \left(5\sqrt{7}\right)^2}$

$= 15$ cm.



Volume of the pyramid

$= \frac{1}{3} \times (700) \times 15 = 3500 \text{ cm}^3$.

29

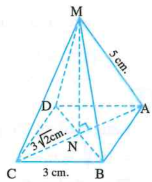
Base area $= 9 \text{ cm}^2$.

$\because$ The base side length $= 3$ cm.

$\therefore$ AC $= 3\sqrt{2}$

$\therefore$ AN $= 1.5\sqrt{2}$

$\therefore \Delta$ MAN :



$\therefore$ MN (The height) $= \sqrt{5^2 - \left(1.5\sqrt{2}\right)^2} = \frac{1}{2}\sqrt{82}$

The volume $= \frac{1}{3} \times 9 \times \frac{1}{2}\sqrt{82} = \frac{3}{2}\sqrt{82} \simeq 13.6 \text{ cm}^3$.

30

Volume of the pyramid

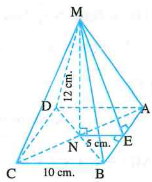
$= \frac{1}{3}$ base area $\times$ the height

$400 = \frac{1}{3} \times$ base area $\times 12$

$\therefore$ Base area $= 100$

$\therefore$ The base side length

$= 10$ cm.



The slant height

$= \sqrt{5^2 + (12)^2} = 13$ cm.

, $\because$ its lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (4 \times 10) \times 13 = 260 \text{ cm}^2$.

31

Volume of the pyramid $= \frac{1}{3}$ base area $\times$ height

$1296 = \frac{1}{3} \times (18)^2 \times$ height

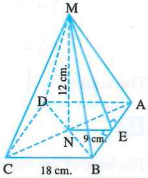
height $= 12$ cm.

The slant height $= \sqrt{9^2 + (12)^2}$

$= 15$ cm.

The lateral area

$= \frac{1}{2} \times (4 \times 18) \times 15 = 540$ cm^2.



32

Total area = lateral area + base area

$384 = \frac{1}{2}$ base perimeter $\times$ slant height + base area

$= \frac{1}{2} \times (4 \times 12) \times$ slant height $+ 12 \times 12$

Slant height $= 10$ cm.

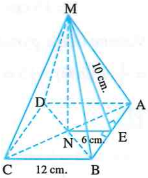
Pyramid height $= \sqrt{(10)^2 - (6)^2}$

$= 8$ cm.

Volume of the pyramid

$= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times (12)^2 \times 8 = 384$ cm^3.



33

$\because$ Length of the diagonal of the squared base

$= 10\sqrt{2}$ cm.

$\therefore$ The side length $= 10$ cm.

lateral area $= \frac{1}{2} \times$ base perimeter $\times$ slant height

$260 = \frac{1}{2} \times (4 \times 10) \times$ slant height

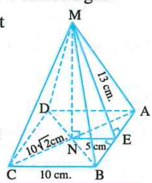
Slant height $= 13$ cm.

Pyramid height $= \sqrt{(13)^2 - 5^2}$

$= 12$ cm.

The volume $= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3} \times (10)^2 \times 12 = 400$ cm^3.



34

Length of the slant height

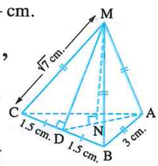
$MD = \sqrt{\left(\sqrt{7}\right)^2 - (1.5)^2} = \frac{\sqrt{19}}{2}$ cm.

$\therefore AD = \sqrt{3^2 - (1.5)^2} = \frac{3\sqrt{3}}{2}$ cm. ,

, $\because ND = \frac{1}{3} AD = \frac{\sqrt{3}}{2}$ cm.

$\therefore$ Height of the Pyramid

$= MN = \sqrt{\left(\frac{\sqrt{19}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = 2$ cm.



$\therefore$ The volume $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times \frac{1}{2} (3)^2 \sin 60° \times 2 = \frac{3\sqrt{3}}{2}$ cm^3.

, lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times (3 \times 3) \times \frac{\sqrt{19}}{2} = \frac{9}{4}\sqrt{19}$ cm^2.

35

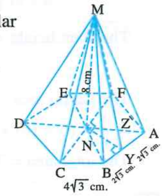
$\because$ The hexagon figure is a regular

$\therefore NB = BC = 4\sqrt{3}$ cm.

$\therefore$ In Δ MNB :

$MB = \sqrt{8^2 + \left(4\sqrt{3}\right)^2}$

$= 4\sqrt{7}$ cm.



In Δ MYB :

$MY = \sqrt{\left(4\sqrt{7}\right)^2 - \left(2\sqrt{3}\right)^2} = 10$ cm.

$\therefore$ The slant height $= 10$ cm.

The lateral area $= \frac{1}{2}$ base perimeter $\times$ slant height

$= \frac{1}{2} \times 24\sqrt{3} \times 10 = 120\sqrt{3}$ cm^2.

The total area = lateral area + base area

$= 120\sqrt{3} + \left[\frac{6}{4} \times \left(4\sqrt{3}\right)^2 \times \cot\frac{\pi}{6}\right]$

$= 192\sqrt{3}$ cm^2.

36

$\because$ The base is an equilateral triangle

passing through its vertices

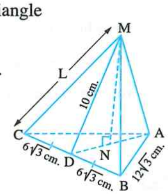
a circle of radius length 12 cm.

, $\because \frac{a}{\sin A} = 2$ r (sin law)

$\therefore \frac{a}{\sin 60°} = 2 \times 12$

$\therefore a = 12\sqrt{3}$

$\therefore$ The base side length $= 12\sqrt{3}$



, from ΔABD : $\therefore AD = \sqrt{\left(12\sqrt{3}\right)^2 - \left(6\sqrt{3}\right)^2} = 18$ cm.

$\therefore ND = \frac{1}{3} AD = 6$ cm. , from Δ MND :

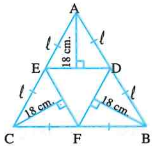
$\therefore MN = \sqrt{(10)^2 - (6)^2} = 8$ cm.

, $\because$ volume of the pyramid $= \frac{1}{3}$ base area $\times$ height

$\therefore$ Volume of the pyramid

$= \frac{1}{3} \times \frac{1}{2} (12\sqrt{3})^2 \sin 60° \times 8 = 288\sqrt{3}$ cm^3.

37



Let BF = ℓ

$\therefore$ DE = $\frac{1}{2}$ BC = ℓ

similary :

DF = EF = ℓ

$\therefore$ DEF is an equilateral triangle of side length ℓ

$\therefore$ The net for a triangular regular faces pyramid and $\frac{18}{\ell} = \sin 60°$

$\therefore \ell = 12\sqrt{3}$ cm.

Total area = 4 × face area = $4 \times \frac{1}{2} \times (12\sqrt{3})^2 \sin 60°$

$= 432\sqrt{3}$ cm^2.

38

The figure after folding it gives a quadrilateral regular pyramid its slant height $= \sqrt{(13)^2 - (5)^2}$

= 12 cm.

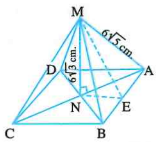
Area of one container = $\frac{1}{2} \times (4 \times 10) \times 12 + (10)^2$

= 240 + 100 = 340 cm^2.

(1) area of 1000 containers = 340000 cm^2 = 34 m^3.

(2) the cost = 34 × 15 = 510 pounds

39



Δ MNA is right angled at N

$\therefore$ NA $= \sqrt{\left(6\sqrt{5}\right)^2 - \left(6\sqrt{3}\right)^2}$

$= 6\sqrt{2}$ cm.

$\therefore$ AC $= 12\sqrt{2}$

$\therefore$ The length of square diagonal $= 12\sqrt{2}$

$\therefore$ The square side length = 12 cm.

, $\because$ EN = $\frac{1}{2}$ BC = $\frac{1}{2} \times 12 = 6$ cm.

$\therefore$ In Δ MEN :

$\therefore$ ME $= \sqrt{6^2 + \left(6\sqrt{3}\right)^2} = 12$ cm.

The slant height = 12 cm.

The lateral area = $\frac{1}{2}$ × base perimeter × slant height

$= \frac{1}{2} \times (4 \times 12) \times 12 = 288$ cm^2.

The total area = 288 + $(12)^2$ = 432 cm^2.

The volume = $\frac{1}{3} \times (12)^2 \times 6\sqrt{3} = 288\sqrt{3}$ cm^3.

40

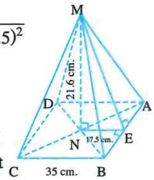
Volume of the model = $\frac{1}{3}$ base area × height

$= \frac{1}{3} \times (11.5)^2 \times 7 \simeq 308.58$ cm^3.

The mass = density × volume

$= 3.2 \times 308.58 \simeq 987.5$ gm.

41



The slant height $= \sqrt{(21.6)^2 + (17.5)^2}$

$\simeq 27.8$ m.

Area of the glass

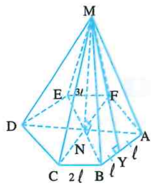
= The lateral area of the pyramid

$= \frac{1}{2}$ base perimeter × slant height

$= \frac{1}{2} \times (4 \times 35) \times 27.8 \simeq 1946$ m^2.

Third Higher skills

1



$\because$ BN = BC = 2ℓ

$\therefore$ In Δ MNB :

MB $= \sqrt{(3\ell)^2 + (2\ell)^2} = \sqrt{13}\,\ell$

, in Δ MBY :

MY $= \sqrt{\left(\sqrt{13}\,\ell\right)^2 - \ell^2} = 2\sqrt{3}\,\ell$

$\therefore$ The slant height $= 2\sqrt{3}\,\ell$

$\therefore$ The lateral area = $\frac{1}{2}$ base perimeter × slant height

$= \frac{1}{2} \times (2\ell \times 6) \times 2\sqrt{3}\,\ell = 12\sqrt{3}\,\ell^2$ cm^2.

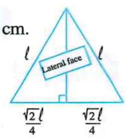
Base area = $\frac{6}{4} \times (2\ell)^2 \times \cot\frac{\pi}{6} = 6\sqrt{3}\,\ell^2$ cm^2.

$\therefore$ The lateral area = 2 base area

2

$\because$ Length of base diagonal = ℓ

$\therefore$ The side length of the base $= \frac{\ell\sqrt{2}}{2}$ cm.



The slant height

$= \sqrt{\ell^2 - \left(\frac{\ell\sqrt{2}}{4}\right)^2} = \frac{\sqrt{14}}{4}\,\ell$

The lateral area = $\frac{1}{2}$ × base perimeter × slant height

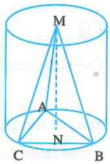
$= \frac{1}{2} \times \frac{\sqrt{2}\,\ell}{2} \times 4 \times \frac{\sqrt{14}}{4}\,\ell = \frac{\sqrt{7}}{2}\,\ell^2$

The total area = lateral area + base area

$= \frac{\sqrt{7}}{2}\ell^2 + \left(\frac{\ell}{2}\sqrt{2}\right)^2$

$= \left(\frac{\sqrt{7}}{2} + \frac{1}{2}\right)\ell^2 = \frac{\ell^2}{2}\left(1 + \sqrt{7}\right)$

3

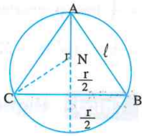


∵ The pyramid is right

∴ Its height meets the base ABC at the centre N which is the point of intersection of the medians , let the radius of the base (r) , height of cylinder = height of pyramid = h

base area of the cylinder = πr^2

, ∵ $\sin 60° = \frac{1.5\,r}{\ell}$



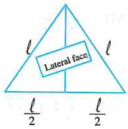
∴ ℓ (side length of pyramid base) $= r\sqrt{3}$

∴ Base area of the pyramid

$= \frac{1}{2}(r\sqrt{3})^2 \sin 60° = \frac{3\sqrt{3}}{4}r^2$

∴ $\frac{\text{pyramid volume}}{\text{cylinder volume}} = \frac{\frac{1}{3} \times \frac{3\sqrt{3}}{4}\cancel{r^2} \times \cancel{h}}{\pi \cancel{r^2}\cancel{h}} = \frac{\sqrt{3}}{4\pi}$

4



Let the base side length

= the lateral edge length = ℓ

Base area = ℓ^2

the slant height $= \frac{\ell}{2}\sqrt{3}$

∴ The lateral area $= \frac{1}{2}$ base perimeter × slant height

$= \frac{1}{2} \times (4 \times \ell) \times \frac{\ell}{2}\sqrt{3} = \ell^2\sqrt{3}$

∴ Total area $= \ell^2\sqrt{3} + \ell^2 = \ell^2\left(\sqrt{3} + 1\right)$

∴ $\ell^2\left(\sqrt{3}+1\right) = \left(\sqrt{3}+1\right)A$

∴ $\ell^2 = A$ ∴ $\ell = \sqrt{A}$

∴ The edge length $= \sqrt{A}$

Exercise 8

First Multiple choice questions

(1) d	(2) b	(3) b	(4) c	(5) c
(6) d	(7) a	(8) b	(9) a	(10) d
(11) a	(12) d	(13) a	(14) b	(15) b
(16) c	(17) c	(18) d	(19) c	(20) c
(21) c	(22) d	(23) b	(24) a	(25) d
(26) b	(27) c	(28) b		

(29) First : b Second : c Third : d
Fourth : b Fifth : b

(30) d	(31) d	(32) b	(33) a	(34) c
(35) c	(36) d	(37) c	(38) c	(39) b
(40) a	(41) c	(42) c		

Second Essay questions

1

(1) The volume $= \frac{1}{3} \times \pi \times (9)^2 \times 14$
$= 378\,\pi$ cm^3.

(2) $r = \sqrt{(26)^2 - (24)^2} = 10$ cm.

The volume $= \frac{1}{3} \times \pi \times (10)^2 \times 24 = 800\,\pi$ cm^3.

(3) The height $= \sqrt{(13)^2 - (5)^2} = 12$ cm.

The volume $= \frac{1}{3} \times \pi \times (5)^2 \times 12 = 100\,\pi$ cm^3.

2

(1) The lateral area $= \pi \times (6) \times 12 = 72\,\pi$ cm^2.

, the total area $= \pi\,(6)\,(6 + 12) = 108\,\pi$ cm^2.

(2) Length of the drawer $(\ell) = \sqrt{(12)^2 + 9^2} = 15$ cm.

The lateral area $= \pi\,(9) \times 15 = 135\,\pi$ cm^2.

, the total area $= \pi\,(9)\,(9 + 15) = 216\,\pi$ cm^2.

(3) $r = \sqrt{(15)^2 - (13)^2} = 2\sqrt{14}$ cm.

The lateral area $= \pi\left(2\sqrt{14}\right) \times 15 = 30\sqrt{14}\,\pi$ cm^2.

The total area $= \pi\left(2\sqrt{14}\right)\left(2\sqrt{14} + 15\right)$
$= \left(56 + 30\sqrt{14}\right)\pi$ cm^2.

3

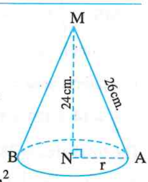
$r = \sqrt{\ell^2 - h^2} = \sqrt{(17)^2 - (15)^2} = 8$ cm.

The lateral area $= \pi r \ell = \pi\,(8) \times 17 = 136\,\pi$ cm^2.

The total area $= \pi r(\ell + r) = \pi\,(8)\,(17 + 8) = 200\,\pi$ cm^2.

The volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 8^2 \times 15 = 320\,\pi$ cm^3.

4



∵ The cone is right

∴ $\overline{MN} \perp \overline{AN}$

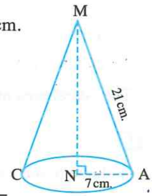
∵ $r = \sqrt{(26)^2 - (24)^2} = 10$ cm.

The base circumference

$= 2\,\pi\,r = 20\,\pi$ cm.

The base area $= \pi r^2 = 100\,\pi$ cm^2.

5 The opposite figure is a cone its base circumference = 44 cm.



$\therefore 2 \times \frac{22}{7} \times r = 44$

$\therefore r = 7$ cm.

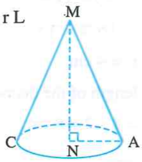
, $\because$ the cone is right

$\therefore \overline{MN} \perp \overline{AN}$

$\because$ The height of the cone

$= MN = \sqrt{(21)^2 - (7)^2} = 14\sqrt{2}$ cm.

6 $\because$ The area of the sector $= \frac{1}{2}$ r L



$\therefore 20 \cancel{\pi} = \frac{1}{2} r \times 8 \cancel{\pi}$

$\therefore r = 5$ cm. and it represent the drawer of the solid.

, $\because$ the length of the arc of the sector

= the circumference of the base

$\therefore 8\pi = 2\pi r`$

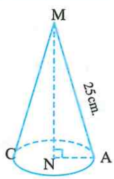
$\therefore r` = 4$ cm. (and it represent the radius of the base of the cone)

, $\because$ the cone is right $\therefore \overline{MN} \perp \overline{AN}$

$\therefore$ The height of the solid

$= MN = \sqrt{(5)^2 - (4)^2} = 3$ cm.

7 The net represents a right circular cone



, $\because$ the area of the circle

$= 49\pi$ cm^2.

$\therefore \pi r^2 = 49\pi$ $\therefore r = 7$ cm.

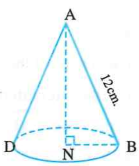
, $\because$ the cone is right

$\therefore \overline{MN} \perp \overline{AN}$

$\therefore MN = \sqrt{(25)^2 - (7)^2} = 24$ cm.

$\therefore$ The height = 24 cm.

8 $\because$ The length of the radius of the circle of the sector r = 12 cm. represents the drawer of the cone



, $\because$ the area of the sector = 150 cm^2.

$\therefore \frac{1}{2} r \ell = 150$

$\therefore \frac{1}{2} \times 12 \times \ell = 150$

$\therefore \ell = 25$ cm. and it represents the circumference of the base

$\therefore 25 = 2\pi r`$

$\therefore$ The radius length of the base of the cone $= \frac{25}{2\pi}$ cm.

, $\because$ the cone is right $\therefore \overline{AN} \perp \overline{BN}$

$\therefore$ The height of the cone $= \sqrt{(12)^2 - \left(\frac{25}{2\pi}\right)^2}$

$\simeq 11.3$ cm.

9

$\because r = 5$ cm.

length of its drawer $(\ell) = \sqrt{(5)^2 + (12)^2} = 13$ cm.

The total area $= \pi (5)(5 + 13) = 90\pi \simeq 282.7$ cm^2.

10

$\because$ The base circumference = 44

$\therefore 2\pi r = 44$

$\therefore r = \frac{22}{\pi}$

Volume of the cone $= \frac{1}{3} \times \pi \times \left(\frac{22}{\pi}\right)^2 \times 25$

$\simeq 1283.8$ cm^3.

11

Δ ABM is a right-angled at B , m ($\angle$ AMB) = 30°

$\therefore AM = 2 \times 5 = 10$ cm.

$\therefore \ell = 10$ cm.

The lateral area $= \pi (5) \times 10 = 50\pi$ cm^2.

The total area $= \pi (5)(5 + 10) = 75\pi$ cm^2.

12

$\because$ The lateral area $= \pi r \ell$

$\therefore 96\pi = \pi (8) \ell$

$\therefore \ell = 12$ cm.

$\therefore h = \sqrt{(12)^2 - 8^2} = 4\sqrt{5}$ cm.

The volume $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \pi (8)^2 \times 4\sqrt{5} \simeq 599.5$ cm^3.

13

The First (A) :

Its capacity $= \frac{1}{3} \pi \left(\frac{5}{2}\right)^2 \times 11 = \frac{275}{12} \pi$ cm^3.

The Second (B) :

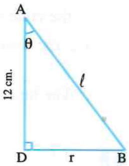
Its capacity $= \frac{1}{3} \pi \left(\frac{11}{2}\right)^2 \times 5 = \frac{605}{12} \pi$ cm^3.

$\therefore$ The capacity of B is the greater

The difference between their capacity

$= \frac{605}{12}\pi - \frac{275}{12}\pi = \frac{55}{2}\pi\ cm^3$.

14



$\cos\theta = \frac{AD}{AB} = \frac{12}{\ell} = \frac{4}{5}$

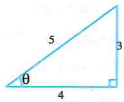
$\therefore \ell = 15$ cm.

$r = \sqrt{(15)^2 - (12)^2} = 9$ cm.

The total area $= \pi r (\ell + r)$

$= \pi (9)(15 + 9)$

$= 216\pi\ cm^2$.



15

$\because$ The volume of the tank $= \frac{1}{3}\pi r^2 h$

$\therefore 32\cancel{\pi} = \frac{1}{3}\cancel{\pi} r^2 \times 6$

$r^2 = 16$ $\qquad \therefore r = 4$ m.

$\ell = \sqrt{4^2 + 6^2} = 2\sqrt{13}$ m.

Its total area $= \pi r (\ell + r)$

$= \pi \times 4 \times (2\sqrt{13} + 4)$

$= \left(16 + 8\sqrt{13}\right)\pi \simeq 140.9\ m^2$.

16

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (15)^2 \times 20 = 1500\pi\ cm^3$.

$\simeq 4712.4\ cm^3$.

, $\because$ the side length of the pyramid base $= 48 \div 4$

$= 12$ cm.

$\therefore$ Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times (12)^2 \times 40$

$= 1920\ cm^3$.

$\therefore$ Volume of the cone > volume of the pyramid

17

Volume of the cone $= \frac{1}{3}$ the base area $\times$ h

$\pi h^3 = \frac{1}{3} \times \pi r^2 \times h \quad \therefore r^2 = 3h^2$

The lateral area $= \pi r \ell = \pi r \sqrt{r^2 + h^2}$

$= \pi r \sqrt{3h^2 + h^2} = 2\pi r h$

= the circumference of the cylinder base × height

= The lateral area of the cylinder

18

$\because$ Volume of the cone $= \frac{1}{3} \times \pi (2)^2 \times 12 = 16\pi\ cm^3$.

$\therefore$ Volume of the dislodges water in cylinder form

$= 16\pi\ cm^3$.

$\therefore 16\pi = \pi r'^2 \times 1 \qquad \therefore r'^2 = 16$

$r' = 4$ cm.

length of the diameter of the base of the vessal

$= 4 \times 2 = 8$ cm.

19

Volume of the wax = Volume of cube

$= (20)^3 = 8000\ cm^3$.

, $\because$ 12% of wax had been lost during the melting and reforming

$\therefore$ The volume of the cone $= 88\% \times 8000$

$= \frac{88}{100} \times 8000 = 7040\ cm^3$.

, $\because$ volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 21 = 7040$

$\therefore r^2 = 320 \qquad \therefore r = 8\sqrt{5}$ cm.

20

Capacity of the cone = 2.2 litres

$= 2.2 \times 1000 = 2200\ cm^3$.

The volume $= \frac{1}{3}\pi r^2 h$

$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 21 = 2200$

$\therefore r^2 = 100 \Rightarrow r = 10$ cm.

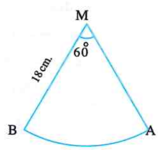
21

The length of the drawer of the cone = 18 cm.

, $\because$ the circumference of circle of the cone $= \overset{\frown}{AB} = r \times \theta^{rad}$

$= 18 \times \frac{60° \times \pi}{180°}$

$\therefore 2\pi r' = 6\pi \qquad \therefore r' = 3$ cm.



$\therefore h = \sqrt{\ell^2 - r^2} = \sqrt{(18)^2 - 3^2}$

$= 3\sqrt{35}$

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (3)^2 \times 3\sqrt{35}$

$\simeq 167.3 \text{ cm}^3.$

22

The length of the drawer of the cone = 20 cm.

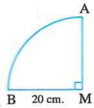
, the perimeter of the circle of the cone

= the length of $\widehat{AB} = r\,\theta^{rad}$

$= 20 \times \frac{90° \times \pi}{180°} = 10\pi$

$\therefore 2\pi r = 10\pi \qquad \therefore r = 5$ cm.

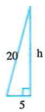
, $h = \sqrt{(20)^2 - (5)^2} = 5\sqrt{15}$



Volume of the cone $= \frac{1}{3}\pi r^2 h$

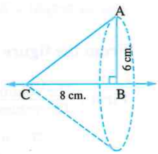
$= \frac{1}{3} \times \pi \times (5)^2 \times 5\sqrt{15}$

$= \frac{125\sqrt{15}}{3}\pi \text{ cm}^3.$



23

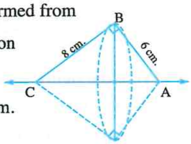
(1) The formed solid is a right cone whose base radius = 6 cm. and its height = 8 cm.



The volume $= \frac{1}{3} \times \pi \times 6^2 \times 8$

$= 96\pi \text{ cm}^3.$

(2) The formed solid is formed from two cones with common base whose radius $= \frac{6 \times 8}{10} = 4.8$ cm.



$\because$ length of the 1st drawer = 8 cm.

$\therefore h_1 = \sqrt{8^2 - 4.8^2} = 6.4$ cm.

$\because$ length of the 2nd drawer = 6 cm.

$\therefore h_2 = \sqrt{6^2 - 4.8^2} = 3.6$ cm.

$\therefore$ Volume of resultant solid

$= \frac{1}{3} \times \pi \times (4.8)^2 \times 6.4 + \frac{1}{3} \times \pi \times (4.8)^2 \times 3.6$

$= 76.8\pi \text{ cm}^3.$

24

(1) About X-axis form a cone

$r_1 = 3$ length units , $h_1 = 4$ length units

$\therefore$ The volume $= \frac{1}{3}\pi (3)^2 \times 4 = 12\pi$ cubic units

(2) About y-axis

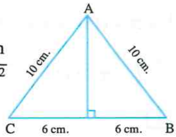
$r_2 = 4$ length units , $h_2 = 3$ length units

$\therefore$ The volume $= \frac{1}{3}\pi (4)^2 \times 3 = 16\pi$ cubic units

25

The formed solid as two congruent cones with common base whose radius $= \sqrt{10^2 - 6^2}$

$= 8$ cm.



, height of each = 6 cm.

The volume $= 2 \times \frac{1}{3}\pi (8)^2 \times 6 = 256\pi \text{ cm}^3.$

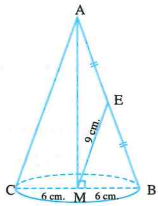
26

$\because \overline{ME}$ is a median drawn from the right vertex in Δ AMB

$\therefore$ AB (The length of drawer of the cone) $= 9 \times 2 = 18$ cm.

$\therefore$ The lateral area $= \pi r \ell$

$= \pi \times 6 \times 18 = 108\pi \text{ cm}^2.$



, the total area $= \pi r (r + \ell)$

$= \pi \times 6 \times (6 + 18) = 144\pi \text{ cm}^2.$

The height $= \sqrt{(18)^2 - 6^2} = 12\sqrt{2}$ cm.

The volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (6)^2 \times 12\sqrt{2} = 144\sqrt{2}\pi \text{ cm}^3.$

27

* **First cone :** $\ell_1 = 80$ cm. , $r_1 = 50$ cm.

$\therefore$ The lateral area $= \pi \times 80 \times 50 = 4000\pi \text{ cm}^2.$

* **Second cone :** $h_2 = 120$ cm. , $r_2 = 50$ cm.

$\therefore \ell_2 = \sqrt{(120)^2 + (50)^2} = 130$ cm.

$\therefore$ The lateral area $= \pi \times 130 \times 50 = 6500\pi \text{ cm}^2.$

The total area wanted for painting is the sum of two lateral areas of the two cones $= 4000\pi + 6500\pi$

$= 10500\pi \simeq 32987 \text{ cm}^2 \simeq 3.3 \text{ m}^2.$

The cost $= 3.3 \times 300 = 990$ pounds

28

Volume of the pentagonal pyramid

$= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3}\left(\frac{5}{4} \times 10^2 \times \cot \frac{\pi}{5} \times 42\right)$

$= 2408.67 \text{ cm}^3.$

Volume of the cone = 90% of volume of pyramid

$= \frac{90}{100} \times 2408.67 = 2167.8 \text{ cm}^3.$

$\therefore \frac{1}{3} \pi r^2 h = 2167.8$

$\frac{1}{3} \times \pi \times (15)^2 \times h = 2167.8$

$h = \dfrac{2167.8}{\frac{1}{3} \times \pi \times 15^2} \simeq 9.2 \text{ cm.}$

29

The volume of the cone $= \frac{1}{3} \pi r^2 h = 100 \text{ cm}^3.$

(1) After doubling its height

$\therefore$ The volume of the resultant cone

$= \frac{1}{3} \pi r^2 (2h) = 2\left[\frac{1}{3} \pi r^2 h\right]$

$= 2 \times 100 = 200 \text{ cm}^3.$

(2) After doubling its radius length

$\therefore$ The volume of the resultant cone

$= \frac{1}{3} \pi (2r)^2 (h) = \frac{1}{3} \pi \times 4 r^2 h$

$= 4\left[\frac{1}{3} \pi r^2 h\right] = 4 \times 100 = 400 \text{ cm}^3.$

(3) After doubling its height and radius length

$\therefore$ The volume of the resultant cone

$= \frac{1}{3} \pi (2r)^2 (2h)$

$= \frac{1}{3} \pi \times 4 r^2 \times 2h$

$= 8\left[\frac{1}{3} \pi r^2 h\right] = 8 \times 100 = 800 \text{ cm}^3.$

Third Higher skills

1

(1) b (2) c (3) d

(4) First : c Second : b

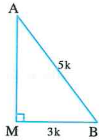
(5) a (6) (d) (7) c

Instructions to solve 1 :

(1) $\because$ The volume of hemisphere = The volume of the cone.

$\therefore \frac{1}{2}\left(\frac{4}{3} \pi r^3\right) = \frac{1}{3} \pi r^2 h \qquad \therefore h = 2r$

(2) $\because AM = \sqrt{(5k)^2 - (3k)^2} = 4k$



, $\because$ the volume of the cone $= 96\pi$

$\therefore \frac{1}{3} \pi \times (3k)^2 \times 4k = 96\pi$

$\therefore 12k^3 = 96 \qquad \therefore k^3 = 8$

$\therefore k = 2$

$\therefore r = 6$ cm. , $h = 8$ cm. , $\ell = 10$ cm.

$\therefore$ The total area $= \pi r (r + \ell) = \pi \times 6 \times (6 + 10)$

$= 96\pi \text{ cm}^2.$

(3) $\because$ The volume of the cone $= 49\pi$

$\therefore \frac{1}{3} \pi r^2 h = 49\pi \qquad \therefore \frac{1}{3} \pi r^2 \times 3 = 49\pi$

$\therefore r^2 = 49 \qquad \therefore r = 7$ cm.

$\therefore$ The length of the arc of the folded sector

$= 2\pi r = 2\pi \times 7 = 14\pi$ cm.

(4) Let the radius of the smallest cone $= r_1$

and its height $= h_1$ and its drawer $= \ell_1$

Let the radius of the greatest cone $= r_2$

and its height $= h_2$ and its drawer $= \ell_2$

From the figure we find : $\frac{h_1}{h_2} = \frac{\ell_1}{\ell_2} = \frac{r_1}{r_2} = \frac{1}{2}$

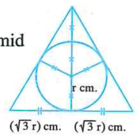
First : $\dfrac{\text{The volume of the smallest cone}}{\text{The volume of the greatest cone}}$

$= \dfrac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$

Second : $\dfrac{\text{The lateral area of the smallest cone}}{\text{The lateral area of the greatest cone}}$

$= \dfrac{\pi r_1 \ell_1}{\pi r_2 \ell_2} = \frac{r_1}{r_2} \times \frac{\ell_1}{\ell_2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(5) **From the figure we find :**



the area of the base of the pyramid

$= \frac{1}{2} \times \left(2\sqrt{3}\, r\right) \times 3r$

$= 3\sqrt{3}\, r^2$

, the area of the base of the cone $= \pi r^2$

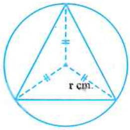
$\therefore \dfrac{\text{The volume of the regular triangular pyramid}}{\text{The volume of the greatest cone can be put inside the pyramid}}$

$= \dfrac{\frac{1}{3} \times 3\sqrt{3}\, r^2 \times h}{\frac{1}{3} \times \pi r^2 \times h} = \dfrac{3\sqrt{3}}{\pi}$

(6) **From the figure we find :**

the area of the base of the pyramid

$= 3 \times \left(\frac{1}{2} r^2 \sin 120°\right)$



$= \frac{3\sqrt{3}}{4} r^2$

, the area of the base of the cone $= \pi r^2$

$\therefore \frac{\text{The volume of the regular triangular pyramid}}{\text{The volume of the smallest cone that contain it}}$

$= \frac{\frac{1}{3} \times \frac{3\sqrt{3}}{4} r^2 \times h}{\frac{1}{3} \times \pi r^2 \times h} = \frac{3\sqrt{3}}{4\pi}$

(7) Let the radius of the base of the cone = r and its height = h and its volume = v

after increasing we find that the radius of the base of the cone $= \frac{3}{2}$ r , its height $= \frac{3}{2}$ h and its volume $\grave{v}$

$\therefore \frac{\grave{v}}{v} = \frac{\frac{1}{3}\pi\left(\frac{3}{2} r\right)^2 \times \left(\frac{3}{2} h\right)}{\frac{1}{3}\pi \times r^2 \times h} = \frac{9}{4} \times \frac{3}{2} = \frac{27}{8}$

$\therefore \grave{v} = \frac{27}{8} v$ $\qquad \therefore \grave{v} = 337.5 \% \ v$

2

$L_1 : 3x - \sqrt{3}\, y = -6$ dividing by (-6)

$\therefore \frac{x}{-2} + \frac{y}{2\sqrt{3}} = 1$

$L_2 : \sqrt{3}\, x + y = 2\sqrt{3}$ dividing by $\left(2\sqrt{3}\right)$

$\therefore \frac{x}{2} + \frac{y}{2\sqrt{3}} = 1$

$\therefore A\left(0 , 2\sqrt{3}\right) , B(2 , 0) , C(-2 , 0)$

by rotation about x-axis make two cones with the same base of radius length $= 2\sqrt{3}$ unit and the same height (h = 2 units)

$\therefore$ The two cones are congruent

The volume of the resultant solid

$= 2 \times \frac{1}{3} \times \pi \times \left(2\sqrt{3}\right)^2 \times 2 = 16\pi$ cubic unit.

3

(1) The formed solid from the rotation is a right circular cylinder added to it a right circular cone of height (7 – 4) = 3 cm. and base radius = 3.5 cm.

$\therefore$ The volume $= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \pi \times (3.5)^2 \times 4 + \frac{1}{3}\pi \times (3.5)^2 \times 3$

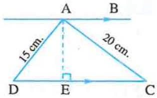
$= \frac{245}{4}\pi \simeq 192.4 \text{ cm}^3.$

(2) The formed solid is a right circular cone formed from the rotation of Δ CAB , its radius length of the base = 6 cm. , its height = 8 cm. subtracted from it a right circular cone formed from the rotation of the unshaded triangle and the radius length of its base = 3 cm. and its height = 8 cm.

$\therefore$ The volume $= \frac{1}{3}\pi (6)^2 \times 8 - \frac{1}{3}\pi (3)^2 \times 8$

$= 72\pi \simeq 226.2 \text{ cm}^3.$

(3) The formed solid from the rotation around $\overleftrightarrow{AB}$ is a right circular cylinder its base radius = AE and height CD subtracted from it two right circular cones the base radius of each of them = AE and their heights are CE , ED



From the figure we notice that :

$CD = \sqrt{(15)^2 + (20)^2} = 25$ cm.

$, AE = \frac{15 \times 20}{25} = 12$ cm.

$\therefore$ The volume $= \pi (12)^2 \times 25 - \left(\frac{1}{3}\pi (12)^2 \times CE + \frac{1}{3}\pi (12)^2 \times ED\right)$

$= \pi \times (12)^2 \times 25 - \frac{1}{3}\pi (12)^2 \times (CE + ED)$

$= \pi \times (12)^2 \times 25 - \frac{1}{3}\pi \times (12)^2 \times 25$

$= \pi \times (12)^2 \times 25 \times \frac{2}{3} = 2400\pi$

$\simeq 7539.8 \text{ cm}^3.$

Exercise 9

First Multiple choice questions

(1) b	(2) c	(3) b	(4) b	(5) c
(6) c	(7) b	(8) b	(9) c	(10) c
(11) c	(12) d	(13) a	(14) d	(15) d
(16) c	(17) c	(18) c	(19) b	(20) a
(21) a	(22) c	(23) b	(24) d	(25) a
(26) b	(27) b	(28) a	(29) d	(30) d
(31) d	(32) b	(33) b	(34) a	(35) b
(36) d	(37) d	(38) a	(39) d	(40) d
(41) c	(42) b	(43) a	(44) a	(45) b
(46) b	(47) d	(48) b	(49) c	(50) b
(51) d	(52) a	(53) a	(54) b	(55) d
(56) d	(57) c	(58) a	(59) d	(60) a
(61) d	(62) d	(63) c	(64) a	(65) d
(66) a	(67) d	(68) c	(69) b	(70) d
(71) c	(72) a	(73) a	(74) c	(75) b
(76) c	(77) b	(78) a	(79) d	(80) b

Second Essay questions

1

(1) $(x-2)^2+(y-3)^2=25$

(2) $x^2+y^2=9$

(3) $(x-0)^2+(y+1)^2=12$

(4) $(x+4)^2+(y+3)^2=\frac{9}{4}$

2

(1) $(x+2)^2+(y-3)^2=4^2$

i.e. $x^2+y^2+4x-6y-3=0$

(2) $r=\sqrt{(5-0)^2+(-12-0)^2}=13$

$\therefore (x-5)^2+(y+12)^2=(13)^2$

i.e. $x^2+y^2-10x+24y=0$

(3) $r=\sqrt{(7-3)^2+(-5-2)^2}=\sqrt{65}$

$\therefore (x-7)^2+(y+5)^2=(\sqrt{65})^2$

i.e. $x^2+y^2-14x+10y+9=0$

(4) The centre of the circle $M=\left(\frac{6+0}{2}, \frac{-4+2}{2}\right)=(3,-1)$

$r=\sqrt{(6-3)^2+(-4+1)^2}=3\sqrt{2}$

$\therefore (x-3)^2+(y+1)^2=(3\sqrt{2})^2$

i.e. $x^2+y^2-6x+2y-8=0$

(5) $\ell=3, k=2$

$\because$ The circle touches x-axis

$\therefore r=|k|=2, c=\ell^2=9$

$\therefore$ The equation of the circle is :

$x^2+y^2+6x+4y+9=0$

(6) $\ell=-3, k=0$

$\because$ The circle touches y-axis

$\therefore r=|\ell|=3, c=k^2=0$

$\therefore$ The equation of the circle is : $x^2+y^2-6x=0$

(7) $\ell=-5, k=5$

$\because$ The circle touches the two coordinate axes

$\therefore r=|\ell|=|k|=5, c=25$

$\therefore$ The equation of the circle is :

$x^2+y^2-10x+10y+25=0$

(8) $\because$ The two tangents at A and B are parallel

$\therefore \overline{AB}$ is a diameter in the circle

$\therefore$ The centre of the circle $M=\left(\frac{6+0}{2}, \frac{2-1}{2}\right)=\left(3, \frac{1}{2}\right)$

$\therefore r=\sqrt{(6-3)^2+\left(2-\frac{1}{2}\right)^2}=\frac{3\sqrt{5}}{2}$

$\therefore$ The equation of the circle is

$(x-3)^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{3\sqrt{5}}{2}\right)^2$

i.e. $x^2+y^2-6x-y-2=0$

(9) The centre of the circle $M=(5, 0), r=3$

$\therefore$ The equation of the circle is

$(x-5)^2+(y-0)^2=(3)^2$

i.e. $x^2+y^2-10x+16=0$

(10) $\because$ The circle touches the two coordinate axes, and it lies in the 4th quadrant.

$\therefore c=\ell^2=k^2=r^2=36$

, The centre of the circle $M=(6,-6)$

$\because$ The equation of the circle is :

$(x-6)^2+(y-6)^2=(6)^2$

i.e. $x^2+y^2-12x+12y+36=0$

3

(1) The centre of the circle $=(0, 0)$

, $r=\sqrt{8}=2\sqrt{2}$ length units

(2) The centre of the circle $=(-3, 5)$

, $r=7$ length units

(3) The centre of the circle $=(-4, 0)$

, $r=3$ length units

(4) The centre of the circle $=(0, -7)$

, $r=\sqrt{24}=2\sqrt{6}$ length units

(5) $\because \ell=-2, k=3, c=-12$

$\therefore$ The centre of the circle $=(2, -3)$

, $r=\sqrt{\ell^2+k^2-c}=\sqrt{4+9+12}=5$ length units

(6) $\because \ell=0, k=2, c=-8$

$\therefore$ The centre of the circle $=(0, -2)$

, $r=\sqrt{\ell^2+k^2-c}=\sqrt{0+4+8}=2\sqrt{3}$ length units

(7) $\because \ell=-2, k=-1, c=0$

$\therefore$ The centre of the circle $=(2, 1)$

, $r=\sqrt{\ell^2+k^2-c}=\sqrt{4+1-0}=\sqrt{5}$ length units

(8) $\because \ell=-4, k=0, c=-12$

$\therefore$ The centre of the circle $=(4, 0)$

, $r=\sqrt{\ell^2+k^2-c}=\sqrt{16+0+12}=2\sqrt{7}$ length units

4

(1) $\because x^2+y^2-4x+8y=0$

$\therefore \ell=-2 , k=4 , c=0$

$\therefore r_1=\sqrt{\ell^2+k^2-c}=\sqrt{4+16}=2\sqrt{5}$ length units

, $\because x^2+y^2+12y+16=0$

$\therefore \ell=0 , k=6 , c=16$

$\therefore r_2=\sqrt{\ell^2+k^2-c}=\sqrt{0+36-16}$

$=2\sqrt{5}$ length units

$\therefore r_1=r_2$

$\therefore$ The two circles are congruent.

(2) $\because x^2+y^2+14y=1$

$\therefore \ell=0 , k=7 , c=-1$

$\therefore r_1=\sqrt{\ell^2+k^2-c}=\sqrt{0+49+1}$

$=5\sqrt{2}$ length units

, $\because x^2+y^2+10x-25=0$

$\therefore \ell=5 , k=0 , c=-25$

$\therefore r_2=\sqrt{\ell^2+k^2-c}=\sqrt{25+25}$

$=5\sqrt{2}$ length units

$\because r_1=r_2$

$\therefore$ The two circles are congruent

(3) $\because x^2+y^2-2x+4y-3=0$

$\therefore \ell=-1 , k=2 , c=-3$

$\therefore r_1=\sqrt{\ell^2+k^2-c}=\sqrt{1+4+3}$

$=2\sqrt{2}$ length units

, $\because x^2+y^2+6x-11=0$

$\therefore \ell=3 , k=0 , c=-11$

$\therefore r_2=\sqrt{\ell^2+k^2-c}=\sqrt{9+0+11}$

$=2\sqrt{5}$ length units

, $\because r_1 \neq r_2$

$\therefore$ The two circles are not congruent

5

(1) C_1 : its centre is $(12 , -1)$, $r=2$

$\therefore$ The equation of the circle C_1

is $(x-12)^2+(y+1)^2=4$

C_2 : its centre is $(6 , 2)$, $r=2$

$\therefore$ the equation of the circle C_2

is $(x-6)^2+(y-2)^2=4$

C_3 : its centre is $(1 , 5)$, $r=1$

$\therefore$ The equation of the circle C_3

is $(x-1)^2+(y-5)^2=1$

C_4 : its centre is $(0 , 0)$, $r=3$

$\therefore$ The equation of the circle C_4

is $x^2+y^2=9$

C_5 : its centre is $(-9 , 1)$, $r=5$

$\therefore$ The equation of the circle C_5

is $(x+9)^2+(y-1)^2=25$

(2) $\because r_1=r_2=2$ $\therefore C_1$ and C_2 are congruent.

6 $\because r_1=r_2=2$ length units

$\therefore$ The two circles are congruent

The centre of the circle $C_1=(0 , 0)$

The centre of the circle $C_2=(5 , 2)$

$\therefore$ The equation of the circle C_1 is $x^2+y^2=4$

The equation of the circle C_2

is $(x-5)^2+(y-2)^2=4$

The equation of the circle C_3

is $(x+4)^2+(y-3)^2=4$

7

(1) $\because$ The equation include xy

$\therefore$ The equation does not represent a circle.

(2) $\because$ coefficient of x^2 = coefficient of $y^2=1$

and the equation has no terms of xy ,

$\ell^2+k^2-c=(4)^2+(-8)^2+1=81>0$

$\therefore$ The equation represents a circle.

(3) $\because$ The coefficient of $x^2 \neq$ the coefficient of y^2

$\therefore$ The equation does not express a circle.

(4) $x^2+y^2+\frac{3}{2}y-4=0$

$\because$ The coefficient of x^2 = the coefficient of $y^2=1$

and the equation is empty from xy ,

$\ell^2+k^2-c=(0)^2+\left(\frac{3}{2}\right)^2+4=6\frac{1}{4}>0$

$\therefore$ The equation expresses a circle.

(5) $x^2+y^2+2xy-3x+6y-4=0$

$\because$ The equation includes the term $2xy$

$\therefore$ The equation does not express a circle.

(6) $\because$ The coefficient of x^2 = the coefficient of $y^2 = 1$ and the equation is empty from xy

$, \ell^2 + k^2 - c = \left(\frac{1}{2}\right)^2 + (1)^2 - 7 = -\frac{23}{4} < 0$

$\therefore$ The equation does not represent a circle.

(7) $\because$ The coefficient of x^2 = the coefficient of $y^2 = 1$ and the equation does not include xy

$, \ell^2 + k^2 - c = (1)^2 + (-2)^2 - 5 = 0$

$\therefore$ The equation does not represent a circle.

(8) $x^2 + y^2 + 4x - 32 = 0$

$\because$ The coefficient of x^2 = the coefficient of $y^2 = 1$ and the equation has no term in xy.

$, \ell^2 + k^2 - c = (2)^2 + 32 = 36 > 0$

$\therefore$ The equation represents a circle.

(9) $x^2 - y^2 + x - y - 7 = 0$

$\because$ The coefficient of $x^2 \neq$ the coefficient of y^2

$\therefore$ The equation does not express a circle.

8 $r = \sqrt{(2+1)^2 + (-1-3)^2} = 5$ length unit

$\therefore$ The equation of the first circle C_1 is

$(x-2)^2 + (y+1)^2 = 25$

, the equation of the second circle C_2 is

$(x+1)^2 + (y-3)^2 = 25$

9 $\because C_1 : x^2 + y^2 - 2x + 6y + 1 = 0$

$\therefore M_1 = (1, -3)$, $r_1 = \sqrt{(-1)^2 + (3)^2 - 1}$

$= 3$ length unit

$\because C_2 : x^2 + y^2 - 2x + 6y + \frac{15}{4} = 0$

$\therefore M_2 = (1, -3)$, $r_2 = \sqrt{(-1)^2 + (3)^2 - \frac{15}{4}}$

$= 2.5$ length unit

, the two circles are concentric

10 $(x-6)^2 + (y+1)^2 = 25$ substituting by the coordinates of the points A , B , C and D we get

- A (9 , 3) $\therefore (9-6)^2 + (3+1)^2 = 25 = r^2$

$\therefore$ The point A lies on the circle

- B (7 , 5) $\therefore (7-6)^2 + (5+1)^2 = 37 > r^2$

$\therefore$ The point B is outside the circle

- C (3 , 3) $\therefore (3-6)^2 + (3+1)^2 = 25 = r^2$

$\therefore$ The point C lies on the circle

- D (2 , –3) $\therefore (2-6)^2 + (-3+1)^2 = 20 < r^2$

$\therefore$ The point D lies inside the circle

11 $r = \sqrt{(2+1)^2 + (-1-3)^2} = 5$ and the equation of the circle is $(x-2)^2 + (y+1)^2 = 25$

substituting by the coordinates of the points B , C and D

- B (2 , 4) $\therefore (2-2)^2 + (4+1)^2 = 25 = r^2$

$\therefore$ The point B lies on the circle

- C (–3 , 1) $\therefore (-3-2)^2 + (1+1)^2 = 29 > r^2$

$\therefore$ The point C lies outside the circle

- D (1 , 2) $\therefore (1-2)^2 + (2+1)^2 = 10 < r^2$

$\therefore$ The point D lies inside the circle

12 The centre of the circle = (– 3 , 4)

, r = 3 length unit

(1) The length of the perpendicular

$= \frac{|3 \times -3 - 4 \times 4 + 5|}{\sqrt{3^2 + 4^2}} = 4 > r$

$\therefore$ The straight line L_1 is outside the circle

(2) The length of the perpendicular

$= \frac{|6 \times -3 - 8 \times 4 + 23|}{\sqrt{6^2 + (-8)^2}} = 2.7 < r$

$\therefore$ The straight line L_2 is secant to the circle

(3) The length of the perpendicular

$= \frac{|3 \times -3 - 4 \times 4 + 10|}{\sqrt{3^2 + (-4)^2}} = 3 = r$

$\therefore$ The straight line L_3 is a tangent to the circle

13 The centre of the circle M = (3 , – 2)

$r = \sqrt{(-3)^2 + 2^2 + 12} = 5$ length unit

The length of the perpendicular

$= \frac{|5 \times 3 - 12 \times -2 + 13|}{\sqrt{5^2 + (-12)^2}} = 4 < r$

$\therefore$ The straight line is a secant to the circle

14 • $C_1 : (x-5)^2 + (y+2)^2 = 4$

$\therefore M_1 = (5, -2)$, $r_1 = 2$ length unit

- $C_2 : (x+7)^2 + (y-3)^2 = 1$

$\therefore M_2 = (-7, 3)$, $r_2 = 1$ length unit

$\therefore r_1 + r_2 = 3$ length unit

, $M_1M_2 = \sqrt{(5+7)^2 + (-2-3)^2} = 13$ length unit

$\because M_1M_2 > r_1 + r_2$

$\therefore$ The two circles are disjoint (distant circles)

15 • $C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$

$\therefore M_1 = (5 , 4)$

$, r_1 = \sqrt{(-5)^2 + (-4)^2 - 16} = 5$ length unit

• $C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$

$\therefore M_2 = (-7 , -5)$

$, r_2 = \sqrt{7^2 + 5^2 + 26} = 10$ length unit

$\therefore r_1 + r_2 = 15$ length unit.

$, M_1M_2 = \sqrt{(5+7)^2 + (4+5)^2} = 15$ length unit

$\because M_1M_2 = r_1 + r_2$

$\therefore$ The two circles are touching externally

16 • $C_1 : (x+2)^2 + y^2 = 1$

$\therefore M_1 = (-2 , 0)$ $, r_1 = 1$ length unit

• $C_2 : x^2 + y^2 - 2x - 8y - 19 = 0$

$\therefore M_2 = (1 , 4)$

$, r_2 = \sqrt{(-1)^2 + (-4)^2 + 19} = 6$ length unit

$, M_1M_2 = \sqrt{(-2-1)^2 + (0-4)^2} = 5$ length unit

$\therefore r_2 - r_1 = 5$ $\qquad \therefore r_2 - r_1 = M_1M_2$

$\therefore$ The two circles are touching internally.

17 • $C_1 : (x+2)^2 + (y+11)^2 = K$

$\therefore M_1 = (-2 , -11)$ $, r_1 = \sqrt{K}$ length unit

• $C_2 : (x-3)^2 + (y-1)^2 = 16$

$\therefore M_2 = (3 , 1)$ $, r_2 = 4$ length unit

If the two circles are touching externally

$\therefore M_1M_2 = r_1 + r_2$

$\therefore \sqrt{(3+2)^2 + (1+11)^2} = 4 + \sqrt{K}$

$\therefore 13 = 4 + \sqrt{K}$ $\qquad \therefore K = 81$

If the two circles are touching internally

$\therefore |r_2 - r_1| = M_1M_2$ $\qquad \therefore |4 - \sqrt{K}| = 13$

$\therefore 4 - \sqrt{K} = 13$ refused or $4 - \sqrt{K} = -13$

$\therefore \sqrt{K} = 4 + 13 = 17$ $\qquad \therefore K = 289$

18 The point of tangency :

$x^2 + y^2 - 6x - 4y + 12 = x^2 + y^2 + 2x - 4y - 4$

$\therefore -6x - 4y + 12 = 2x - 4y - 4$

$\therefore -6x - 2x = -12 - 4$

$\therefore -8x = -16$

$\therefore x = 2$

By substitution in the first circle equation by $x = 2$

$\therefore (2)^2 + y^2 - 6 \times 2 - 4y + 12 = 0$

$\therefore y^2 - 4y + 4 = 0$ $\qquad \therefore y = 2$

$\therefore$ The two circles intersected at one point $(2 , 2)$

$\therefore$ The two circles touch each other

, the equation of the circle whose center $(2 , 2)$ and passes through the center of the second circle $(-1 , 2)$

$r = \sqrt{(2+1)^2 + (0)^2} = 3$

$\therefore$ The equation : $(x-2)^2 + (y-2)^2 = 9$

19 $x^2 + y^2 = 1$

$\because (2a\cos\theta , 2a\sin\theta) \in$ the circle

$\therefore (2a\cos\theta)^2 + (2a\sin\theta)^2 = 1$

$\therefore 4a^2(\cos^2\theta + \sin^2\theta) = 1$

$\therefore 4a^2 = 1$ $\qquad \therefore a = \pm \frac{1}{2}$

20 (1) $\because \ell^2 + k^2 - c > 0$

$\therefore (-1)^2 + (-2)^2 + h - 2 > 0$

$\therefore h > -3$ $\qquad \therefore h \in]-3 , \infty[$

(2) $\because \ell^2 + k^2 - c > 0$

$\therefore (2)^2 + (-3)^2 + h^2 - 4 > 0$ $\qquad \therefore h^2 + 9 > 0$

$\therefore$ For all values of $h \in R$ the expression $h^2 + 9 > 0$

$\therefore h \in \mathbb{R}$

(3) $\because \ell^2 + k^2 - c > 0$

$\therefore (-2h)^2 + (-h)^2 - 10(h-1) > 0$

$\therefore 4h^2 + h^2 - 10h + 10 > 0$

$\therefore h^2 - 2h + 2 > 0$

$\therefore (h-1)^2 + 1 > 0$ this will be satisfied for all values of h which belongs to $\mathbb{R}$

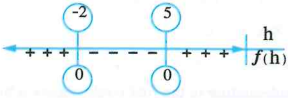
(4) $\because \ell^2 + k^2 - c > 0$

$\therefore (3)^2 + (4)^2 - h^2 + 3h - 15 > 0$

$\therefore 9 + 16 - h^2 + 3h - 15 > 0$

$\therefore h^2 - 3h - 10 < 0$

$\therefore (h+2)(h-5)<0$



$\therefore h \in]-2 , 5[$

(5) $\because \ell^2 + k^2 - c > 0$

$\therefore h^2 + (-3h)^2 + 2h^2 - 12h + 3 > 0$

$\therefore h^2 + 9h^2 + 2h^2 - 12h + 3 > 0$

$\therefore 12h^2 - 12h + 3 > 0$

$\therefore 4h^2 - 4h + 1 > 0$ $\qquad \therefore (2h-1)^2 > 0$

This will be satisfies for all $h \in \mathbb{R}$

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21 **(1)** $\because \ell^2 + k^2 - c > 0$

$\therefore (-1)^2 + (2)^2 - 2a + 3 > 0$

$\therefore -2a > -8$

$\therefore a < 4$ $\qquad \therefore a \in]-\infty , 4[$

(2) $\because$ The circle passes through the origin point

$\therefore c = 0$

$\therefore 2a - 3 = 0$ $\qquad \therefore a = \frac{3}{2}$

(3) $\because$ The circle touches X-axis

$\therefore c = \ell^2$

$\therefore 2a - 3 = (-1)^2$ $\qquad \therefore a = 2$

(4) $\because$ The circle touches y-axis

$\therefore c = k^2$ $\quad \therefore 2a - 3 = 2^2$ $\qquad \therefore a = \frac{7}{2}$

(5) $\because M = (1 , -2)$

$\because$ The circle touches the straight line

$3X + 4y + 15 = 0$

$\therefore r = \frac{|3 \times 1 + 4 \times -2 + 15|}{\sqrt{3^2 + 4^2}} = 2$ length units

$\because \sqrt{\ell^2 + k^2 - c} = r$

$\therefore \sqrt{(-1)^2 + (2)^2 - 2a + 3} = 2$ squaring the two sides

$\therefore 1 + 4 - 2a + 3 = 4$ $\qquad \therefore a = 2$

(6) $\because r = 7$ length unit

$\therefore \sqrt{(-1)^2 + (2)^2 - 2a + 3} = 7$

squaring the two sides

$\therefore 1 + 4 - 2a + 3 = 49$ $\qquad \therefore a = -\frac{41}{2}$

22

(1) $\because$ The circle touches the straight line $X = 2$

$\therefore r = 3$ length units

$\therefore$ The equation of the circle

is $(X-5)^2 + (y-4)^2 = 9$

i.e. $X^2 + y^2 - 10X - 8y + 32 = 0$

(2) The equation of the straight line

is $\frac{y-7}{X-3} = \frac{7-3}{3+1}$ $\qquad$ *i.e.* $-X + y - 4 = 0$

$\therefore r = \frac{|-1 \times 5 + 1 \times 3 - 4|}{\sqrt{(-1)^2 + (1)^2}} = 3\sqrt{2}$ length units

$\therefore$ The equation of the circle

is $(X-5)^2 + (y-3)^2 = 18$

i.e. $X^2 + y^2 - 10X - 6y + 16 = 0$

(3) $r = 3 , M = (4 , 5)$

$\therefore$ The equation of the circle

is $(X-4)^2 + (y-5)^2 = 9$

i.e. $X^2 + y^2 - 8X - 10y + 32 = 0$

(4) $\because$ The circle touches X-axis at $(4 , 0)$, $r = 5$

$\therefore M = (4 , 5)$ or $(4 , -5)$

$\therefore$ The equation of the circle

is $(X-4)^2 + (y-5)^2 = 25$

i.e. $X^2 + y^2 - 8X - 10y + 16 = 0$

or $(X-4)^2 + (y+5)^2 = 25$

i.e. $X^2 + y^2 - 8X + 10y + 16 = 0$

(5) $\because$ The circle touches y-axis

$\therefore M = (3\frac{1}{2} , -4)$ or $(-3\frac{1}{2} , -4)$

$\therefore$ The equation of the circle

is $\left(X - 3\frac{1}{2}\right)^2 + (y+4)^2 = 12.25$

i.e. $X^2 + y^2 - 7X + 8y + 16 = 0$

or the equation of the circle

is $\left(X + 3\frac{1}{2}\right)^2 + (y+4)^2 = 12.25$

i.e. $X^2 + y^2 + 7X + 8y + 16 = 0$

(6) $\because$ The circle touches the two coordinate axes

, $\because$ the point $(-2 , -4)$ in the third quadrant.

$\therefore$ The centre $M = (-r , -r)$

$\therefore$ The equation of the circle

is $(X+r)^2 + (y+r)^2 = r^2$

$\because (-2 , -4) \in$ the circle

$\therefore (-2+r)^2 + (-4+r)^2 = r^2$

$\therefore 4 + r^2 - 4r + 16 + r^2 - 8r = r^2$

$\therefore r^2 - 12r + 20 = 0$

$\therefore (r-2)(r-10) = 0$

$\therefore r = 2$, then $M = (-2 , -2)$

or $r = 10$, then $M = (-10 , -10)$

$\therefore$ There exist two circles , they are

$(x+2)^2 + (y+2)^2 = 4$

i.e. $x^2 + y^2 + 4x + 4y + 4 = 0$

or $(x+10)^2 + (y+10)^2 = 100$

i.e. $x^2 + y^2 + 20x + 20y + 100 = 0$

(7) $\because$ The circle touches x-axis at $(-3 , 0)$ and touches y-axis.

$\therefore r = 3$ length units

$\therefore$ There exist two circles whose centres are $(-3 , 3)$ $(-3 , -3)$

The equation of the first equation is $(x+3)^2 + (y-3)^2 = 9$

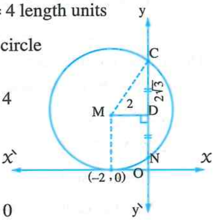
i.e. $x^2 + y^2 + 6x - 6y + 9 = 0$

The equation of the second equation

$(x+3)^2 + (y+3)^2 = 9$

i.e. $x^2 + y^2 + 6x + 6y + 9 = 0$

(8) $r = \sqrt{2^2 + (2\sqrt{3})^2} = 4$ length units

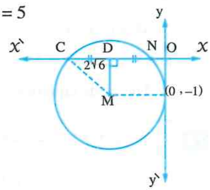


$\therefore$ The centre of the circle $M = (-2 , 4)$

$\therefore c = 4 + 16 - 16 = 4$

$\therefore$ The equation of the circle is $x^2 + y^2 + 4x - 8y + 4 = 0$

(9) $r = \sqrt{(1)^2 + (2\sqrt{6})^2} = 5$



$\therefore$ The centre of the circle M is $(-5 , -1)$

$\therefore c = k^2 = 1$

$\therefore$ The equation of the circle is $x^2 + y^2 + 10x + 2y + 1 = 0$

(10) $\because$ The circle touches x-axis.

$c = \ell^2$

$\therefore$ The equation of the circle is $x^2 + y^2 + 2\ell x + 2ky + \ell^2 = 0$

$\because (2 , 1)$ satisfies the equation.

$\therefore 4 + 1 + 4\ell + 2k + \ell^2 = 0$

$\therefore 4\ell + 2k + \ell^2 = -5$ (1)

$\because (-5 , 2)$ satisfies the equation

$\therefore 25 + 4 - 10\ell + 4k + \ell^2 = 0$

$\therefore -10\ell + 4k + \ell^2 = -29$ (2)

Multiplying (1) by 2

$\therefore 8\ell + 4k + 2\ell^2 = -10$ (3)

Subtracting (2) from (3) :

$\therefore \ell^2 + 18\ell - 19 = 0$

$\therefore (\ell - 1)(\ell + 19) = 0$

$\therefore \ell = 1$ or $\ell = -19$

, then $k = -5$ or $k = -145$

$\therefore$ There are two circles

The equation of the first circle is $x^2 + y^2 + 2x - 10y + 1 = 0$

and the equation of the other is $x^2 + y^2 - 38x - 290y + 361 = 0$

(11) $\because$ The circle touches y-axis. $\therefore c = k^2$

$\therefore$ The equation of the circle is $x^2 + y^2 + 2\ell x + 2ky + k^2 = 0$

$\because (-4 , 2)$ verifies the equation

$\therefore 16 + 4 - 8\ell + 4k + k^2 = 0$

i.e. $-8\ell + 4k + k^2 = -20$ (1)

$\because (-1 , 2)$ verifies the equation.

$\therefore 1 + 4 - 2\ell + 4k + k^2 = 0$

i.e. $-2\ell + 4k + k^2 = -5$ (2)

Subtracting (1) from (2) :

$\therefore 6\ell = 15 \quad \therefore \ell = \frac{5}{2}$

and from the equation (2)

$\therefore -5 + 4k + k^2 = -5 \quad \therefore k^2 + 4k = 0$

$\therefore k(k+4) = 0 \quad \therefore k = 0$ or $k = -4$

$\therefore$ There are two circles

The equation of the first circle is $x^2 + y^2 + 5x = 0$

The equation of the second circle is $x^2 + y^2 + 5x - 8y + 16 = 0$

(12) $\because$ The centre of the circle lies on x-axis

$\therefore$ The equation of the circle is $x^2 + y^2 + 2\ell x + c = 0$

$\because$ (1 , 3) verifies the equation

$\therefore 1 + 9 + 2\ell + c = 0$

, then $2\ell + c = -10$ (1)

$\because$ (2 , – 4) verifies the equation.

$\therefore 4 + 16 + 4\ell + c = 0$

, then $4\ell + c = -20$ (2)

From (1) and (2) :

$\therefore \ell = -5$, c = zero

$\therefore$ The equation of the circle

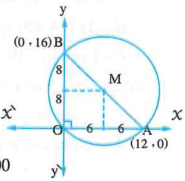
is $x^2 + y^2 - 10x = 0$

(13) M = (6 , 8)

, $AB = \sqrt{12^2 + 16^2} = 20$

$\therefore$ r = 10 length units

$\therefore$ The equation of the circle is



$(x - 6)^2 + (y - 8)^2 = 100$

i.e. $x^2 + y^2 - 12x - 16y = 0$

(14) $\because$ The centre of the circle lies on the straight line $y - x = 1$

$\therefore$ The centre $(-\ell , -k)$ verifies the equation.

$\therefore -k + \ell = 1$ (1)

$\therefore$ The equation of the circle

is $x^2 + y^2 + 2\ell x + 2ky + c = 0$

$\because$ (– 2 , 4) verifies the equation of the circle.

$\therefore 4 + 16 - 4\ell + 8k + c = 0$, then

$-4\ell + 8k + c = -20$ (2)

$\because$ (6 , 8) verifies the equation of the circle

$\therefore 36 + 64 + 12\ell + 16k + c = 0$, then

$12\ell + 16k + c = -100$ (3)

Subtracting (2) from (3) :

$\therefore 16\ell + 8k = -80$

$\therefore 2\ell + k = -10$ (4)

Adding (4) and (1) :

$\therefore 3\ell = -9 \quad \therefore \ell = -3$

From (1) : $\therefore$ k = – 4 and from (2) c = zero

$\therefore$ The equation of the circle

is $x^2 + y^2 - 6x - 8y = 0$

(15) $\because r^2 = \ell^2 + k^2 - c \quad \therefore 85 = \ell^2 + k^2 - c$ (1)

The equation of the circle

is $x^2 + y^2 + 2\ell x + 2ky + c = 0$

$\because$ (– 1 , 2) verifies the equation of the circle.

$\therefore 1 + 4 - 2\ell + 4k + c = 0$

i.e. $-2\ell + 4k + c = -5$ (2)

$\because$ (3 , 4) verifies the equation of the circle.

$\therefore 9 + 16 + 6\ell + 8k + c = 0$

i.e. $6\ell + 8k + c = -25$ (3)

Subtract (2) from (3) :

$8\ell + 4k = -20$

$\therefore k = -5 - 2\ell$ (4) substituting in (2)

$\therefore -2\ell + 4(-5 - 2\ell) + c = -5$

$\therefore c = 15 + 10\ell$ (5)

Substituting from (4) and (5) in the equation (1)

$\therefore 85 = \ell^2 + (-5 - 2\ell)^2 - 15 - 10\ell$

$\therefore \ell^2 + 25 + 20\ell + 4\ell^2 - 15 - 10\ell - 85 = 0$

$\therefore 5\ell^2 + 10\ell - 75 = 0$

$\therefore \ell^2 + 2\ell - 15 = 0$

$\therefore (\ell - 3)(\ell + 5) = 0 \qquad \therefore \ell = 3$ or $\ell = -5$

$\therefore$ k = – 11 or k = 5 , c = 45 or c = – 35

$\therefore$ There are two equations :

$x^2 + y^2 + 6x - 22y + 45 = 0$

or $x^2 + y^2 - 10x + 10y - 35 = 0$

(16) To find the intersection poins with x-axis , put y = 0 in the circle equation

$\therefore x^2 + 2x = 0$

$\therefore x(x + 2) = 0 \qquad \therefore x = 0$ or $x = -2$

$\therefore$ The intersection points are (0 , 0) , (– 2 , 0)

$\therefore$ M = the center of the required circle is

the mid point of $\overline{AB} = \left(\frac{0-2}{2} , \frac{0}{2}\right)$

$\therefore$ M = (– 1 , 0)

and its radius $r = MA = \sqrt{(-1)^2 + (0)^2} = 1$

$\therefore$ The cirde equation : $(x + 1)^2 + y^2 = 1$

23 $\because \ell = \frac{1}{2}$, k = – 2 , c = – 2

$\therefore r^2 = \frac{1}{4} + 4 + 2 = 6\frac{1}{4}$

$\therefore$ The area of $\Delta = \frac{n}{2} r^2 \sin\left(\frac{360°}{n}\right)$

$= \frac{3}{2} \times 6\frac{1}{4} \times \sin 120°$

$= \left(\frac{75\sqrt{3}}{16}\right)$ square units

24 $\because \ell = 3 , k = -6 , c = 5$

$\therefore r^2 = 9 + 36 - 5 = 40$

$\therefore$ The area of the regular pentagon

$= \frac{n}{2} r^2 \sin\left(\frac{360°}{n}\right)$

$= \frac{5}{2} \times 40 \times \sin 72 \simeq 95.10565$ square units

$\because$ The square unit represents an area of $(5)^2 = 25\ cm^2$

$\therefore$ The area of the required pentagon

$= 95.10565 \times 25 \simeq 2378\ cm^2$

25 $\ell = -5 , k = 3 , c = 25$

$\therefore r^2 = 25 + 9 - 25 = 9$

$\therefore$ The area of the regular hexagon

$= \frac{n}{2} r^2 \sin\left(\frac{360°}{n}\right)$

$= \frac{6}{2} \times 9 \sin 60° = \left(\frac{27\sqrt{3}}{2}\right)$ square units

26 $r^2 = 16$

$\therefore$ The area of the regular polygon

$= \frac{n}{2} r^2 \sin\left(\frac{360°}{n}\right)$

$= \frac{12}{2} \times 16 \times \sin 30° = 48$ square unit

27 The centre of the circle is the point of intersection of the two diameters.

$3X + y = -2$ (1)

$, 4X - y = 16$ (2)

From (1) and (2) :

$\therefore X = 2 , y = -8$

$\therefore$ The centre of the circle $M = (2 , -8)$

$\therefore$ The equation of the circle is $(X - 2)^2 + (y + 8)^2 = 25$

i.e. $X^2 + y^2 - 4X + 16y + 43 = 0$

Substituting by $X = 5$ and $y = -4$

$\therefore$ The left hand side $= (5)^2 + (-4)^2 - 4(5) + 16(-4) + 43 = 0$

= the right hand side

$\therefore (5 , -4) \in$ the circle.

28 $\because \vec{r} = (1 , 5) + k(1 , 2)$ *i.e.* $\frac{y-5}{X-1} = 2$

$\therefore 2X - 2 = y - 5$

$\therefore 2X - y = -3$ (1) $, X + y = 0$ (2)

From (1) and (2)

$\therefore$ The point of intersection of the two diameters is $(-1 , 1)$

$\therefore$ The centre of the circle $M = (-1 , 1)$

$\because X^2 + y^2 - 2X\cos\theta - 2y\sin\theta - 8 = 0$

$\therefore \ell = -\cos\theta , k = -\sin\theta , c = -8$

$r = \sqrt{\cos^2\theta + \sin^2\theta + 8} = \sqrt{1 + 8}$

$= 3$ length units

$\therefore$ The equation of the required circle is $(X + 1)^2 + (y - 1)^2 = 9$

29 The intersection points of the two circles

$X^2 + y^2 - 10X = X^2 + y^2 + 2X - 12$

$\therefore -10X - 2X = -12$

$\therefore -12X = -12$ $\therefore X = 1$

By substitution in the first circle equation to find the value of y by $X = 1$

$\therefore (1)^2 + y^2 - 10(1) = 0$

$\therefore y^2 = 9$ $\therefore y = \pm 3$

$\therefore$ The intersection points are $(1 , 3) , (1 , -3)$

(1) The centre of the circle is $(0 , 0)$:

distance between first point and the centre

$= \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}$

, distance between second point and the centre

$= \sqrt{(1-0)^2 + (-3-0)^2} = \sqrt{10}$

$\therefore$ The two points lie on a circle of centre $(0 , 0)$ and radius length $\sqrt{10}$ length unit

$\therefore$ Equation of the circle is : $X^2 + y^2 = 10$

(2) The centre of the circle is $(2 , 0)$:

distance between first point and the centre

$= \sqrt{(1-2)^2 + (3-0)^2} = \sqrt{10}$

, distance between second point and the centre

$= \sqrt{(1-2)^2 + (-3-0)^2} = \sqrt{10}$

$\therefore$ The two points lie on a circle of centre $(2 , 0)$ and radius length $\sqrt{10}$ length unit

$\therefore$ Equation of the circle is : $(X - 2)^2 + y^2 = 10$

30 $\because$ $MA = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$ length units

$MB = \sqrt{(-4)^2 + (-5)^2} = \sqrt{41}$ length units

$MC = \sqrt{(-4)^2 + 5^2} = \sqrt{41}$ length units

$\therefore$ $MA = MB = MC$

$\therefore$ A , B and C lie on the same circle.

The equation of the circle

is $(x+5)^2 + (y+5)^2 = 41$

---

31 The equation of the circle

is $x^2 + y^2 + 2\ell x + 2ky + c = 0$

$\because$ The points A , B and C lie on the circle.

$\therefore$ $9 + 4 + 6\ell - 4k + c = 0$ (1)

, $9 + 64 + 6\ell + 16k + c = 0$ (2)

, $1 - 2\ell + c = 0$ (3)

From (1) , (2) and (3)

$\therefore$ The centre of the circle M = (3 , 3)

$\therefore$ The equation of the circle

is $x^2 + y^2 - 6x - 6y - 7 = 0$

$\because$ The midpoint of $\overline{AB} = \left(\frac{3+3}{2}, \frac{-2+8}{2}\right)$

$= (3 , 3) = M$

$\therefore$ $\overline{AB}$ is a diameter in the circle.

---

32 $AB = \sqrt{(8-0)^2 + (0-6)^2} = 10$

, $BC = \sqrt{0 + (6-0)^2} = 6$

, $AC = \sqrt{(8-0)^2 + 0} = 8$

$\because$ $(AB)^2 = (BC)^2 + (AC)^2$

$\therefore$ ΔABC is right angled triangle at $\angle C$

$\therefore$ $\overline{AB}$ is a diameter in the circle which passes through its vertices

The center $M = \left(\frac{8+0}{2}, \frac{0+6}{2}\right) = (4 , 3)$, $r = 5$

$\therefore$ The equation : $(x-4)^2 + (y-3)^2 = 25$

---

33 $AB = \sqrt{6^2 + (0)^2} = 6$, $BC = \sqrt{3^2 + \left(-3\sqrt{3}\right)^2} = 6$

, $CA = \sqrt{(-3)^2 + \left(-3\sqrt{3}\right)^2} = 6$

$\therefore$ ΔABC is an equilateral triangle.

$\therefore$ The centre of its circumcircle

$= \left(\frac{-2+4+1}{3}, \frac{0+0+3\sqrt{3}}{3}\right) = \left(1 , \sqrt{3}\right)$

$\therefore$ $r = MA = \sqrt{(-3)^2 + \left(-3\sqrt{3}\right)^2}$

$= 2\sqrt{3}$ length units

$\therefore$ The equation of the circle

is $(x-1)^2 + \left(y - \sqrt{3}\right)^2 = 12$

---

34 $x^2 + y^2 + 2\ell x + 2ky + c = 0$

$\because$ (2 , – 1) , (– 2 , 0) , (0 , – 9)

Verify the equation of the circle.

$\therefore$ $4 + 1 + 4\ell - 2k + c = 0$

i.e. $4\ell - 2k + c = -5$ (1)

, $4 - 4\ell + c = 0$ (2) , $81 - 18k + c = 0$ (3)

From (1) , (2) and (3) :

$\therefore$ $\ell = 1$, $c = 0$, $k = \frac{9}{2}$

$\therefore$ The equation is $x^2 + y^2 + 2x + 9y = 0$

, the centre of the circle M = (– 1 , – 4.5)

, $r = \sqrt{1 + (4.5)^2} = \frac{\sqrt{85}}{2}$ length unit.

---

35 let the circle which passes through the points A , B and C be

$x^2 + y^2 + 2\ell x + 2ky + c = 0$

$\therefore$ The points verify the equation of the circle.

$\therefore$ $9 + 6\ell + c = 0$ (1)

, $81 + 18k + c = 0$ (2)

, $1 + 2k + c = 0$ (3)

From (1) , (2) and (3) :

$\therefore$ $\ell = -3$, $k = -5$, $c = 9$

$\therefore$ The equation of the circle

is $x^2 + y^2 - 6x - 10y + 9 = 0$

Substituting by the coordinates of the point D

$\therefore$ The left hand side $= (-1)^2 + (2)^2 - 6(-1) - 10(2) + 9$

$= 0$

= The right hand side.

$\therefore$ The point D lies on the circle.

i.e. The quadrilateral ABCD is cyclic.

---

36 (1) $x^2 + y^2 - 8x + 6y = 0$

(2) $x^2 + y^2 - 4x + 4y + 4 = 0$

(3) $x^2 + y^2 - 12y + 20 = 0$

(4) $x^2 + y^2 - 12x - 4y + 36 = 0$

(5) $x^2 + y^2 - 12x - 15y + 36 = 0$

(6) $x^2 + y^2 - 4x - 2y - 11 = 0$

(7) $x^2 + y^2 - 24x - 8\sqrt{3}y + 144 = 0$

(8) $x^2 + y^2 - 8x - 4y + 16 = 0$

(9) $x^2 + y^2 - 2x - 8y + 16 = 0$

37 $\because$ The equation of the straight line is $\frac{x}{3} + \frac{y}{4} = 1$

$\therefore$ DO = 3 , OE = 4

ED = 5

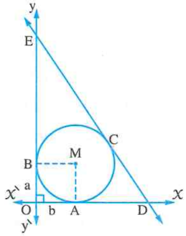
let AO = b

, OB = a

$\therefore$ DA = 3 − b

BE = 4 − a

From the geometry of the figure we get.



b = a

, AD = DC , BE = EC

$\therefore$ DC = 3 − b

, EC = 4 − a = 4 − b

$\because$ DC + EC = 5 $\qquad \therefore$ 3 − b + 4 − b = 5

$\therefore$ b = 1 , a = 1 , r = 1

$\therefore$ The centre of the circle is (1 , 1)

$\therefore$ The equation of the circle is $(x-1)^2 + (y-1)^2 = 1$

i.e. $x^2 + y^2 - 2x - 2y + 1 = 0$

38 Let the centre of the circle = $(x , 0)$

, $\because$ slope of $\overline{MA}$ × slope of $\overline{AB}$ = −1

$\therefore \frac{3}{2-x} \times \frac{3}{4} = -1 \qquad \therefore x - 2 = \frac{9}{4}$

$\therefore x = 4\frac{1}{4}$

$\therefore$ The centre of the circle M = $\left(4\frac{1}{4} , 0\right)$

, $\because r = MA = \sqrt{\left(4\frac{1}{4} - 2\right)^2 + (0-3)^2} = 3\frac{3}{4}$

$\therefore$ The equation of the circle is :

$\left(x - 4\frac{1}{4}\right)^2 + y^2 = \left(3\frac{3}{4}\right)^2$

i.e. $x^2 + y^2 - \frac{17}{2}x + 4 = 0$

i.e. $2x^2 + 2y^2 - 17x + 8 = 0$

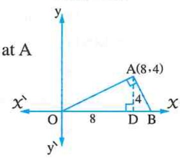
39 From the geometry of the figure :

$(4)^2 = 8 \times BD \qquad \therefore BD = 2$

$\therefore$ The point B (10 , 0)

$\because \Delta$ ABC is right angled at A

$\therefore \overline{BO}$ is a diameter in the circle which passes through the points A , B and O



$\therefore$ The centre of the circle is (5 , 0) , r = 5 units.

$\therefore$ The equation of the circle is $(x-5)^2 + y^2 = 25$

i.e. $x^2 + y^2 - 10x = 0$

Third Higher skills

1

(1) (d)	(2) (c)	(3) (b)	(4) (d)
(5) (b)	(6) (c)	(7) (b)	(8) (b)

Instructions to solve 1 :

(1) Put k − 2 = 2 − k $\qquad \therefore$ 2 k = 4 $\qquad \therefore$ k = 2

, then the equation be $-2x + 6y - 25 = 0$

«equation of straight line»

, at k ≠ 2 , then the coefficient of $x^2 \neq$ the coefficient of y^2

$\therefore$ The equation does not express a circle whatever the value of k

(2) $\because$ The equation of the base of the cone is $x^2 + y^2 = 64$

$\therefore$ The radius length of the base of the cone = 8 length unit.

$\therefore$ The volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 64 \times 6$

$= 128\pi$ cubic unit.

(3) $\because$ The centre of the circle = (7 , 5)

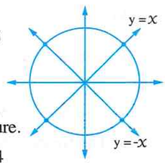
, r = 4 length unit.

$\therefore$ The distance between the centre and the y-axis = 7 length unit.

$\therefore$ The least distance between the y-axis and any point on the circle = 7 − 4 = 3 length unit.

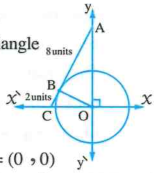
(4) $\because$ The centre of the circle that touches the two coordinates axes must be lies on the straight line $y = x$ or $y = -x$

, $\because$ the circle $x^2 + y^2 = 25$ intersects the two lines $y = x$ and $y = -x$ at four points as in the opposite figure.



$\therefore$ The number of circles = 4

(5) From the opposite figure :



$\because$ Δ AOC is a right-angled triangle at O and $\overline{OB} \perp \overline{AC}$

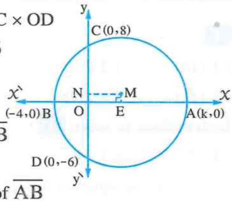
$\therefore (OB)^2 = 8 \times 2 = 16$

$\therefore$ OB = 4 length unit.

, $\because$ the centre of the circle = (0 , 0)

$\therefore$ The equation of the circle is $x^2 + y^2 = 16$

(6) $\because$ OB × OA = OC × OD



$\therefore 4 \times OA = 8 \times 6$

$\therefore$ OA = 12

$\therefore$ A = (12 , 0)

, draw $\overline{ME} \perp \overline{AB}$

, $\overline{MN} \perp \overline{DC}$

$\therefore$ E is midpoint of $\overline{AB}$

$\therefore$ E (4 , 0) , N is midpoint of $\overline{DC}$

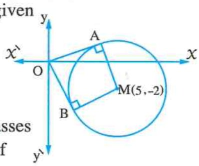
$\therefore$ N (0 , 1) $\qquad \therefore$ M = (4 , 1)

$\therefore r = MA = \sqrt{(12-4)^2 + (0-1)^2} = \sqrt{65}$

$\therefore$ The equation of the circle is

$(x-4)^2 + (y-1)^2 = 65$

(7) $\because$ The centre of the given circle (M) = (5 , − 2)



, $\because$ $\overline{MA} \perp \overline{AO}$

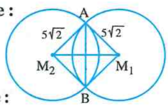
, $\overline{MB} \perp \overline{OB}$

$\therefore$ There is a circle passes thtough the vertices of the figure AMBO and its diameter $\overline{OM}$ and its centre is the midpoint of $\overline{MO}$

$\therefore$ The centre is $\left(\frac{5+0}{2} , \frac{-2+0}{2}\right) = \left(\frac{5}{2} , -1\right)$

and it is the same circle that passes through the vertices of the Δ AOB

(8) According to the first circle :



$M_1 = (5 , 5)$

, $r_1 = 5\sqrt{2}$

According to the second circle :

$M_2 = (-3 , -1)$, $r_2 = 5\sqrt{2}$

, $\because$ the length of $\overline{M_1M_2} = \sqrt{(5+3)^2 + (5+1)^2}$

= 10 length unit

, $\because (M_1M_2)^2 = (AM_1)^2 + (AM_2)^2$

$\therefore$ The figure A M_1 B M_2 is a square

$\therefore$ AB = 10 length unit.

2 $\because$ The circle N touches the two coordinates axes in the 3[rd] quadrant.

$\therefore$ N is $(-\ell , -\ell)$ satisfies the equation of the straight line $y = 2x + 1$

$\therefore -\ell = -2\ell + 1 \qquad \therefore \ell = 1$

$\therefore$ The centre of the circle N is (− 1 , − 1)

$\therefore$ The equation of the circle N

is $(x+1)^2 + (y+1)^2 = 1$

, then $x^2 + y^2 + 2x + 2y + 1 = 0$

$\because$ The circle M touches the two coordinate axes in the 2[nd] quadrant.

$\therefore$ M is (− k , k) verifies the equation of the straight line $y = 2x + 1$

$\therefore k = -2k + 1 \qquad \therefore k = \frac{1}{3}$

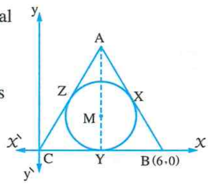
$\therefore$ The centre of the circle M is $\left(-\frac{1}{3} , \frac{1}{3}\right)$

$\therefore$ The equation of the circle M

is $\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2$

i.e. $x^2 + y^2 + \frac{2}{3}x - \frac{2}{3}y + \frac{1}{9} = 0$

3 $\because$ Δ ABC is equilateral of side length 6 length units



$\therefore$ CY = 3 length units

, $AY = 3\sqrt{3}$ length units.

$\therefore$ M is the point of intersecting of the medians of Δ ABC

$\therefore MY = \frac{3\sqrt{3}}{3} = \sqrt{3}$ length unit.

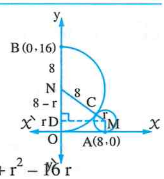
$\therefore$ The centre of the circle is $\left(3 , \sqrt{3}\right)$

, $r = \sqrt{3}$ length units

$\therefore$ The equation of the circle

is $(x-3)^2 + \left(y - \sqrt{3}\right)^2 = 3$

4 Draw $\overline{MD} \perp \overleftrightarrow{OY}$



let the radius length of the circle M be r length unit

In Δ MND :

$(8+r)^2 = 8^2 + (8-r)^2$

$\therefore 64 + r^2 + 16r = 64 + 64 + r^2 - 16r$

$\therefore 32r = 64 \qquad \therefore r = 2$

$\therefore$ The centre of the circle M is (8 , 2)

$\therefore$ The equation of the circle M

is $(x-8)^2+(y-2)^2=4$

i.e. $x^2+y^2-16x-4y+64=0$

Life Applications

1

$r=\sqrt{\ell^2+k^2-c}=\sqrt{(3)^2+(-4)^2-11}$

$=\sqrt{14}$ length unit

$\therefore$ The area of the circle $=\pi r^2=14\pi$

$\because$ The square unit in the plane represents $(5)^2=25\ cm^2$

$\therefore$ The area of the square $=14\times\frac{22}{7}\times 25=1100\ cm^2$

2 $\because$ The equation of the circle

is $(x-7)^2+(y+9)^2=(30)^2$

$BA=\sqrt{(25-7)^2+(-30+9)^2}\simeq 27.66$

$\because BA<r$

$\therefore$ The radar can observe the ship whose position is at (B)

3 $\because \ell=-2$, $k=6$, $c=-60$

$\therefore r^2=4+36+60=100$

$\therefore$ The area of the regular octagon

$=\frac{n}{2}r^2\sin\left(\frac{360°}{n}\right)=\frac{8}{2}\times 100\times\sin 45°$

$=200\sqrt{2}$ square units

4

(1) The pulley A touches the coordinate axes and its radius length = 5 units.

$\therefore$ The centre of its circle M (5 , 5)

$\therefore$ Its equation is $(x-5)^2+(y-5)^2=25$

i.e. $x^2+y^2-10x-10y+25=0$

(2) $\because$ The equation of the circle of the pulley (B)

is $x^2+y^2+14x+45=0$

$\therefore \ell=7$, $k=0$

Its centre N = (– 7 , 0)

$\therefore$ The distance between the two centres.

$=\sqrt{(5+7)^2+5^2}=13$ length unit.

$\because$ Each unit in the coordinates plane represents 6 cm.

$\therefore$ The distance between the two centres of the two pullies = 13 × 6 = 78 cm.

5 $\because$ The maximum height between the two edges.

= 10 units , let r_1 is the radius of the great gear and r_2 is the radius of the small gear

$\therefore 2r_1+2r_2=10$

$\therefore r_1+r_2=5$

$\therefore$ The centre of the great gear is (5 , 4) and the centre of the small gear is (5 , 9)

$r_1^2=25+16-32=9$

$\therefore r_1=3$ length units

$\therefore r_2=2$ length units

$\therefore$ The equation of the circle of the small gear

is $(x-5)^2+(y-9)^2=4$

i.e. $x^2+y^2-10x-18y+102=0$